Financial Derivatives Costs and Risks in a One-Period Model

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Abstract

We present a one-period XVA model encompassing bilateral and centrally cleared trading in a unified framework with explicit formulas for most quantities at hand. We illustrate possible uses of this framework for running stress test exercises on a financial network from a clearing member's perspective or for optimizing the porting of the portfolio of a defaulted clearing member.

1 Introduction

In the wake of the 2008–09 global financial crisis, clearing through central counterparties (CCPs) has become mandatory for standardized derivatives, other ones remaining under bilateral setup with higher capital requirements. One role of the CCPs[1] is to provide to their clearing members fully collateralized hedges of their market risk with their clients. But this comes at a cost to the clearing members, which pass it to their corporate clients in the form of XVA (cross-valuation adjustments) add-ons. Bearing in mind that the risks of a hedge are, by definition, of the same magnitudes as the ones of the originating position and that standardized derivatives usable as hedging assets have to be traded through CCPs, the XVA footprint of not only bilateral but also centrally cleared trading is significant and should be analyzed in detail, which is the topic of this paper.

More precisely, the trades of a clearing member bank with a CCP are partitioned between proprietary trades, which are in effect hedges of the bilateral trading exposure of the bank, and back-to-back hedges of so-called cleared client trades, through which

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[1]See Gregory (2015) and Gregory (2014) for general XVA and CCP references, as well as Menkveld and Vuillemey (2021) for a recent CCP survey.

non-member clients gain access to the clearing services of a CCP. Albanese et al. (2020) focus on the XVA analysis of a bank only acting as a clearing member of one CCP, without proprietary trading. The present paper provides an integrated XVA analysis in the realistic situation of a bank dealing with many clients and CCPs, through both proprietary (also dubbed house) accounts and client accounts. For the sake of tractability, this is achieved in a stylized one-period setup.

Applications of the proposed framework include a sounder risk assessment in the context of stress test exercises[2] or optimizing the porting of the portfolio of defaulted clearing members.

The first type of application is motivated by the default in 2020 of Ronin Capital, a broker/dealer firm with proprietary trading operations, that had clearing exposures on both CCP services Fixed Income Clearing Corporation (FICC) GSD[3] segment (123 members) and CME Futures (56 members of which 24 common with FICC GSD). If all members are assumed to be only exposed to these CCPs and their cleared clients, we can illustrate these relationship by the network depicted in Figure 1.1. Any common member on those two CCPs needs to ensure conservative risk assessment that can be achieved in the proposed framework by accounting for common memberships on the two CCPs. If such common memberships are ignored, they can lead to lower loss estimates giving wrong risk view on potential losses.

The second type of application is an illustration of the results of defaulted portfolio porting as it has been the case for the trader Einer Aas on NASDAQ OMX[4] that has defaulted on 2018 with loss spill-over effect on surviving members.

The paper is outlined as follows. Section 2 sets the stage. Section 3 develops the corresponding XVA analysis. Section 4 develops two applications. Section 7 concludes.

2 Abstract Setup

We consider a finite set of market participants, also susceptible to serve as clearing members of CCPs. Derivative transactions can then be concluded between two individual participants, or between a set of participants[5], pooled in the form of a CCP, and a clearing member of this CCP.

CCPs are typically siloed into different services, each devoted to a specific class of derivatives. We first consider a setup with a single CCP service[6]: see Figure 2.1, where $\mathcal{P}$ and $\overline{\mathcal{P}}$ represent the contractual cash flows from cleared and bilateral clients to a reference clearing member, dubbed the bank hereafter, hence promised, in successive turns[7], from the bank to the CCP, from the CCP to other clearing members, and from

[2] As required by Article 302 of the CRR document The European Parliament and of the Council and OICV-IOSCO (2013).

[3] Government Securities Division.

[4] **O**ptions**m**äklarna/Helsinki Stock E**x**change

[5] Two or more, in practice from a few units to a few hundreds.

[6] The extension to several CCPs is done in Section 3.3.

[7] Ignoring at this stage the presence of additional "exchanges".

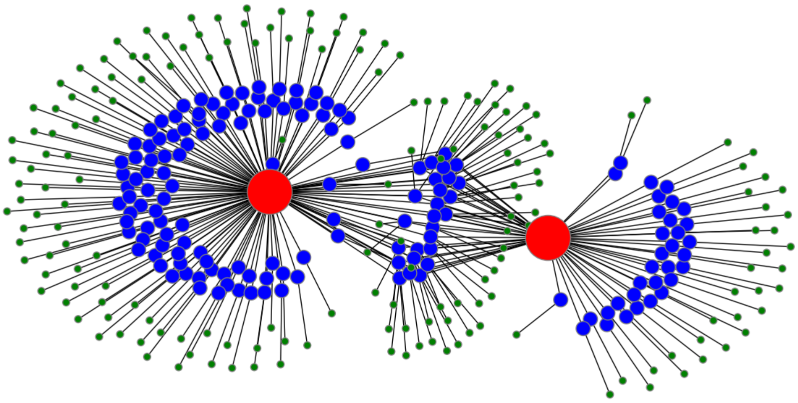

Figure 1.1: Network consisting of two CCPs (in red), 123 members for CCP1 seen on the left hand side, and 56 members for CCP2 on the right hand side, with 24 common members displayed as the group of members in the middle of the two CCPs (155 members in total, in blue), and with 179 cleared clients (in green), each member portfolio exposure toward a CCP being mirrored with a corresponding cleared client portfolio.

the latter to their own clients. As a consequence, the CCP is flat in terms of market risk, as is also each of the clearing members.

2.1 Defaults Settlement Rule

As reasserted in the wake of the 2008–09 global financial crisis by the Volcker rule, a dealer bank should be hedged as much as possible, at least in terms of market risk[8]. Jump-to-default risk, on the other hand, is hardly hedgeable in practice. Instead it is mitigated through netting and collateralization. Namely, designated netting sets of transactions between two given counterparties (two individual participants or a participant and the CCP) are jointly collateralized, i.e. guaranteed against the default of one or the other party. The collateral (or guarantee) comprises a variation margin, which tracks the mark-to-market (counterparty-risk-free value) of the netting set between the two parties, and nonnegative amounts of initial margin posted by each party to the other, which provide a defense against the risk of slippage of the value of the netting set away from its (frozen) variation margin during its liquidation period. In the case of transactions with a CCP, there is an additional layer of collateral in the form of the (funded) default fund contributions of the clearing members, which is meant as a defense against extreme and systemic risk. For each participant, variation margin is rehypothecable and

[8]cf. paragraph number 1851 in section 619 from The United States Congress (2010).

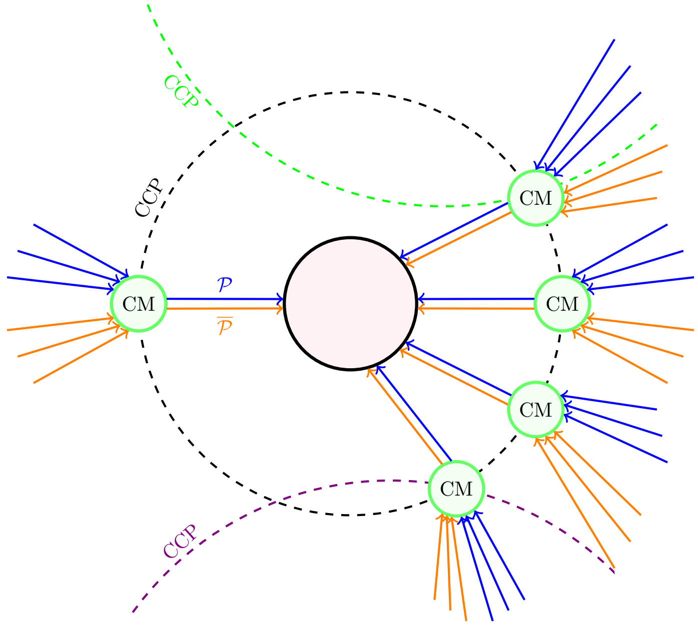

Figure 2.1: Contractual cash flows between market participants. The reference clearing member bank is on the left.

fungible across all its netting sets. Initial margin is segregated at the netting set level. default fund contributions are segregated at the clearing member level, and all of them must be borrowed entirely by members.

The general rule regarding the settlement of contracts of a defaulted netting set is that:

Assumption 2.1 If a counterparty in default is indebted toward the other beyond its posted margin, then this debt is only reimbursed at the level of this posted margin (assuming zero recovery rate of the defaulted party for simplicity in this paper); otherwise the debt between the two parties is fully settled. Here debt is understood on a counterparty-risk-free basis.

This rule also applies to a netting set of transactions between a clearing member and a CCP. However, in our stylized setup, a CCP is nothing but the collection of its clearing members. Our CCP has no resources of its own (in particular, it cannot post any default fund contribution, or "skin-in-the-game"[9]). As long as it is non-default, i.e. as long as at least one of its clearing members is non-default, our CCP can only handle the losses triggered by the defaults of some of its clearing members by redirecting these losses on

[9]Such additional protection layer, though quite common in practice, is of marginal magnitude compared to the other protection layers. By omitting skin-in-the-game component, the obtained results are conservative in terms of risk management and the various formulations are simplified.

the surviving ones. This participation of the surviving members to the losses triggered by the defaults of the other members corresponds in our framework to the usage by the CCP of their default fund contributions, both funded (as already introduced above) and unfunded. As will be detailed in equations below, the funded (unless otherwise stated) default fund contributions are used in priority for covering losses triggered by the defaults of clearing members over their margins. The unfunded default fund contributions correspond to additional refills that can be required by the CCP, often up to some cap in principle, without bounds in our model, in case the funded default fund contributions of the surviving members are not enough.

2.2 XVA Framework

Assume that at time 0 all the banking participants, including the reference clearing member bank, in Figure 2.1, with no prior endowments, enter transactions with their clients and hedge their positions, both bilaterally between them and through the CCP and exchanges[10]. As seen above, the CCP and each bank are flat in terms of market risk. However, as market participants are assumed to be defaultable with zero recovery, in order to account for counterparty credit risk and its funding and capital consequences, each banking participant requires from its corporate clients a pricing rebate (considering conventionally the bank as the "buyer") with respect to the mark-to-market (counterparty-risk-free) valuation of the deals. The corporate clients of the bank are assumed to absorb the ensuing prices via their corporate business, which is their primary motivation for these deals.

A reference probability measure $\mathbb{Q}^*$, with corresponding expectation operator denoted by $\mathbb{E}^*$, is used for the linear valuation of cash flows, using the risk-free asset as our numéraire everywhere. This choice of a numéraire simplifies equations by removing all terms related to the (assumed risk-free) remuneration of all cash and collateral accounts. The funding issue is then refocused on the risky funding side, i.e. funding costs in what follows really means excess funding costs with respect to a theoretical situation where the bank could equally borrow and lend at the risk-free rate.

More precisely, as detailed in Albanese et al. (2021, Remark 2.3), we take $\mathbb{Q}^*$ equal to a given risk-neutral measure on the financial sub σ algebra $\mathfrak{B}$ of the full model σ algebra $\mathfrak{A}$, while $\mathbb{Q}^*$ is equal to the physical probability measure conditionally on $\mathfrak{B}$, the physical probability measure being defined on $\mathfrak{A}$ and equivalent to $\mathbb{Q}^*$ on $\mathfrak{B}$.

Following the general XVA guidelines of Crépey (2021a, Section 2), the above-mentioned pricing rebate required by the reference clearing member bank, dubbed funds transfer price (FTP), comes in two parts: first, the expected counterparty default losses and funding expenditures of the bank, an amount that flows into the bank liabilities and which we refer to as contra-asset valuation (CA); second, a cost of capital risk premium (KVA), which instead is loss-absorbing[11] and is also used by the management of the bank as retained earnings for remunerating the shareholders of the bank for their capital

[10] cf. Section 3.1.

[11] Hence, not a liability.

at risk within the bank. All in one, the bank buys the deals from its clients at the (aggregated) price (MtM − FTP), where MtM is their counterparty-risk-free value and

$$\mathrm{FTP} = \underbrace{\mathrm{CA}}_{\text{Expected costs}} + \underbrace{\mathrm{KVA}}_{\text{Risk premium}} . \tag{1}$$

Assumption 2.2 At time 0 the amounts CA and KVA sourced from the corporate clients of the bank are deposited on the reserve capital and capital at risk accounts of the bank.

