Risk-based portfolio strategies and estimation risk

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The aim of this research project is to assess the influence of the estimation risk on Risk-based portfolio strategies such as Minimum Variance, Maximum Diversification, Equal Risk Contribution, or Risk Parity. These widely used strategies in the asset management industry have the salient feature of being free of expected return estimates, which are the largest source of parameter uncertainty. At the same time, they still depend on the estimates of the risk and dependencies between assets, and hence are subject to an estimation risk. In this respect, the first objective of our research project consists in evaluating the expected loss function associated to the standard plug-in approach (which consists in replacing the population parameters by their sample estimated) generally used to implement these strategies. To the best of our knowledge, it is the first attempt to quantify the impact of the estimation risk on the risk-based portfolio strategies. The second objective of the project is to propose an original approach, based on a formal statistical testing framework, which is designed to take into account the estimation risk within the risk-based allocation strategy. This inference procedure allows to determine a set of portfolio weights that satisfies the risk-based strategy. In contrast to the standard plug-in approach, the optimal allocation is not uniquely defined. This provides an opportunity for the investor to choose in this set the portfolio which maximizes the performance, the diversification or any other secondary objective.

The risk-based portfolio strategies (Minimum Variance, Maximum Diversification, Equal Risk Contribution, Risk Parity, etc.) are largely used by the asset management industry (Jurczencko et al. (2015)). Indeed, these strategies are used in the multi-asset allocation for more robustness in strategic decisions; in equities as alternatives to market capitalization benchmarks, which are heavily concentrated in a few stocks and significantly biased towards overvalued stocks and sectors; in the smart beta exchange-traded funds (ETFs) industry, and so on.

One advantage of risk-based strategies is that they imply a lower parameter estimation risk than alternative strategies based on the estimates of expected returns. It is well-known that the traditional mean-variance optimization turns out to be an “estimation-error maximisation" (Michaud, 1989). In particular, it relies on the estimation of expected returns which are notoriously unstable and hard to predict (Merton, 1980). In contrast, risk-based strategies only require the estimation of risk and dependencies between assets, through the variance-covariance matrix. Hence, the Minimum Variance (Clarke et al. 2006) or the Risk Parity (Roncalli, 2014) strategies have proven to be robust in practice and are used in many popular investment vehicles (smart beta ETF, mutual fund, etc.).

However, risk-based portfolio strategies are based on risk measure estimates, and thus are still affected by an estimation risk. The problem is particularly accurate when correlations among portfolio assets are high and thus benefits from risk diversification

are lowered. In this case, small estimation risk leads to a large weight instability. Furthermore, the problem is further amplified in the presence of large-scale portfolios in which the curse of dimensionality strengthens parameter instability. A solution consists in using techniques like shrinkage of the covariance matrix or thresholding methods. However, even in these cases, the estimation risk cannot be overlooked. To the best of our knowledge, no academic study has been devoted to this issue.

The aim of our research project is to assess the influence of the estimation risk on the Risk-based portfolio strategies, with a particular focus on the Risk Parity strategy. The first objective consists in evaluating the expected loss function associated to the standard plug-in approach (which consists in replacing the population parameters by their sample estimates) generally used to implement these strategies. To the best of our knowledge, it is the first attempt to precisely quantify the impact of the estimation risk on the risk-based portfolio strategies. The second objective of the project is to propose an original approach, based on a formal statistical testing framework, which is designed to take into account the estimation risk within the risk-based allocation strategy. This inference procedure will allow to determine a set of portfolio weights that satisfies the risk-based strategy. Contrary to the standard plug-in approach, the optimal allocation is not uniquely defined. This property gives the investor the opportunity to choose in this set the portfolio which maximizes the performance, the diversification, or any other secondary objective.

Our study is also related to the current literature devoted to the estimation risk affecting the risk measures. This risk is generally assessed through asymptotic confidence intervals. For instance, Chan et al. (2007) and Francq and Zakoïan (2015) derive the asymptotic confidence intervals for the conditional VaR estimator in the specific context of heavy-tailed GARCH models. Gouriéroux and Zakoïan (2013) consider a different approach based on an Estimation adjusted VaR (EVaR). Alternatively, several papers propose resampling methods to carry out inference on risk measures. Danielson and Zhou (2016) in their paper untitled "Why risk is so hard to measure?", assess the estimation error of VaR and ES forecasts. Here, our approach is different since we do not measure the estimation risk, but we propose a test of equality of risk measures that takes into account the estimation risk. The logic is similar to Hurlin et al. (2016) who propose a bootstrap-based test of the null hypothesis of equality of two firms' conditional risk measures at a single point in time.

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