Let EC denote an economic capital of the bank corresponding to the minimum level of capital at risk that the bank should hold from a regulatory (i.e. solvency) perspective. If $\mathrm{KVA} < \mathrm{EC}$, then the bank shareholders need to provide the missing amount $(\mathrm{EC} - \mathrm{KVA})$ of capital at risk, so that the actual level of capital at risk of the bank is

$$\max(\mathrm{EC}, \mathrm{KVA})$$

and *shareholder* capital at risk reduces to

$$\max(\mathrm{EC}, \mathrm{KVA}) - \mathrm{KVA} = (\mathrm{EC} - \mathrm{KVA})^{+}. \tag{2}$$

3 Theoretical XVA Analysis

In this section we detail each term in the equations above, in the realistic setup of a bank involved into an arbitrary combination of bilateral and centrally cleared portfolios, in a tractable one-period setup with T denoting the period length. In the one-period XVA model of Albanese et al. (2021, Section 3), there were no CCPs and the bank was assumed to have access to a “fully collateralized back-to-back hedge of its market risk”, ensuring by definition and for free to the bank a cash-flow $(\mathcal{P} - \mathrm{MtM})$ at time 1, irrespective of the default status of the bank and its client. There, $\mathcal{P}$ denoted the contractual cash flows from the (assumed unique) client to the bank and MtM was the corresponding counterparty-risk-free value. In the present paper we reveal the mechanism of “a fully collateralized hedge of the market risk” of the bank, which can be achieved through central clearing, but at a certain cost that we analyze.

3.1 Cash Flows

Given disjoint sets of indices $I \ni 0$, C, and B for the clearing members (including the reference bank labeled by 0) and for the respective cleared and bilateral netting sets of the bank with its individual clients/counterparties[12], we denote by:

- $\mathrm{MtM}_\iota = \mathbb{E}^*\mathcal{P}_\iota$, for any $\iota \in I \cup C \cup B$;

[12]e use the terms client for cleared clients and counterparty for bilateral counterparties.

- J_0, shortened as J, and J_i, $i \in I \setminus \{0\}$, the survival indicator random variables of the bank and of the other clearing members at time 1; $\gamma = \mathbb{Q}^*(J = 0)$, the default probability of the bank;

- $\mathcal{J} = \max_i J_i$, the survival indicator random variable of the CCP (i.e. of at least one clearing member),

- $\mathcal{P}_i$, MtM_i, and IM_i, $i \in I$, the contractual cash flows, variation margin, and initial margin from the clearing member i to the CCP corresponding to the cleared clients account of the member i;

- $\overline{\mathcal{P}}_i$, $\overline{\mathrm{MtM}}_i$, and $\overline{\mathrm{IM}}_i$, , $i \in I$, the contractual cash flows, variation margin, and initial margin from the clearing member i to the CCP corresponding to the house account of the clearing member i;

- DF_i, $i \in I$, the default fund contribution posted by the clearing member i to the CCP;

- J_b, $b \in B$, the survival indicator random variable of the counterparty of the bilateral netting set b of the reference bank; $\mathcal{P}_b$, VM_b, and IM_b, the associated contractual cash flows, variation margin[13], and initial margin from the corresponding counterparty to the bank; and $\overline{\mathrm{IM}}_b$, the initial margin from the bank to the counterparty;

- J_c, $c \in C$, the survival indicator random variable of the client of the cleared trading netting set c of the bank, and $\mathcal{P}_c$, MtM_c, and IM_c, the associated contractual cash flows, variation margin, and initial margin from the corresponding client to the bank[14];

- $\mathcal{L}$, the loss of the CCP, i.e. the loss triggered by the defaults of its clearing members beyond their posted collateral[15], which is borne by the surviving members (if any);

- $\mu = J\mu$, the proportion of these losses allocated to the reference clearing member bank.

Assumption 3.1 We have $\sum_i (\mathcal{P}_i + \overline{\mathcal{P}}_i) = 0$ (the CCP is flat in terms of market risk), $\sum_c \mathcal{P}_c = \mathcal{P}_0$ (by definition of cleared trades and of their mirroring trades), and $\sum_b \mathcal{P}_b = \overline{\mathcal{P}}_0 + \mathcal{P}_e$ (the reference bank is flat in terms of market risk), where $\mathcal{P}_e$ denotes the contractual cash-flows from the bank to exchanges that are used by the bank as an alternative hedging venue (on top of the CCP).

We emphasize that $\sum_b \mathcal{P}_b$ may differ from $\overline{\mathcal{P}}_0$, as part of its bilateral derivatives exposure can be hedged by the bank via futures-style instruments traded on exchanges. That is,

[13] We implicitly assume that members of CCPs are fully collateralized hence VM = MtM for any cleared position. On the contrary, for counterparty positions, we do not make any assumption about the variation margin requirements hence simply denote them for a counterparty b by VM_b.

[14] Note that a bank does not post any initial margin on its cleared client netting sets.

[15] variation margin, initial margin, and (funded) default fund contributions.

on the hedge side of the bilateral derivative trading of the bank, $\overline{\mathcal{P}}_0$ is promised by the bank to the CCP and $\mathcal{P}_e = \sum_b \mathcal{P}_b - \overline{\mathcal{P}}_0$ to exchanges. Futures-style instruments traded on exchanges can be rehypothecated, hence trigger no XVA exposure to the bank. On the other hand, trading on exchanges[16], may entail liquidity impact costs, which are ignored in this paper[17].

Assumption 3.1 is the clearing condition regarding the contractually promised cash flows, which applies to each banking participant (written there for the reference bank) and to the CCP. Assumption 2.1 is monitoring the default cash flows. We need one more condition, regarding the funding side of the problem:

Assumption 3.2 The bank can use the amounts CA and max(EC, KVA) on its reserve capital and capital at risk accounts for its variation margin borrowing purposes. Funds needed beyond CA + max(EC, KVA) for variation margin posting purposes are borrowed by the bank at its credit spread γ above OIS. The initial margin and default fund contributions, instead, must be borrowed entirely, but at some blended funding spread $\widetilde{\gamma} \leq \gamma$.

The rationale for funding variation margin but not initial margin from CA+max(EC, KVA) is set out before Equation (15) in Albanese et al. (2017). The motivation for the assumption $\widetilde{\gamma} \leq \gamma$ is provided in Albanese et al. (2020, Section 5), along with numerical experiments suggesting that $\widetilde{\gamma}$ can be several times lower than γ.

Lemma 3.1 *The borrowing needs of the bank for reusable and segregated collateral amount to, respectively,*

$$\begin{aligned}&\Big(\sum_b(\mathrm{MtM}_b - \mathrm{VM}_b) - \mathrm{CA} - \max(\mathrm{EC},\mathrm{KVA})\Big)^+\\&\mathrm{IM} + \overline{\mathrm{IM}} + \mathrm{DF} + \sum_b \mathrm{IM}_b.\end{aligned} \tag{3}$$

Proof. On the bilateral trades of the bank and their hedges, the Treasury of the bank receives $\sum_b \mathrm{VM}_b$ of variation margin from the counterparties and has to post an aggregated amount $\sum_b \mathrm{MtM}_b$ of variation margin. Note that this holds whatever the split of the hedges of the trades belonging to each bilateral netting set b between proprietary cleared trades of the bank, for which the Treasury needs to post the corresponding mark-to-market amount as variation margin to the CCP, and hedges on exchanges, for which, as detailed in the continuous-time setup of Crépey (2021b, Section 3), the Treasury needs to lend the corresponding mark-to-market amount to the trading desks of the bank, internally to the bank at the risk-free rate, in order to fund their trading at the risk-free rate. The assumption stated before the lemma then leads to (3). ∎

[16] or more generally open markets, meaning any economic system with very limited regulatory constraints such as tarifs, taxes, licensing requirements, subsidies, unionization, and any other regulations or practices that interfere with free-market activity. This is based on definition in Troy and Boyle (2021).

[17] For a possible Radner equilibrium approach to the latter, see Bastide et al. (2021).

Lemma 3.2 *On the bank survival event $\{J=1\}$, the counterparty default losses $\mathcal{C}$ and the funding expenses $\mathcal{F}$ of the bank are given by*

$$\begin{aligned}\mathcal{C} = \sum_c (1-J_c)(\mathcal{P}_c - \mathrm{MtM}_c - \mathrm{IM}_c)^+ + \mu\mathcal{L} \\ + \sum_b (1-J_b)(\mathcal{P}_b - \mathrm{VM}_b - \mathrm{IM}_b)^+,\end{aligned} \tag{4}$$

where

$$\mathcal{L} = \sum_i (1-J_i)\big((\mathcal{P}_i - \mathrm{MtM}_i - \mathrm{IM}_i)^+ + (\overline{\mathcal{P}}_i - \overline{\mathrm{MtM}}_i - \overline{\mathrm{IM}}_i)^+ - \mathrm{DF}_i\big)^+, \tag{5}$$

and

$$\begin{aligned}\mathcal{F} = \widetilde{\gamma}\big(\mathrm{IM} + \overline{\mathrm{IM}} + \mathrm{DF}\big) + \\ + \widetilde{\gamma}\sum_b \overline{\mathrm{IM}}_b + \gamma\Big(\sum_b (\mathrm{MtM}_b - \mathrm{VM}_b) - \mathrm{CA} - \max(\mathrm{EC}, \mathrm{KVA})\Big)^+.\end{aligned} \tag{6}$$

Proof. On the CCP survival event $\{\mathcal{J}=1\}$, the CCP receives, by Assumption 2.1,

$$\begin{aligned}\sum_i \Bigg(J_i(\mathcal{P}_i + \overline{\mathcal{P}}_i) + (1-J_i)\Big(\mathcal{P}_i \wedge (\mathrm{MtM}_i + \mathrm{IM}_i) + \overline{\mathcal{P}}_i \wedge (\overline{\mathrm{MtM}}_i + \overline{\mathrm{IM}}_i) + \\ \big((\mathcal{P}_i - (\mathrm{MtM}_i + \mathrm{IM}_i))^+ + (\overline{\mathcal{P}}_i - (\overline{\mathrm{MtM}}_i + \overline{\mathrm{IM}}_i))^+\big) \wedge \mathrm{DF}_i\Big)\Bigg),\end{aligned} \tag{7}$$

which, using the CCP clearing condition Assumption 3.1:

$$0 = \sum_i (\mathcal{P}_i + \overline{\mathcal{P}}_i) = \sum_i \big(J_i(\mathcal{P}_i + \overline{\mathcal{P}}_i) + (1-J_i)(\mathcal{P}_i + \overline{\mathcal{P}}_i)\big)$$

in Assumption 3.1, is equal to

$$-\sum_i (1-J_i)\big((\mathcal{P}_i - \mathrm{MtM}_i - \mathrm{IM}_i)^+ + (\overline{\mathcal{P}}_i - \overline{\mathrm{MtM}}_i - \overline{\mathrm{IM}}_i)^+ - \mathrm{DF}_i\big)^+ = -\mathcal{L},$$

which is (5).

On the bank survival event $\{J=1\}$ ($\subseteq \{\mathcal{J}=1\}$), by the respective Assumptions 2.1 and 3.1, the bank receives from its clients and counterparties

$$\sum_c \Big(J_c\mathcal{P}_c + (1-J_c)\big(\mathcal{P}_c \wedge (\mathrm{MtM}_c + \mathrm{IM}_c)\big)\Big) + \sum_b \Big(J_b\mathcal{P}_b + (1-J_b)\big(\mathcal{P}_b \wedge (\mathrm{VM}_b + \mathrm{IM}_b)\big)\Big), \tag{8}$$

respectively pays $\sum_c \mathcal{P}_c + \overline{\mathcal{P}}_0$ to the CCP and $\mathcal{P}_e = \sum_b \mathcal{P}_b - \overline{\mathcal{P}}_0$ to the exchanges, summing up to

$$\sum_c \mathcal{P}_c + \sum_b \mathcal{P}_b = \sum_c \big(J_c\mathcal{P}_c + (1-J_c)\ \mathcal{P}_c\big) + \sum_b \big(J_b\mathcal{P}_b + (1-J_b)\ \mathcal{P}_b\big). \tag{9}$$

Subtracting (8) from (9), we obtain

$$\sum_c (1-J_c)(\mathcal{P}_c - \mathrm{MtM}_c - \mathrm{IM}_c)^+ + \sum_b (1-J_b)(\mathcal{P}_b - \mathrm{VM}_b - \mathrm{IM}_b)^+.$$

On top of this comes the participation $\mu\mathcal{L}$ of the bank to the CCP default losses, which yields (4). Moreover, in view of Lemma 3.1 and Assumption 3.2, the (risky) funding expenses of the bank are given by (6). ∎

3.2 Valuation

Let $\mathbb{E}$ denote the expectation with respect to the bank survival measure $\mathbb{Q}$ associated with $\mathbb{Q}^*$, i.e., for any random variable $\mathcal{Y}$,

$$\mathbb{E}\mathcal{Y} = (1-\gamma)^{-1}\mathbb{E}^*(J\mathcal{Y}). \tag{10}$$

(expectation of $\mathcal{Y}$ conditional on the survival of the bank). As easily seen in Albanese et al. (2021, Section 3):

Lemma 3.3 *For any random variable $\mathcal{Y}$ and constant Y, we have*

$$Y = \mathbb{E}^*(J\mathcal{Y} + (1-J)Y) \Longleftrightarrow Y = \mathbb{E}\mathcal{Y}.$$

Under a cost-of-capital XVA approach, the bank charges its future losses to its corporate clients at a CA level making $J(\mathcal{C}+\mathcal{F}-\mathrm{CA})$, the trading loss of the shareholders of the bank, $\mathbb{Q}^*$ centered. In addition, given a target hurdle rate h assumed in $[0,1]$ (and typically of the order of 10%), the management of the bank ensures to the bank shareholders dividends at the height of h times their capital at risk $(\mathrm{EC}-\mathrm{KVA})^+$ (cf. (2)), where we model EC as $\mathbb{ES}\big(J(\mathcal{C}+\mathcal{F}-\mathrm{CA})\big)$, the expected shortfall of the trading loss $\ell = J(\mathcal{C}+\mathcal{F}-\mathrm{CA})$ computed under the bank survival measure at a quantile level[18] of $\alpha = 99.75\%$, i.e. under the primal and dual representations of the expected shortfall[19], $\mathbb{V}\mathrm{a}\mathbb{R}_a(\ell)$ denoting the $\mathbb{Q}$ value-at-risk (left quantile) of level a of ℓ:

$$\begin{aligned}\mathrm{EC} &= \tfrac{1}{1-\alpha}\int_{a=\alpha}^{1} \mathbb{V}\mathrm{a}\mathbb{R}_a(\ell)da \\ &= \sup\left\{\mathbb{E}[\ell\chi]\ ;\ \chi \text{ is measurable, } 0\le\chi\le(1-\alpha)^{-1}, \text{ and } \mathbb{E}[\chi]=1\right\},\end{aligned} \tag{11}$$

which for atomless ℓ also coincides[20] with $\mathbb{E}[\ell|\ell\ge\alpha]$. Note that, in view of the dual representation, an expected shortfall of a centered random variable is nonnegative.

Accordingly (as detailed after the definition):

[18] Under normal distribution assumptions, such ES at percentile level 99.75% allows reaching similar loss level as with a VaR (quantile) risk metric at the level 99.9%. In practice, regulatory and economic capital indeed aims at capturing extreme losses that can occur once every 1000 years, cf. paragraph 5.1 from Basel Committee on Banking Supervision (2005) for the detailed instructions.

[19] see e.g. Föllmer and Schied (2010, Equations (2.1) and (5.5)).

[20] See Corollary 5.3 and representation thanks to expression (3.7) from Acerbi and Tasche (2002).

Definition 3.1 We let

$$
\begin{aligned}
\mathrm{CA} = \underbrace{\mathrm{CVA}}_{\mathrm{BCVA+CCVA}} + \underbrace{\mathrm{MVA}}_{\mathrm{BMVA+CMVA}} + \mathrm{FVA}, \text{ where}\\
\mathrm{CCVA} &= \mathbb{E}^*\Big(J\big(\sum_c (1-J_c)(\mathcal{P}_c - \mathrm{MtM}_c - \mathrm{IM}_c)^+ + \mu\mathcal{L}\big) + (1-J)\mathrm{CCVA}\Big),\\
\mathrm{CMVA} &= \mathbb{E}^*\Big(J\widetilde{\gamma}\big(\mathrm{IM} + \overline{\mathrm{IM}} + \mathrm{DF}\big) + (1-J)\mathrm{CMVA}\Big),\\
\mathrm{BCVA} &= \mathbb{E}^*\big(J\sum_b (1-J_b)(\mathcal{P}_b - \mathrm{VM}_b - \mathrm{IM}_b)^+ + (1-J)\mathrm{BCVA}\big), \qquad (12)\\
\mathrm{BMVA} &= \mathbb{E}^*\big(J\widetilde{\gamma}\sum_b \overline{\mathrm{IM}}_b + (1-J)\mathrm{BMVA}\big),\\
\mathrm{FVA} &= \mathbb{E}^*\Big(J\gamma\big(\sum_b (\mathrm{MtM}_b - \mathrm{VM}_b) - \mathrm{CA} - \max(\mathrm{EC},\mathrm{KVA})\big)^+ + (1-J)\mathrm{FVA}\Big),\\
\mathrm{KVA} &= \mathbb{E}^*\big(Jh(\mathrm{EC}-\mathrm{KVA})^+ + (1-J)\mathrm{KVA}\big), \text{ where } \mathrm{EC} = \mathbb{ES}\big(J(\mathcal{C}+\mathcal{F}-\mathrm{CA})\big).
\end{aligned}
$$

Hence in view of (4) and (6):

$$
\mathrm{CA} = \mathbb{E}^*\Big(J\big(\mathcal{C}+\mathcal{F}\big) + (1-J)\mathrm{CA}\Big), \qquad (13)
$$

i.e. $\mathbb{E}^*\Big(J\big(\mathcal{C}+\mathcal{F}-\mathrm{CA}\big)\Big) = 0$, as desired[21]. The terminal cash flows of the form $(1-J)\times\ldots$ in (12) or (13) thus yields the desired shareholder centric perspective. They can also be interpreted as the amounts of reserve capital and risk margin lost by the bank shareholders, as their property is transferred to the liquidator of the bank, if the bank defaults.

Due to these terminal cash flows, the above definition is in fact a fix-point system of equations. The split of the underlying CA equation (13) into the collection of equations (12) is motivated by both interpretation and numerical considerations. From an interpretation viewpoint, it is useful to provide the more granular view on the costs of the bank provided by the split of the global CA amount between, on the one hand, bilateral and centrally cleared trading default risk components BCVA and CCVA and, on the other hand, bilateral and centrally cleared trading funding risk components BMVA and CMVA for segregated initial margin, whereas the FVA cost of funding variation margin is holistic in nature, via the feedback impact of CA+max(EC, KVA) into the FVA. From a numerical viewpoint, the collection (12) of smaller problems may be easier to address than the global equation (13). Each of the smaller problems can also be handled by a dedicated desk of the bank, namely the CVA desk, for the BCVA and CCVA, and the Treasury of the bank, for the BMVA, CMVA and the FVA.

Passing in the above equations to the bank survival measure $\mathbb{Q}^*$ based on Lemma 3.3 shows that the corresponding fixed point problem is in fact well-posed and yields explicit formulas for all the quantities at hand.

[21] see after (3.3).

Proposition 3.1 *We have*

$$
\begin{aligned}
&\mathrm{CCVA} = \mathbb{E}\big[\sum_c (1-J_c)(\mathcal{P}_c - \mathrm{MtM}_c - \mathrm{IM}_c)^+ + \mu\mathcal{L}\big], \textit{ where}\\
&\qquad \mathcal{L} = \sum_i (1-J_i)\big((\mathcal{P}_i - \mathrm{MtM}_i - \mathrm{IM}_i)^+ + (\overline{\mathcal{P}}_i - \overline{\mathrm{MtM}}_i - \overline{\mathrm{IM}}_i)^+ - \mathrm{DF}_i\big)^+,\\
&\mathrm{CMVA} = \widetilde{\gamma}\big(\mathrm{IM} + \overline{\mathrm{IM}} + \mathrm{DF}\big),\\
&\mathrm{BCVA} = \mathbb{E}\big(\sum_b (1-J_b)(\mathcal{P}_b - \mathrm{VM}_b - \mathrm{IM}_b)^+\big),\\
&\mathrm{BMVA} = \widetilde{\gamma}\sum_b \overline{\mathrm{IM}}_b\\
&\mathrm{EC} = \mathbb{ES}(J(\mathcal{C} - \mathrm{CVA})) \geq 0, \ \textit{where}\\
&\qquad J(\mathcal{C} - \mathrm{CVA}) = J\Big(\sum_c (1-J_c)(\mathcal{P}_c - \mathrm{MtM}_c - \mathrm{IM}_c)^+ + \mu\mathcal{L} - \mathrm{CCVA}\\
&\qquad\qquad + \sum_b (1-J_b)(\mathcal{P}_b - \mathrm{VM}_b - \mathrm{IM}_b)^+ - \mathrm{BCVA}\Big),\\
&\mathrm{FVA} = \frac{\gamma}{1+\gamma}\big(\sum_b (\mathrm{MtM}_b - \mathrm{VM}_b) - (\mathrm{CCVA} + \mathrm{CMVA} + \mathrm{BCVA} + \mathrm{BMVA}) - \mathrm{EC}\big)^+,\\
&\mathrm{KVA} = \frac{h}{1+h}\mathrm{EC}.
\end{aligned}
\tag{14}
$$

All the above XVA numbers are nonnegative.

Proof. By the result recalled after (11), EC is nonnegative as an expected shortfall under $\mathbb{Q}$ of the random variable $J(\mathcal{C}+\mathcal{F}-\mathrm{CA})$, which is centered under $\mathbb{Q}^*$ and therefore under $\mathbb{Q}$, by (10). The first four formulas in (14) directly follow from Definition 3.1 and Lemma 3.3, which also implies that $\mathrm{KVA} = \mathbb{E}\big(h(\mathrm{EC}-\mathrm{KVA})^+\big) = h(\mathrm{EC}-\mathrm{KVA})^+$. As h is nonnegative, this KVA semilinear equation is equivalent to the KVA formula (last line) in (14). In particular (as $h \in [0,1]$ and $\mathrm{EC} \geq 0$), $\mathrm{KVA} \leq \mathrm{EC}$, i.e. $\max(\mathrm{EC}, \mathrm{KVA}) = \mathrm{EC}$. This and Lemma 3.3 yield

$$
\mathrm{FVA} = \mathbb{E}\Big(\gamma\big(\sum_b (\mathrm{MtM}_b - \mathrm{VM}_b) - \mathrm{CA} - \mathrm{EC}\big)^+\Big) = \gamma\big(\sum_b (\mathrm{MtM}_b - \mathrm{VM}_b) - \mathrm{CA} - \mathrm{EC}\big)^+.
$$

As $\mathrm{CA} = \mathrm{CCVA} + \mathrm{CMVA} + \mathrm{BCVA} + \mathrm{BMVA} + \mathrm{FVA}$, this is an FVA semilinear equation, which, as γ is nonnegative, is equivalent to the FVA formula

$$
\mathrm{FVA} = \frac{\gamma}{1+\gamma}\big(\sum_b (\mathrm{MtM}_b - \mathrm{VM}_b) - (\mathrm{CCVA} + \mathrm{CMVA} + \mathrm{BCVA} + \mathrm{BMVA}) - \mathrm{EC}\big)^+.
$$

Last, we have $\mathrm{EC} = \mathbb{ES}(J(\mathcal{C} + \mathcal{F} - \mathrm{CA}))$, where the identity $\mathcal{C} + \mathcal{F} - \mathrm{CA} = \mathcal{C} - \mathrm{CVA}$ and the formula for $J(\mathcal{C} - \mathrm{CVA})$ in (14) are obtained by substituting the already derived XVA formulas in (4) and (6). ∎

Remark 3.1 *The reason why funding disappears from the bank trading loss, i.e.* $J(\mathcal{C} + \mathcal{F} - \mathrm{CA}) = J(\mathcal{C} - \mathrm{CVA})$*, is because, in a one-period setup, the collateral borrowing requirements* (3) *of the bank are simply constants. Hence funding triggers no risk to the bank, but only a deterministic cost. In the dynamic setup of Crépey (2021b), funding generates both costs and risk.*

Corollary 3.1 *If members, clients and counterparties are default free, all considered XVAs are zero.*

We detail the calculation of economic capital under the member survival measure.

Lemma 3.4 *Denoting by* q^α *the* $\mathbb{Q}$ *upper*[22] *quantile (value-at-risk) of level* α *of the bank trading loss* $J(\mathcal{C} - \mathrm{CVA})$*, if* $\mathbb{Q}(\mathcal{C} - \mathrm{CVA} = q^\alpha) = 0$*, then*

$$\mathrm{EC} = \mathbb{E}^*\big(\mathcal{C} - \mathrm{CVA}\big|\mathcal{C} - \mathrm{CVA} \geq q^\alpha, J = 1\big). \tag{15}$$

Proof. If $\mathbb{Q}(\mathcal{C} - \mathrm{CVA} = q^\alpha) = 0$, then, using equations (3.4) and (3.7) from Acerbi and Tasche (2002) to represent the tail mean expression[23] for the ES, we have by (14):

$$\begin{aligned}\mathrm{EC} = \mathbb{ES}\big(J(\mathcal{C} - \mathrm{CVA})\big) &= \mathbb{E}\big(J(\mathcal{C} - \mathrm{CVA})\big|J(\mathcal{C} - \mathrm{CVA}) \geq q^\alpha)\big) \\ &= \frac{\mathbb{E}\left(J(\mathcal{C} - \mathrm{CVA})\mathbb{1}_{\left\{J(\mathcal{C}-\mathrm{CVA})\geq q^\alpha\right\}}\right)}{\mathbb{Q}\big(J(\mathcal{C} - \mathrm{CVA}) \geq q^\alpha\big)} \\ &= \frac{\mathbb{E}^*\left((\mathcal{C} - \mathrm{CVA})\mathbb{1}_{\left\{(\mathcal{C}-\mathrm{CVA})\geq q^\alpha\right\}}\mathbb{1}_{\{J=1\}}\right)}{\mathbb{Q}^*\big((\mathcal{C} - \mathrm{CVA}) \geq q^\alpha, J = 1\big)},\end{aligned}$$

using (10) on both numerator and denominator expressed in expectation form, which yields (15). ∎

3.3 Extension to Several CCPs or CCP Services

In the realistic case where the reference bank is a clearing member of several services of one or several CCPs, we index all the CCP related quantities in the above by an additional index *ccp* in a finite set disjoint from $I \cup C \cup B$. Then, with $\mathrm{CA} = \mathrm{CCVA} + \mathrm{CMVA} + \mathrm{BCVA} + \mathrm{BMVA} + \mathrm{FVA}$ as before:

[22] In the sense of definition 2.1 from Acerbi and Tasche (2002).
[23] As per Definition 2.6 and further representation (3.7) from Acerbi and Tasche (2002).

Proposition 3.2 *We have*

$$
\begin{aligned}
\mathcal{C} &= \sum_{ccp,c}(1-J_c)(\mathcal{P}_c^{ccp} - \mathrm{MtM}_c^{ccp} - \mathrm{IM}_c^{ccp})^+ + \sum_{ccp}\mu^{ccp}\mathcal{L}^{ccp} \\
&\quad + \sum_b (1-J_b)(\mathcal{P}_b - \mathrm{VM}_b - \mathrm{IM}_b)^+, \text{ where} \\
\mathcal{L}^{ccp} &= \sum_i (1-J_i)\big((\mathcal{P}_i^{ccp} - \mathrm{MtM}_i^{ccp} - \mathrm{IM}_i^{ccp})^+ + (\overline{\mathcal{P}}_i^{ccp} - \overline{\mathrm{MtM}}_i^{ccp} - \overline{\mathrm{IM}}_i^{ccp})^+ - \mathrm{DF}_i^{ccp}\big)^+, \\
\mathcal{F} &= \widetilde{\gamma}\sum_{ccp}\widetilde{\gamma}\big(\mathrm{IM}^{ccp} + \overline{\mathrm{IM}}^{ccp} + \mathrm{DF}^{ccp}\big) + \\
&\quad + \widetilde{\gamma}\sum_b \overline{\mathrm{IM}}_b + \gamma\big(\sum_b(\mathrm{MtM}_b - \mathrm{VM}_b) - \mathrm{CA} - \max(\mathrm{EC},\mathrm{KVA})\big)^+, \\
\mathrm{CCVA} &= \mathbb{E}\Big[\sum_{ccp,c}(1-J_c)(\mathcal{P}_c^{ccp} - \mathrm{MtM}_c^{ccp} - \mathrm{IM}_c^{ccp})^+ + \sum_{ccp}\mu^{ccp}\mathcal{L}^{ccp}\Big], \\
\mathrm{CMVA} &= \sum_{ccp}\widetilde{\gamma}\big(\mathrm{IM}^{ccp} + \overline{\mathrm{IM}}^{ccp} + \mathrm{DF}^{ccp}\big), \\
\mathrm{BCVA} &= \mathbb{E}\Big(\sum_b(1-J_b)(\mathcal{P}_b - \mathrm{VM}_b - \mathrm{IM}_b)^+\Big), \\
\mathrm{BMVA} &= \widetilde{\gamma}\sum_b \overline{\mathrm{IM}}_b, \\
\mathrm{FVA} &= \frac{\gamma}{1+\gamma}\Big(\big(\sum_b(\mathrm{MtM}_b - \mathrm{VM}_b) - (\mathrm{CCVA} + \mathrm{CMVA} + \mathrm{BCVA} + \mathrm{BMVA}) - \mathrm{EC}\big)^+\Big), \\
\mathrm{CA} &= \mathrm{CCVA} + \mathrm{CMVA} + \mathrm{BCVA} + \mathrm{BMVA} + \mathrm{FVA}, \\
J(\mathcal{C} + \mathcal{F} - \mathrm{CA}) &= J(\mathcal{C} - \mathrm{CVA}) \\
&= J\Big(\sum_{ccp,c}(1-J_c)(\mathcal{P}_c^{ccp} - \mathrm{MtM}_c^{ccp} - \mathrm{IM}_c^{ccp})^+ + \sum_{ccp}\mu^{ccp}\mathcal{L}^{ccp} - \mathrm{CCVA} \\
&\qquad + \sum_b(1-J_b)(\mathcal{P}_b - \mathrm{VM}_b - \mathrm{IM}_b)^+ - \mathrm{BCVA}\Big), \\
\mathrm{KVA} &= \frac{h}{1+h}\mathbb{ES}(J(\mathcal{C} - \mathrm{CVA})).
\end{aligned}
\tag{16}
$$

***Proof*.** In the case of several CCP services, the second line in (3) must be turned into $\sum_{ccp}\big(\mathrm{IM}^{ccp} + \overline{\mathrm{IM}}^{ccp} + \mathrm{DF}^{ccp}\big) + \sum_b \overline{\mathrm{IM}}_b$; the terms in the first lines of (4) and (6) must now be summed over the various CCP services in which the bank is involved as a clearing member. The rest of the analysis proceeds as before. ∎

4 Concrete Setup

We describe two possible applications of our XVA framework which will be illustrated by numerical case studies. To these ends, we introduce a market and credit model with parameters that can capture dependence (in the sense of correlation) between portfolio changes, joint defaults and possible averse exacerbated changes of the portfolio due to their owner default known as wrong-way risk. Then two networks will be defined to serve the numerical illustrations, one rather educational on the use of the XVA metrics and the other one reflecting the more realistic situation depicted by Figure 1.1.

The CVA and KVA computations require a Monte-Carlo routine run under $\mathbb{Q}^*$ in combination with a rejection technique in order to yield simulations under the survival measures associated with different clearing members, which, in the numerical applications that follow, play in turn the role of the reference bank in the theoretical XVA framework of Sections 2-3. For obtaining confidence intervals regarding the expected shortfalls that are embedded in the KVA computations, the simulations are split into several batches, from which the mean of the estimated ESs yields the final ES estimate, while the standard deviation of the estimated ESs is used to define a confidence interval.

The default time for member i is generated based on Student t copulas with correlated credit and market components, where credit components are reflected through the member's default times and the market components through their portfolio variation over the liquidation period following the default, proxied in our setup by the difference $\Delta\mathcal{P}_i := \mathcal{P}_i - \mathrm{MtM}_i$. We denote by $\rho^{cr} > 0$ the correlation coefficient of the copula Gaussian factor driving common defaults, by $\rho^{mkt} > 0$ the correlation coefficient of the Gaussian factor driving common portfolio variations, and by $\rho_i^{wwr} > 0$ the correlation coefficient of the Gaussian factor driving both portfolio variation and default for member i. The Student t degree of freedom parameter is assumed to be the same for generating both members' default and portfolio variations. In equations, denoting by F_i the marginal c.d.f. of member i's default time and by S_ν the Student t c.d.f. with degree of freedom ν:

$$\begin{cases} \tau_i = F_i^{-1}\Bigg(S_\nu\Bigg(\sqrt{\frac{\nu}{\mathcal{W}_i^c}}\Bigg(\sqrt{\rho^{cr}}\mathcal{T} - \sqrt{\rho_i^{wwr}}\sqrt[4]{\frac{1-\rho^{cr}}{1-\rho^{mkt}}}\mathcal{X}_i + \\ \qquad \sqrt{1-\rho^{cr}}\sqrt{1-\frac{\rho_i^{wwr}}{\sqrt{1-\rho^{cr}}\sqrt{1-\rho^{mkt}}}}\mathcal{T}_i\Bigg)\Bigg)\Bigg), \\ \\ \frac{\Delta\mathcal{P}_i}{\mathrm{nom}_i\sigma_i\sqrt{\Delta_\ell}} = \sqrt{\frac{\nu}{\mathcal{W}_i^m}}\Bigg(\sqrt{\rho^{mkt}}\mathcal{E} + \sqrt{\rho_i^{wwr}}\sqrt[4]{\frac{1-\rho^{mkt}}{1-\rho^{cr}}}\mathcal{X}_i + \\ \qquad \sqrt{1-\rho^{mkt}}\sqrt{1-\frac{\rho_i^{wwr}}{\sqrt{1-\rho^{cr}}\sqrt{1-\rho^{mkt}}}}\mathcal{E}_i\Bigg) \end{cases} \tag{17}$$

where $\mathrm{nom}_i \in \mathbb{R}$ is a signed nominal of the portfolio of member i, σ_i is its annual relative volatility, Δ_ℓ is a positive liquidation period accounting for the time taken by the CCP to port defaulted portfolios to surviving clearing members, and

- $\mathcal{T}$, $\mathcal{T}_i$, $\mathcal{E}$, $\mathcal{E}_i$ and $\mathcal{X}_i$ are i.i.d. standard normal random variables, where:
  - $\mathcal{T}$ represents the common systemic factor for default times across members

– $\mathcal{E}$ represents the common systemic factor for portfolio variations across members,
– $\mathcal{X}_i$ is the common factor co-driving portfolio variations and default time of member i,
– $\mathcal{T}_i$ is the idiosyncratic factor for member i's default time,
– $\mathcal{E}_i$ is the idiosyncratic factor for member i's portfolio variations;

- $\mathcal{W}_i^c$ and $\mathcal{W}_i^m$ are i.i.d. random variables following χ^2 distribution with degree of freedom ν, independent from the above Gaussian random variables.

Remark 4.1 *In practice, margin computations rely on historical estimates based on several market stressed periods. Our approach, instead, aims at reflecting extreme market shocks with fat tailed Student t distributions of degree of freedom* $\nu = 3$*, and volatility level within a reasonable range of* $[20\%, 40\%]$*. Our static formulation depicts stationary increments of the defaulted portfolios' value over the liquidation period.*

The above setup requires the following constraints on the correlation coefficients to be properly defined[24]:

$$\sqrt{1-\rho^{cr}}\sqrt{1-\rho^{mkt}} \geq \rho_i^{wwr} \tag{18}$$

The "minus" sign in front of the common credit-market factor $-\sqrt{\rho_i^{wwr}}$ for the default time component in (17) ensures that the corresponding common factor accelerates defaults, whilst increasing the market exposure due to the $+\sqrt{\rho_i^{wwr}}$ factor in the second part of (17). As detailed in appendix A, such decomposition in terms of random Gaussian factors can be seen as the result of a Cholesky decomposition on correlated Gaussian factors.

In the examples that follow, market participants are identified by a number and can then be included in one of several of the considered CCPs.

4.1 Single CCP Setup and initial XVA costs

We consider a single CCP service with 20 members labeled by $i \in 0 \ldots n = 19$, only trading for cleared clients (i.e. without bilateral or centrally cleared proprietary trading). Each member faces one client. The ensuing financial network is depicted by Figure 4.1.

All clients are assumed to be risk-free. For any member i, its posted IM to the CCP is calculated based on the idea of a VM call not fulfilled over a time period $\Delta_s < \Delta_\ell$ at a confidence level $\alpha \in (1/2, 1)$, using a VaR metric[25] applied to the non-coverage of VM

[24] otherwise, the model for both default time and portfolio variation factors is undefined due to their idiosyncratic coefficient term $\sqrt{1 - \frac{\rho_i^{wwr}}{\sqrt{1-\rho^{cr}}\sqrt{1-\rho^{mkt}}}}$.

[25] We use conventions and definition 2.2 from Acerbi and Tasche (2002) for defining the quantile of interest of the trading loss r.v.. Such VaR level for a r.v. trading loss L of the member of reference and a confidence level $(1-\alpha) \in (0, 1/2)$ is defined under the member reference surviving measure $\mathbb{Q}$ as $\mathbb{V}\mathrm{a}\mathbb{R}_{1-\alpha}(L) := x^{(\alpha)} = q^{\alpha}(L) = \inf\{x \in \mathbb{R} | \mathbb{Q}(L \leq x) > \alpha\}$.

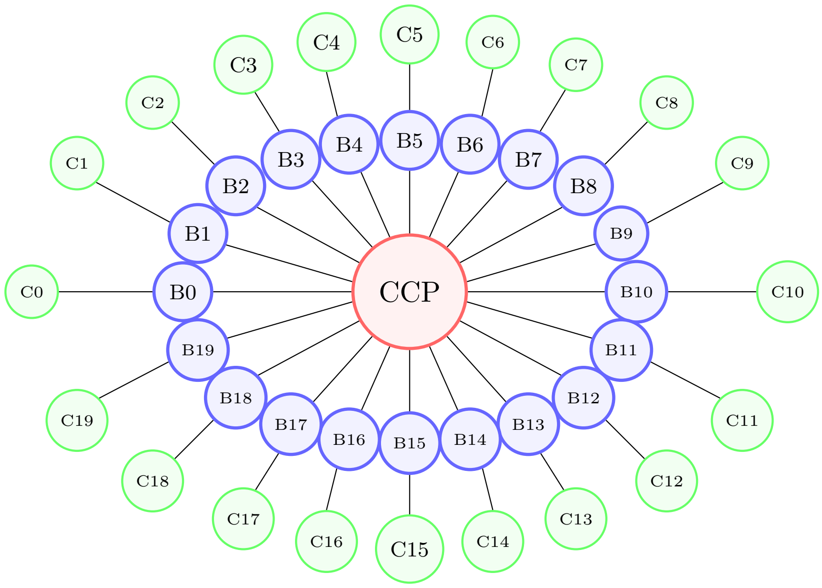

Figure 4.1: Financial network composed of 1 CCP, its 20 members and one client per member

call taken also to follow a scaled Student t distribution $\mathcal{S}_\nu$ with ν degrees of freedom:

$$\mathrm{IM}_i = \mathbb{V}\mathrm{a}\mathbb{R}_{1-\alpha}\big(\mathrm{nom}_i\sigma_i\sqrt{\Delta_s}\mathcal{S}_\nu\big) = |\mathrm{nom}_i|\,\sigma_i\sqrt{\Delta_s}{S_\nu}^{-1}(\alpha) \tag{19}$$

where ${S_\nu}^{-1}$ is the inverse c.d.f. of a Student t distribution with degree of freedom ν. The default fund is calculated at the CCP level as

$$\mathrm{Cover2} = \mathrm{SLOIM}_{(0)} + \mathrm{SLOIM}_{(1)}, \tag{20}$$

for the two largest stressed losses over IM (SLOIM_i) among members, identified with subscripts (0) and (1), where SLOIM is calculated as the upper quantile at confidence level $\alpha' > \alpha$ of the loss over IM, i.e.

$$\mathrm{SLOIM}_i = \mathbb{V}\mathrm{a}\mathbb{R}_{\alpha'}\big(\mathrm{nom}_i\sigma_i\sqrt{\Delta_s}\mathcal{S}_\nu - \mathrm{IM}_i\big) = |\mathrm{nom}_i|\,\sigma_i\sqrt{\Delta_s}\Big({S_\nu}^{-1}(\alpha') - {S_\nu}^{-1}(\alpha)\Big) \tag{21}$$

The total amount (20) is then allocated between the clearing members to define their (funded) default fund contributions as
$\mathrm{DF}_i = \dfrac{\mathrm{SLOIM}_i}{\sum_j \mathrm{SLOIM}_j}\mathrm{Cover2}.$

The quantities nom_j's of other clearing members are not observable by a given one. However, following Murphy and Nahai-Williamson (2014) and Lipton (2018), for $i \in 0 \ldots n = 19$ with $\mathrm{nom}_{(i)}$ denoting the i-th largest absolute nominal amount, a parameterization of the form

$$\mathrm{nom}_{(i)} = \alpha e^{-\beta(i+1)}, \qquad \alpha, \beta > 0 \tag{22}$$

cm id	0	1	2	3	4	5	6	7	8	9
DP (%)	0.5	0.6	0.7	0.8	0.9	2	1.9	1.8	1.7	1.6
size	-242	184	139	105	-80	-61	-46	35	26	-20
vol (%)	20	21	22	23	24	25	26	27	28	29

cm id	10	11	12	13	14	15	16	17	18	19
DP (%)	1.5	1.4	1.3	1.2	1.1	1	0.9	0.8	0.7	0.6
size	-15	-11	-9	-6	5	-4	-3	2	2	-1
vol (%)	30	31	32	33	34	35	36	37	38	39

Table 4.1: Member characteristics and portfolio parameters, ordered by decreasing member size.

can be fit to the total default fund held by the CCP[26] and the sum of its five largest default fund contributions[27], published each quarter for most of the CCPs and that are public data. The inferred parameter α and β from the default fund data are used to depict a similar pattern on the nominal sizes[28]. The participants and portfolios parameter inputs are detailed in Table 4.1 where id is the identifier of the CM, DP stands for the one year probability of default of the member expressed in percentage points, size represent the overall portfolio size of the member detained within the CCP, and vol is the annual volatility used for the portfolio variations.

The portfolios listed in the Table 4.1 relates to the members towards the CCP (which are mirroring the ones between the members and their clients). The sizes sum up to 0, in line with the CCP clearing condition (first identity in Assumption 3.1, here without proprietary trades).

The parameters of the XVA costs calculations are summarized in Table 4.2. Note that the chosen period length of $T = 5$ years covers the bulk (if not the final maturity) of most realistic CCP portfolios.

For each member, the CCVA, CMVA and KVA costs are calculated and reported in Table 4.3. For KVA, two calculations have been performed, one based on ES at 99^{th} percentile level and another one based on 99.75^{th} percentile level. The amount in square bracket is the corresponding quantile level from which average is calculated and numbers

[26]Item referenced as 4.3.15 in Bank of International Settlements and OICV-IOSCO (2021), *Value of pre-funded default resources (excluding initial and retained variation margin) held for each clearing service in total, post-haircut.* in the quantitative disclosure documents.

[27]Item referenced as 18.4.2 in Bank of International Settlements and OICV-IOSCO (2021):*For each segregated default fund with 25 or more members; Percentage of participant contributions to the default fund contributed by largest five clearing members in aggregate.*; or item referenced 18.4.1 for CCP services with less than 25 members

[28]As if the default fund amounts are proportional to the portfolio sizes.

[29]Such confidence level at 97% for SLOIM in DF calibration allows for a ratio of default fund over initial margin of about 10% in our calculations, a ratio (of this level or less) often observed in practice.

One-period length T	5 years
Liquidation period at default Δ_l	5 days
Portfolio variations correlation ρ^{cr}'s	30%
Credit factors correlation ρ^{mkt}'s	20%
Correlation between credit factors and Portfolio variations ρ_i^{wwr}'s	20%
IM covering period (MPoR)	2 days
IM quantile level	95%
Funding blending ratio $\widetilde{\gamma}/\gamma$	25%
SLOIM calculation[29] for DF Cover-2	VaR 97%
DF allocation rule	based on IM
Quantile level used for clearing members EC calculation	99.75%
Hurdle rate h used for KVA computations	10.0%
Number of Monte-Carlo simulation (for CCVA & KVA computations)	5,000,000
Number of batches (for KVA computations)	50

Table 4.2: XVAs calculation configuration

in parenthesis represent the 95% confidence interval in relative difference from calculated metric for both CCVA and KVA.

All the XVA numbers decrease with the member size. To assess the behavior of CCVA and KVA w.r.t. correlations between credit factors, market factors and correlation between credit and portfolio variation factors, we calculate those XVA components making these correlation values equal for all members, varying them between 10% and 90% and display obtained results in Figures 4.2 and 4.3. As expected, the KVA depicts both an increase with respect to the two correlation axes, though the wrong-way risk correlation has more impact compared to the credit factors correlation. Whilst the increase with respect to the wrong-way risk correlation is well portrayed for both CCVA and KVA, there are very marginal changes for CCVA w.r.t. the credit factors correlation and market factors correlation. This is understandable as such metric is a mean so that credit correlation ρ^{cr} and market correlation ρ^{mkt} have limited impact because of the summation across members of excess losses. The various survival measures used for each member, though making the overall expectation not linear, have indeed very limited impact with the expectation form.

4.2 Two CCPs Network Setup

We consider the case of Figure 1.1 where there are two CCPs with some common members and stress test is considered from the perspective of one of these common members. The motivation for this case is to provide a realistic example mimicking in a simpli-

cm id	CMVA	CCVA	KVA (99%)	KVA (99.75%)
0	0.0687	0.0442 (0.6%)	0.2093 [0.0948] (1%)	0.4142 [0.2306] (1.6%)
1	0.0656	0.0412 (0.7%)	0.2087 [0.0855] (1.2%)	0.437 [0.2267] (1.8%)
2	0.0604	0.0327 (0.7%)	0.1683 [0.0679] (1.2%)	0.355 [0.1823] (1.6%)
3	0.0544	0.026 (0.7%)	0.1355 [0.0543] (1.2%)	0.2863 [0.1466] (1.6%)
4	0.0485	0.0191 (0.8%)	0.103 [0.0395] (1.2%)	0.2224 [0.1112] (1.7%)
5	0.0834	0.0133 (0.8%)	0.0809 [0.0295] (1.2%)	0.1772 [0.0886] (1.6%)
6	0.0623	0.0111 (0.8%)	0.0667 [0.0251] (1.1%)	0.1439 [0.0728] (1.6%)
7	0.0467	0.0099 (0.7%)	0.0557 [0.0223] (1.1%)	0.1171 [0.0606] (1.5%)
8	0.0341	0.0078 (0.7%)	0.0432 [0.0174] (1.1%)	0.091 [0.0471] (1.6%)
9	0.0256	0.006 (0.8%)	0.0342 [0.0137] (1.2%)	0.0721 [0.0371] (1.7%)
10	0.0187	0.0048 (0.8%)	0.0266 [0.0107] (1.1%)	0.0561 [0.0289] (1.6%)
11	0.0132	0.0037 (0.7%)	0.0202 [0.0081] (1.1%)	0.0423 [0.0219] (1.5%)
12	0.0104	0.0031 (0.7%)	0.017 [0.0069] (1.2%)	0.0358 [0.0185] (1.6%)
13	0.0066	0.0022 (0.7%)	0.0116 [0.0047] (1.1%)	0.0244 [0.0127] (1.6%)
14	0.0052	0.0019 (0.7%)	0.01 [0.0041] (1.1%)	0.021 [0.0108] (1.6%)
15	0.0039	0.0016 (0.7%)	0.0082 [0.0033] (1.1%)	0.0172 [0.0089] (1.5%)
16	0.0027	0.0012 (0.7%)	0.0063 [0.0026] (1.1%)	0.0132 [0.0068] (1.6%)
17	0.0017	0.0008 (0.7%)	0.0043 [0.0017] (1.1%)	0.009 [0.0047] (1.5%)
18	0.0015	0.0009 (0.7%)	0.0044 [0.0018] (1.1%)	0.0092 [0.0048] (1.6%)
19	0.0007	0.0004 (0.7%)	0.0023 [0.0009] (1.1%)	0.0047 [0.0025] (1.5%)

Table 4.3: Initial XVA costs: estimates, [value-at-risk underlying the KVA estimate] and (95% confidence level errors)

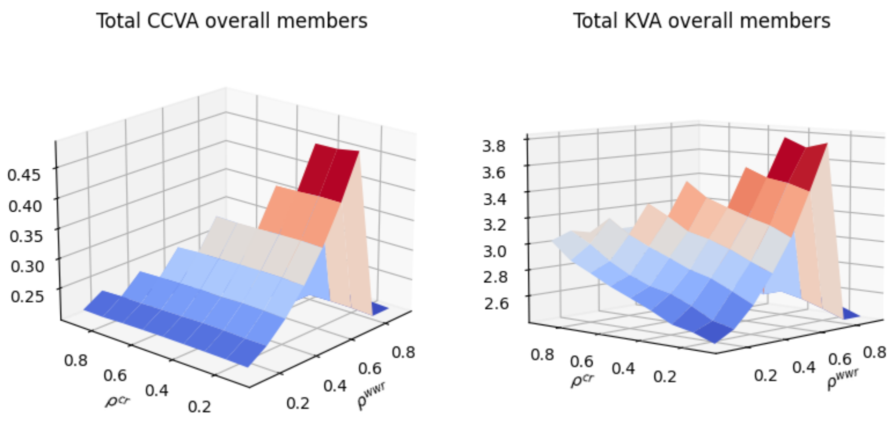

Figure 4.2: CCVA and KVA w.r.t. credit factors correlation and credit and portfolio variation factors correlation

fied way the trading firm Ronin Capital, which had memberships on both FICC GSD[30] segments, hereafter denominated by CCP1 and CME Futures, hereafter denominated by CCP2 in March 2020. It is well known that a VaR type risk measure is not sub-additive, in particular for credit portfolios as illustrated in Example 5.4 in Acerbi and Tasche (2002) and Example 2.25 in McNeil et al. (2015) for a portfolio of defaultable bonds, so that for a common member adding VaR estimates of trading losses on two CCPs separately can lead to underestimated levels with respect to the actual VaR of the global exposition of the member. As such, stress test exercises accounting for common memberships could reveal a larger risk compared to the exercise where stress tests are conducted separately on each CCP. To perform the analysis, the following setup is considered:

- all members have only clearing client positions[31], with 123 members on CCP1 and 56 members on CCP2, out of which 24 are common to both CCPs,
- all clients are assumed default free,
- both CCPs use configuration as per Table 4.2,
- the sizes of the positions are assumed exponentially distributed in the sense that from the most exposed member to the least one, absolute value of positions decrease exponentially with the form in (22) as depicted by Figures 4.4 and 4.5 respectively,

[30] Government Securities Division
[31] Ronin Capital had in fact only a house account and was thus not clearing any client position.

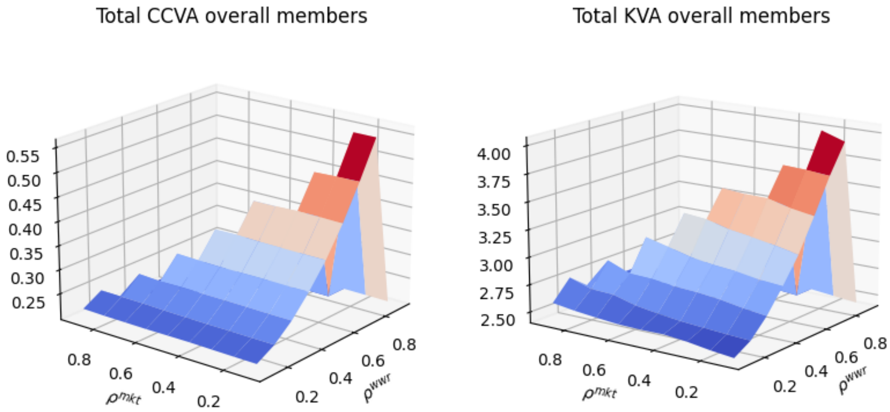

Figure 4.3: CCVA and KVA w.r.t. market factors correlation and credit and portfolio variation factors correlation

- the proportion of the default fund detained by the 5 biggest members is 25% for CCP1 and 61% for CCP2[32],
- the size of the default fund of CCP1 is assumed to be twice the one of the default fund of CCP2.

All data used are either public or have been anonymized. Similar configuration as given in Table 4.2 is used, apart from the number of Monte-Carlo simulations reduced to 2 millions for memory capacity reasons.

The clearing conditions were ensured by setting the sum of the portfolio sizes nom_i to zero on each CCP. The situation of member 3, exposed to both CCPs, as the defaulting member, corresponds roughly to the situation of Ronin Capital in 2018. In particular, an annual probability of default of 0.1% corresponds roughly to a BBB rating, that was assigned to Ronin Capital in 2018 for its issuances[33].

5 Stress Test Exercises

As outlined in the Capital Requirements Regulation detailed in The European Parliament and of the Council and OICV-IOSCO (2013) article 290, financial institutions

[32] taken from the quantitative disclosure of both CCPs as of third quarter of 2020.
[33] `https://www.spglobal.com/marketintelligence/en/news-insights/blog/banking-essentials-newsletter-july-edition-2`

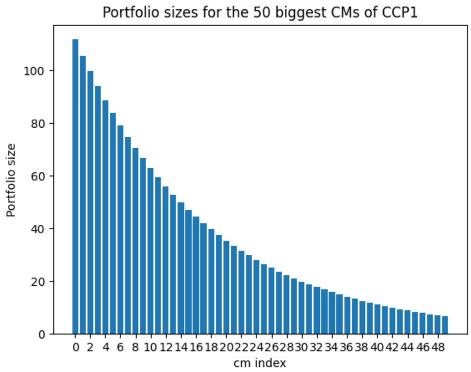

Figure 4.4: Decreasing absolute nom_i per member for CCP1

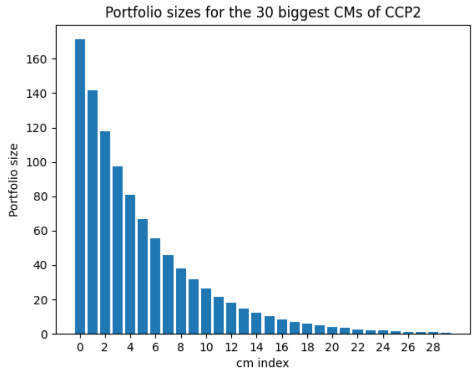

Figure 4.5: Decreasing absolute nom_i per member for CCP2

must conduct regular stress test exercises of their credit and counterparty exposures. Paragraph 8 of this article also stipulates the reverse stress test[34] requirement to

> [...] identify extreme, but plausible, scenarios that could result in significant adverse outcomes.

This is complemented by article 302 on the exposure financial institutions may have towards CCPs:

> Institutions shall assess, through appropriate scenario analysis and stress testing, whether the level of own funds held against exposures to a CCP, including potential future credit exposures, exposures from default fund contributions and, where the institution is acting as a clearing member [...] adequately relates to the inherent risks of those exposures.

In practice, stress test exercises aim at assessing the capacity of financial institutions to absorb financial and economic shocks. In regular exercises, such as the ones conducted by the EBA, the shocks are usually considered under so called *central* or *baseline* macroeconomic scenario corresponding to a median quantile and *adverse* scenario usually taken as a 90^{th} percentile reflecting severe yet plausible scenario that can occur once every 10 years[35]. Additionally extreme scenarios can be considered for measuring the capital adequacy[36] for absorbing extremely severe losses around confidence level at 99.9%. From

[34] See dedicated definition on p.12 in of Governors of the Federal Reserve System (2012) and articles 97, 98 p. 37 in European Central Bank (2018) for official regulatory definitions.

[35] Such confidence level are suggested by the Federal Reserve outlining p.10 in Board of Governors of the Federal Reserve System (Board) (2013) the various recession periods of the United States listed in their Table 1 p. 14. The 2021 EBA stress test instructions in European Bank Authority (2021) also indicate p.72 that stressed market risk factors are based on shocks specified in European Systemic Risk Board (2013), in reference to Dordu et al. (2017, p. 29), which outline the US recessions periods in their Table 1 as stressful economic episodes.

[36] cf. paragraph 5.1 p.11 from Basel Committee on Banking Supervision (2005).

a member perspective, it requires to have the capacity of scanning certain points of its trading loss distribution (compensated of any initial CA). In our static framework, this boils down to identifying particular levels of the distribution of its trading loss $\mathcal{C} - \mathrm{CCVA} - \mathrm{BCVA}$, where $\mathcal{C}$ is given by the multiple CCP services extension of (4), whereas CCVA and BCVA are given by the first and fourth lines of (16).

The other type of stress test exercises, referenced as *reverse stress test*[37] (Bellini et al., 2021), consists in identifying the probability of reaching a given loss level as well as describing the scenario configuration such as projected defaults and loss magnitude leading to such loss levels. Not only the distribution must span a sufficient large spectrum of losses, including the ones targeted by the exercise, but it also has to be sufficiently rich numerically to identify combinations of events leading to such losses. These scenarios can also be considered to identify their contribution to capital requirement similar to what is proposed in Albanese et al. (2021). Confidence intervals of corresponding extreme scenario probabilities should complement the analysis to ensure the reliability of the used model and numerical methods.

Regulators have the ability to challenge financial institutions on these elements and demand for clear plans on developments and implementations for refining these requirements in case such requirements are not deemed sufficiently reflected in their analysis in terms of approaches and estimations[38].

We now briefly explain how to identify and exploit the scenarios leading to contribute the most to economic capital, in the spirit of Albanese et al. (2021). For the reference member bank, we denote by M the number of Monte-Carlo scenario for which $J = 1$, i.e. survival of the reference bank. Its trading loss $\mathcal{C} - \mathrm{CVA}$ for a simulation $m \in 1 \dots M$ is given by $\mathcal{C}^m - \mathrm{CVA}$ where the simulations m are only those for which the reference member bank is under survival state. The simulations are ranked in nondecreasing order and denoted with indices (m) so that $\mathcal{C}^{(1)} - \mathrm{CVA} \leq \dots \leq \mathcal{C}^{(M)} - \mathrm{CVA}$.

To get an estimate of the economic capital based on expected shortfall, relying on Acerbi and Tasche (2002, Definition 2.6 and Proposition 4.1), we calculate, for a high confidence level $\alpha \in (0, 1)$,

$$\widehat{\mathbb{ES}}\left(\mathcal{C} - \mathrm{CVA}\right) := \frac{1}{M - \lfloor \alpha M \rfloor} \sum_{m=\lceil \alpha M \rceil}^{M} \left\{ \mathcal{C}^{(m)} - \mathrm{CVA} \right\}, \tag{23}$$

where $\lceil x \rceil$ and $\lfloor x \rfloor$ denote the integer ceil and floor of $x \in \mathbb{R}$ and with $\mathcal{C}^{\left(\lceil \alpha M \rceil\right)} - \mathrm{CVA} \leq \dots \leq \mathcal{C}^{(M)} - \mathrm{CVA}$ the largest $M - \lfloor \alpha M \rfloor$ trading loss values for the reference member

[37] See also dedicated definition on p.12 in of Governors of the Federal Reserve System (2012) and articles 97, 98 p. 37 in European Central Bank (2018) for official regulatory definitions.

[38] That may entail re-assessment of the Pillar 2 guidance additional capital requirement set in the annual Supervisory Review and Evaluation Process reported by Banks, cf. European Central Bank (2021) for a brief definition and use and Basel Committee on Banking Supervision (2019b) for more extensive details. A similar requirement applies under the Federal Reserve Board rules detailed in Board of Governors of the Federal Reserve System (2020).

bank.

To obtain the contribution of any of these simulation scenarios on the estimated economic capital, we calculate the above quantity by removing the considered scenario. The re-estimated quantity without the scenario ranked (m') is denoted by $\widehat{\mathbb{ES}}_{\alpha}^{-(m')}(\mathcal{C}-\mathrm{CVA})$ and can be formulated as

$$\widehat{\mathbb{ES}}^{-(m')}(\mathcal{C}-\mathrm{CVA}) = \frac{1}{M-1-\lfloor\alpha(M-1)\rfloor}\Big\{\big(M-\lfloor\alpha M\rfloor\big)\widehat{\mathbb{ES}}(\mathcal{C}-\mathrm{CVA})-\Big(\mathcal{C}^{(m')}-\mathrm{CVA}\Big)\Big\} \tag{24}$$

so that the contribution $\delta_{(m')}\widehat{\mathbb{ES}}(\mathcal{C}-\mathrm{CVA})$ of scenario (m') to $\widehat{\mathbb{ES}}(\mathcal{C}-\mathrm{CVA})$ is given by:

$$\delta_{(m')}\widehat{\mathbb{ES}}(\mathcal{C}-\mathrm{CVA}) = \widehat{\mathbb{ES}}(\mathcal{C}-\mathrm{CVA}) - \widehat{\mathbb{ES}}^{-(m')}(\mathcal{C}-\mathrm{CVA}) \tag{25}$$

To illustrate the various flavors of stress test exercises that can be conducted by a CCP member, we report numerical results for the two formerly introduced network examples. We start with a reverse stress test exercise on example covered by Table 4.1. For this first numerical illustration, a specific extreme loss is targeted and the corresponding probability of loss reaching at least such target level is estimated. We then consider the example illustrated by Figure 1.1 where projected loss levels for specific confidence levels are indicated for the members with common memberships on the two CCPs.

In Table 5.1, we report, for the example summarized in Table 4.1, the 99.9^{th} percentile levels, referenced as extreme quantile, with corresponding (asymmetric) confidence intervals based on the approach proposed in Meeker et al. (2017, Section G.2). This is done for every clearing member successively playing the role of the reference bank in the setup of Sections 2 -3. We also compute the probabilities of reaching a loss equal to 1.5 times the obtained extreme quantile level, referenced as the RST scenario, with corresponding confidence levels. The calculation of the latter confidence intervals of the probability of being above a quantile relies on the same numerical approach based on batches used for KVA calculations.

Note that, under the estimation approach from Meeker et al. (2017, Section G.2), the displayed confidence intervals for the quantile at 99.9% are not symmetric. Also the batch approach leads to reasonably tight confidence intervals of the RST scenario probabilities. Our description of the scenarios leading to such losses includes the identified defaulted members, the generated losses and the allocated loss coefficient of the reference clearing member (CM1 in this example). Table 5.2 provides with the description of the 20 worst scenarios contributing to the EC estimation for the second biggest member, that is CM1.

cm id	99.9^{th} quantile	$1.5\times$ 99.9^{th} quantile	RST scenario probability
0	3.9949 (-1.1%, 1.4%)	5.9924	0.0387% (5%)
1	4.1141 (-1.3%, 1.7%)	6.1712	0.0428% (4.4%)
2	3.3409 (-1.6%, 1.4%)	5.0114	0.0435% (4.4%)
3	2.6914 (-1.4%, 1.6%)	4.0371	0.0437% (4.3%)
4	2.0695 (-1.2%, 1.8%)	3.1043	0.0452% (4.5%)
5	1.6715 (-1.7%, 1.5%)	2.5073	0.044% (4.6%)
6	1.3517 (-1.3%, 1.6%)	2.0276	0.0437% (3.9%)
7	1.1032 (-1.4%, 1.7%)	1.6548	0.0434% (4.2%)
8	0.8554 (-1.5%, 1.5%)	1.2831	0.0435% (4.1%)
9	0.6817 (-1.7%, 1.5%)	1.0226	0.0433% (4.2%)
10	0.5287 (-1.7%, 1.4%)	0.7931	0.0429% (4.4%)
11	0.4001 (-1.7%, 1.4%)	0.6002	0.0427% (4.4%)
12	0.3385 (-1.3%, 1.5%)	0.5078	0.0431% (4.2%)
13	0.2314 (-1.5%, 1.5%)	0.3471	0.0428% (4.6%)
14	0.1972 (-1.3%, 1.6%)	0.2958	0.0429% (4.4%)
15	0.1623 (-1.4%, 1.5%)	0.2435	0.0427% (4.3%)
16	0.1249 (-1.4%, 1.5%)	0.1874	0.0427% (4.4%)
17	0.0852 (-1.5%, 1.4%)	0.1278	0.0434% (4%)
18	0.0873 (-1.4%, 1.6%)	0.131	0.0427% (4.1%)
19	0.0447 (-1.6%, 1.3%)	0.0671	0.0431% (4%)

Table 5.1: Stress test (ST) extreme quantile (in parentheses: corresponding 95% confidence intervals), $1.5\times$ ST extreme quantile and RST probability to breach 1.5 times the 99.9^{th} quantile loss level (in parentheses: corresponding 95% confidence level errors), for each member, based on $5,000,000$ simulations.

Rank	Total Loss	n	μ	Defaulters	Losses triggered by defaulters
1	10.97	3	0.22	cm0, 9, 16	544.37, 0, 0
2	8.18	3	0.23	cm0, 7, 11	391.38, 0, 0.55
3	7.84	3	0.24	cm0, 6, 7	356.66, 0, 0
4	6.74	1	0.21	cm0	347.74
5	6.06	4	0.25	cm0, 5, 7, 15	0, 267.65, 0, 0.01
6	6.05	5	0.25	cm0, 7, 8, 10, 11	269.86, 0, 0, 0, 0
7	6.03	4	0.24	cm0, 5, 12, 18	278.62, 0, 0, 0
8	4.85	5	0.21	cm5, 6, 7, 11, 12	257.88, 0, 0, 0, 0
9	4.19	4	0.2	cm2, 12, 13, 18	230.49, 0.48, 0, 0
10	4.11	1	0.18	cm5	250.77
11	4.07	3	0.24	cm0, 6, 8	187.74, 0, 0
12	3.94	4	0.22	cm3, 5, 6, 13	0, 0, 197.59, 0
13	3.93	2	0.2	cm2, 8	214.58, 0
14	3.82	1	0.21	cm0	197.11
15	3.7	1	0.21	cm0	191.08
16	3.66	8	0.32	cm0, 2, 5, 10, 11, 12, 14, 16	127.07, 0, 0, 0, 0, 0, 0, 0
17	3.65	4	0.23	cm0, 9, 10, 18	176.98, 0, 0, 0
18	3.5	2	0.21	cm0, 16	179.86, 0
19	3.48	2	0.23	cm0, 7	170.21, 0
20	3.48	3	0.23	cm0, 6, 15	166.23, 0, 0

Table 5.2: Economic Capital 20 worst scenarios details for member 1 in decreasing order of total loss where column with header μ indicates allocated coefficient loss to member 1 and n is the number of defaults within the scenario.

Its theoretical number of scenarios above the RST loss level should be[39] 2076, which is of course far too many to report, but a focus on the worst ones already illustrates the type of information that can be exploited for such exercises. Most of these scenarios are driven by significant losses stemming from member 0's default, reflecting its high concentrated position. Note that the 16^{th} worst loss scenario for member 1 entails 8 defaults, including the one of CM0, which is the only one to generate losses beyond its posted margin (i.e. to trigger a loss to the surviving members).

From CM1 viewpoint (i.e. for CM1 in the role of the reference clearing member), 15 scenarios depict significant losses over the collateral posted by the defaulted CM0 (positive first entries in the last column of Table 5.2). CM0 bears a very large concentrated positions compared to other members. Even if CM0 has more IM and DF requirements than others, this is still not enough: this example highlights that employed DF allocation rules in this example dilute the DF collateral requirements for concentrated positions. It also illustrates that scenarios with multiple defaults do not necessarily lead to extreme

[39]The number of MC simulations of 5,000,000 multiplied by CM1's survival probability over 5 years and by CM1's RST loss level probability estimated in Table 5.1.

losses, due to the fact that members with medium or small positions have large default fund contributions stemming from others' concentrated positions.

In Table 5.3, we report, for the example illustrated by Figure 1.1 with 2 CCPs, the trading loss levels (value-at-risks) at confidence levels 90% and 99.9%, for the 24 common members on the two CCPs. The corresponding numbers in the case where the two CCPs would be considered separately is reported in the columns labeled "stand-alone".

For quantiles at 90% confidence levels, the loss levels are significantly higher when the common membership are considered compared to the stand-alone quantile loss calculation conducted on each CCP and summed, especially for the first ten members[40] For members with very low size on one of the two CCPs compared to the other, considering the common memberships or not does not affect the loss estimates, as expected[41]. This outlines the importance of taking into account such commonality feature for sizeable members on the CCPs. On the contrary, with quantile loss levels at confidence level 99.9%, the sum of stand-alone loss estimations are well above the loss estimate when common memberships are taken into consideration. For members facing the two CCPs, this leads in particular to over-conservative KVA estimates. This in turn is detrimental for client end-users that support unnecessary additional capital costs.

6 Optimizing the Porting of Defaulted Client Portfolios

In this Section we consider the problem of the optimal resolution of the CCP portfolio of a defaulted clearing member. In case a clearing member defaults, the CCP novates what it can of the CCP portfolio of the defaulted member, through auctions among the surviving clearing members, and it liquidates the residual on exchanges. We endorse a natural assumption that the CCP novates client trades and their mirroring client account positions, whereas the CCP liquidates house account positions.

Remark 6.1 *A next step left for future research would be the optimization for the CCP of which trades should be cleared and which should be auctioned.*

Supposing that the reference clearing member, labeled by 0 in Sections 2-3, defaults at time 0, just after that all portfolios have been settled, we consider the problem of the optimal porting of its cleared client trades. In a first stage, we assume that cleared client trades of the defaulted clearing member 0 and the mirroring trades of the defaulted clearing member 0 with the CCP are proposed as an indivisible package to the surviving members, which are just left with the freedom of proposing a price for global porting, i.e. for replacing the defaulted member in all its future contractual obligations related to these trades. For each surviving member CM* successively considered as a

[40] As illustrated by various examples in McNeil et al. (2015), VaR is not in general a sub-additive risk measure, especially not in the context of credit applications.

[41] As the CCP with the very low size compared to the other should have marginal impact.

I	II	III	IV	V	VI	VII	VIII	IX	X
3	0.1	19.9	21	-97.48	23	0.0227 (-3%, 2.8%)	0.0442 (-1.3%, 1.2%)	3.0505 (-2.2%, 1.9%)	2.6768 (-2%, 2.3%)
4	0.1	80.79	24	-18.79	22	0.0215 (-2.7%, 2.8%)	0.0428 (-1.1%, 1.1%)	2.6739 (-2%, 1.8%)	2.3058 (-2%, 2.2%)
9	3.1	-31.58	29	17.74	23	0.0195 (-1.9%, 1.8%)	0.0399 (-0.9%, 0.9%)	1.631 (-2.1%, 2.3%)	1.2558 (-2.3%, 1.9%)
12	0.1	17.97	21	-16.75	24	0.0175 (-1.3%, 1.2%)	0.0288 (-0.7%, 0.7%)	0.8226 (-1.8%, 2%)	0.5575 (-1.7%, 2.1%)
13	0.1	-14.9	22	15.81	25	0.0172 (-1.2%, 1.2%)	0.0274 (-0.7%, 0.7%)	0.7568 (-1.8%, 2%)	0.5124 (-1.8%, 1.8%)
14	0.2	12.34	23	-14.93	26	0.0168 (-1.2%, 1.2%)	0.0261 (-0.7%, 0.7%)	0.7005 (-1.8%, 2%)	0.4763 (-1.6%, 1.6%)
15	0.1	-10.23	24	14.09	27	0.0164 (-1.1%, 1.1%)	0.0246 (-0.7%, 0.7%)	0.6451 (-1.7%, 2%)	0.4426 (-1.6%, 1.5%)
17	0.3	-7.03	26	-13.3	28	0.016 (-1.1%, 1.1%)	0.0226 (-0.7%, 0.7%)	0.5674 (-1.7%, 2%)	0.3998 (-1.6%, 1.9%)
19	0.2	-4.83	28	12.56	29	0.0155 (-1.1%, 1%)	0.0205 (-0.7%, 0.7%)	0.4998 (-1.6%, 2%)	0.3673 (-1.6%, 1.5%)
22	3.9	2.75	20	-11.86	30	0.0173 (-1%, 1%)	0.0197 (-0.8%, 0.8%)	0.4577 (-1.8%, 1.7%)	0.3856 (-1.4%, 2.1%)
26	0.1	1.3	24	11.2	20	0.0094 (-0.9%, 0.9%)	0.0104 (-0.8%, 0.8%)	0.2448 (-1.6%, 2%)	0.2099 (-1.6%, 1.8%)
27	0.1	1.07	25	-10.57	21	0.0093 (-0.9%, 0.9%)	0.0101 (-0.7%, 0.8%)	0.2377 (-1.6%, 2%)	0.2074 (-1.5%, 2%)
28	1.5	0.89	26	9.98	22	0.0097 (-0.9%, 0.9%)	0.0104 (-0.8%, 0.8%)	0.2433 (-2.1%, 1.6%)	0.215 (-1.8%, 1.8%)
31	0.1	-0.51	29	-9.42	23	0.009 (-0.9%, 0.9%)	0.0094 (-0.8%, 0.8%)	0.2191 (-1.7%, 1.8%)	0.2021 (-1.5%, 1.7%)
34	0.1	0.29	21	8.89	24	0.0089 (-0.9%, 0.8%)	0.009 (-0.8%, 0.9%)	0.2051 (-1.5%, 2%)	0.1979 (-1.4%, 2%)
35	0.1	-0.24	22	-8.4	25	0.0087 (-0.9%, 0.9%)	0.0088 (-0.9%, 0.8%)	0.201 (-1.5%, 2%)	0.1948 (-1.4%, 2%)
36	0.1	0.2	23	7.93	26	0.0086 (-0.9%, 0.8%)	0.0086 (-0.8%, 0.9%)	0.197 (-1.6%, 1.9%)	0.1916 (-1.5%, 1.8%)
39	0.1	-0.11	26	-7.48	27	0.0084 (-0.9%, 0.8%)	0.0084 (-0.8%, 0.9%)	0.1907 (-1.5%, 2%)	0.1873 (-1.4%, 2%)
40	0.5	0.09	27	7.07	28	0.0084 (-0.8%, 0.9%)	0.0084 (-0.8%, 0.9%)	0.1898 (-1.5%, 2%)	0.1869 (-1.4%, 2%)
44	0.1	0.04	20	-6.67	29	0.008 (-0.8%, 0.8%)	0.008 (-0.8%, 0.8%)	0.1806 (-1.5%, 1.9%)	0.1796 (-1.5%, 1.9%)
49	0.1	-0.02	25	6.3	30	0.0078 (-0.9%, 0.8%)	0.0079 (-0.8%, 0.8%)	0.1758 (-1.4%, 2%)	0.1752 (-1.4%, 2%)
50	0.1	0.01	26	-5.95	20	0.0049 (-0.9%, 0.8%)	0.0049 (-0.8%, 0.8%)	0.1109 (-1.6%, 1.9%)	0.1106 (-1.6%, 1.9%)
51	0.1	-0.01	27	5.61	21	0.0049 (-0.8%, 0.8%)	0.0049 (-0.8%, 0.8%)	0.1095 (-1.5%, 2%)	0.1092 (-1.5%, 2%)
55	0.1	-0.01	20	-5.3	22	0.0048 (-0.9%, 0.8%)	0.0048 (-0.8%, 0.8%)	0.1084 (-1.5%, 1.9%)	0.1082 (-1.5%, 1.9%)

Table 5.3: Quantile loss levels (confidence errors) for 90% and 99.9% confidence levels across members for the example with 2 CCPs and 155 members including 24 common members. Legend for column headers: I. Member Id, II. DP (%), III. Size on CCP1, IV. Volatility on CCP1, V. Size on CCP2, VI. Volatility on CCP2, VII. 90^{th} Perc. stand-alone, VIII. 90^{th} Perc., IX. 99.9^{th} Perc. stand-alone, 99.9^{th} Perc.

potential taker of the defaulted (client) positions, one then computes the incremental XVAs of porting the defaulted (client) positions to CM*, for each surviving member (CM* included). The corresponding incremental XVA numbers are then summed over metrics and survivors, resulting in the funds transfer price (FTP*) of porting defaulted client positions to CM*. Following a Pareto optimality principle, the taker is then the surviving member for which the ensuing FTP* is the smallest[42]. See Albanese et al. (2020, Section 5.2) for more details.

Remark 6.2 *In actual auctioning default resolution procedures, predetermined chunks of the portfolio are auctioned to the best bidder without revelation of the bid of the others. Different auction designs commonly used by CCPs, such as first price vs. second price auctions, open vs. sealed auctions, English vs. Dutch auctions, are described in Default Risk Management Working Group (2016) and Basel Committee on Banking Supervision (2019a). Conservative bidding can be disincentivized by juniorization of the corresponding default fund contributions (Ferrara and Li, 2017; Huang and Zhu, 2021; Oleschak et al., 2019). Several auctions are in fact conducted in parallel. LCH SA thus uses either English auctions or Dutch auctions depending on the clearing service.*

Our proposed Pareto optimal porting procedure can be seen as the output of an idealized, efficient auction, assuming a large number of clearing members (Oleschak et al., 2019, Section 3.3).

To illustrate the application of our XVA framework for porting, we start by looking at two porting problems on the example of Table 4.1. We consider a first scenario of a single default on the CCP and the corresponding optimal porting and a second scenario with two defaults and the corresponding optimal porting.

Taking the first case with a single default, we assume the scenario whereby member 0, labeled in Table 4.3 as 0, defaults at time 0, right after all positions have been settled. Table 6.1 summarizes the incremental (Δ) CCVA, CMVA and KVA, across members from 1 to 19, in increasing order of the total indicated in the last column.

Based on the results of Table 6.1, CM1 appears to be the taker leading to the least overall FTP costs across all surviving members. This is understandable as this member's portfolio size (184) nets the most the defaulted member's portfolio size (-242) with similar volatility and has a credit default probability similar to the defaulted member, in particular not significantly higher compared to the defaulted member.

All members are impacted by additional margin to fund due to the re-calibration of their DF by the CCP (whereas only the member taker of the portfolio sees its IM adjusted). As CM1 concentrates more risks due in particular to non-perfect offset between its prior positions and the defaulting one, there is an increase of its IM reflected through an increase of CMVA. An overall[43] CCVA reduction is made possible as there

[42] Or indifferently one of the minimzing FTP* members, in case of multiple minima.

[43] aggregated across surviving members.

Member − Costs	Total ΔCMVA	Total ΔCCVA	Total ΔKVA	Total FTP
1	0.0768 (0.0295)	-0.0025 (0.0018)	0.0743 (-0.0006)	0.1486 (0.0307)
2	0.0921 (0.0428)	0.0007 (0.0047)	0.1418 (0.0387)	0.2346 (0.0862)
19	0.1298 (0.0818)	0.0129 (0.0223)	0.1725 (0.2313)	0.3151 (0.3354)
3	0.1054 (0.0576)	0.007 (0.0075)	0.2333 (0.0735)	0.3457 (0.1387)
18	0.1417 (0.0939)	0.0186 (0.0219)	0.2757 (0.2275)	0.4359 (0.3432)
17	0.1549 (0.107)	0.0228 (0.0218)	0.3215 (0.229)	0.4992 (0.3578)
16	0.1688 (0.1208)	0.0271 (0.0217)	0.4075 (0.2303)	0.6034 (0.3728)
4	0.1525 (0.1022)	0.0116 (0.0122)	0.445 (0.1288)	0.6091 (0.2431)
15	0.1814 (0.1334)	0.0305 (0.0214)	0.4656 (0.2294)	0.6775 (0.3842)
14	0.1903 (0.1426)	0.035 (0.0209)	0.5079 (0.2237)	0.7332 (0.3872)
13	0.2061 (0.1582)	0.0384 (0.0208)	0.5903 (0.2272)	0.8349 (0.4063)
12	0.2171 (0.1692)	0.0406 (0.0202)	0.6277 (0.2252)	0.8854 (0.4147)
11	0.2285 (0.1807)	0.0439 (0.0198)	0.6811 (0.2223)	0.9536 (0.4228)
10	0.2385 (0.1908)	0.0469 (0.019)	0.757 (0.2181)	1.0424 (0.428)
8	0.234 (0.1881)	0.0512 (0.0164)	0.7812 (0.1858)	1.0663 (0.3903)
7	0.2327 (0.1876)	0.0519 (0.0149)	0.809 (0.1696)	1.0936 (0.3721)
9	0.2478 (0.2003)	0.0483 (0.0181)	0.7994 (0.2117)	1.0955 (0.4301)
6	0.2687 (0.2225)	0.0506 (0.0135)	0.9811 (0.1689)	1.3004 (0.405)
5	0.2728 (0.2274)	0.0486 (0.0113)	1.0242 (0.1414)	1.3456 (0.3801)

Table 6.1: FTP* corresponding to the different surviving CM*, i.e. for * other than 0, assuming an instant default of member 0 at time 0 (right after all portfolios have been settled).

ΔCMVA	ΔCCVA	ΔKVA
0.0593	0.0182	0.2846

Table 6.2: Standard deviation across members of incremental XVAs for the example with 1 CCP and 20 members, assuming an instant default of member 0 at time 0.

ΔCMVA	ΔCCVA	ΔKVA
0.0586	0.0178	0.2875

Table 6.3: Standard deviation of incremental XVAs across members for the example with 1 CCP and 20 members with members 0 and 8 considered in default state

is one risky member less in the system as well as some offset of positions by the taker. This only happens when CM1 takes over the defaulting portfolio, other potential takers leading to an overall increase of the CCVA. As for the KVA, though there is a reduction effect for the member taker (see the terms in parentheses in Table 6.1), there is an overall increase in total KVA whichever the member taker is. Having CM1 as a taker allows least increase of such KVA component.

As expected, among the three XVA components, KVA plays the major role of allowing identifying the taker leading to the least overall incremental costs across members. These observations can be supported by looking at the standard deviation of each incremental XVA across members as shown in Table 6.2.

Once the CCP has re-allocated all defaulted client trades and their mirroring client account positions, the resulting financial network formerly depicted in Figure 4.1 becomes the network with 19 members shown in Figure 6.1. The thick lines represent the new portfolio exposures for CM1 and the pale dashed line and node the defaulted CM0.

It is also possible to resolve numerically the re-allocation of two defaulted members. The number of possible combinations of takers in that case is $18^2 = 324$. By putting member 0 into default state, the largest member in portfolio size with size -242, as well as member 8, a middle-sized one with portfolio size 26, we get that CM1 and CM4 taking over the respective portfolios of the defaulted CM0 and CM8 leads to the least FTP (additional costs aggregated across the remaining 18 members of the CCP). The resulting network post defaults of members 0 and 8 is shown in Figure 6.2.

As depicted by the standard deviation of each incremental XVA across members in Table 6.3, the KVA plays again the major role of allowing identifying the taker minimizing the ensuing FTP.

When looking at the signed portfolio sizes and one-year default probability of the two takers, it is aligned with the intuition that the second largest member, which is member

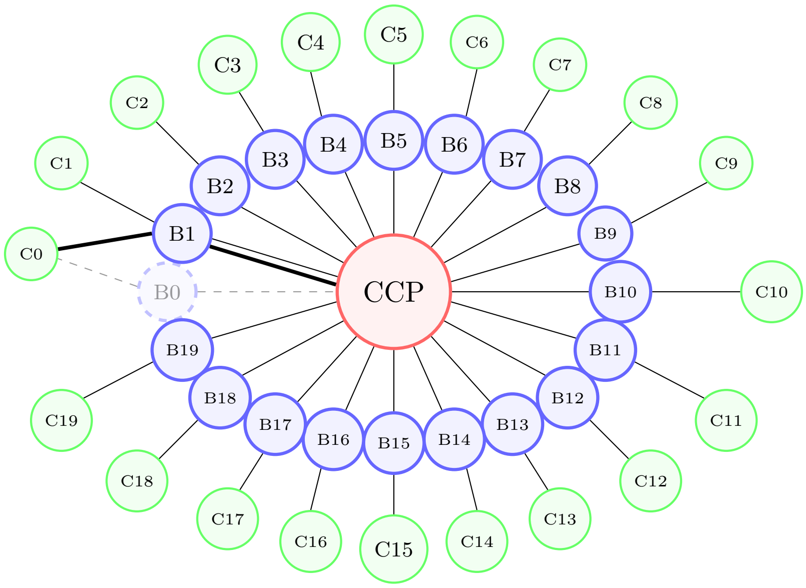

Figure 6.1: The 1-CCP, former 20-member financial network with 19 members post CM0 default. Defaulted CM0, labeled "B0" in the presented network, is represented as pale dashed node with pale dashed links to reflect former exposures to its client and toward the CCP. The optimal porting of CM0 portfolio with CM1, labeled "B1", is outlined with bold links to reflect the new exposures for CM1.

1 with portfolio size 184, should take over the defaulted portfolio of member 0 as it has an opposite portfolio direction, resulting into a strong netting benefit. One can also note that its default probability is quite similar to that of member 1. At first sight, member 4 taking over defaulting portfolio of member 8 seems surprising. Other potential taker with closest opposite portfolios sizes are members 6 (with size −46) and 9 (with size −20). But their default probability levels compared to member 4 are roughly two times larger. the result of member 4 taking over member 8 defaulted portfolio is a significant reduction in terms of KVA compared to the situation where member 6 or 9 takes over member 8's portfolio, as depicted in Table 6.4.

7 Conclusion

We have proposed a fully integrated risk management framework that can serve stress test analysis, including reverse stress test in line with regulatory requirements, as well as porting defaulted portfolios analysis, in a setup encompassing all the trades (bilateral as centrally cleared) of a reference bank. The framework includes dependence features between financial participants portfolios, joint defaults, and a configurable wrong-way

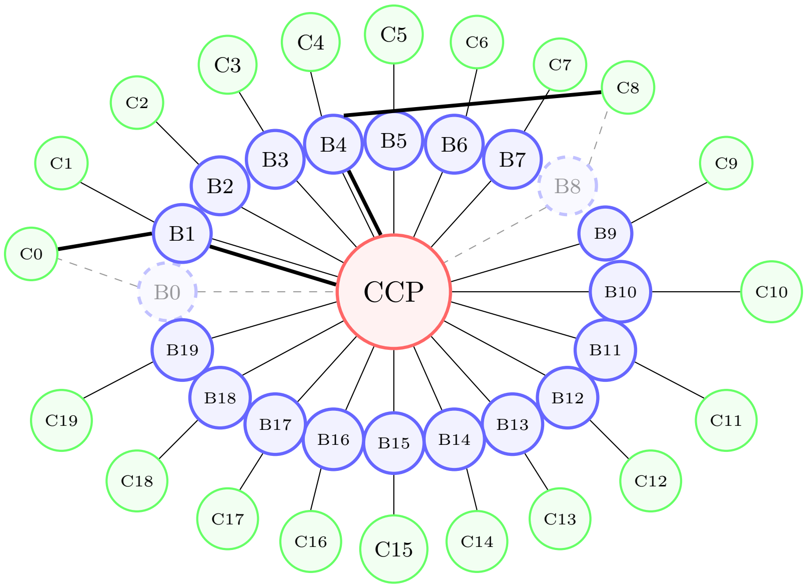

Figure 6.2: The 1-CCP, former 20-member financial network with 18 members post CM0 and CM8 defaults. Defaulted CM0, labeled "B0" and CM8, labeled "B8" in the presented network, are represented as pale dashed node with pale dashed links to reflect former exposures to its client and toward the CCP. The optimal portings of CM0 portfolio with CM1, labeled "B1", and of CM8 portfolio with CM4 are outlined with bold links to reflect the new exposures for both CM1 and CM4.

risk feature. This is done in a numerically tractable static setup. A dynamic extension, as required for traditional front-office XVA applications that require an exact monitoring of all cash flow schedules, is possible along the lines of Crépey (2021b), but of course this would come at a significantly enhanced computational burden. Another improvement would be to add regulatory constraint such as minimum regulatory capital requirements and leverage ratios. We only briefly touch the lack of data transparencies that CCP members experience. Further research could be devoted to exploit further the information of quantitative disclosure to infer from a prior belief of those members' positions informative post distributions reflecting those information.

{taker of 0, taker of 8}	Sizes	Vol (%)	DP (%)	FTP MVA	FTP CCVA	FTP KVA	FTP
{ 1, 4 }	{ 184, -80 }	{ 21, 24 }	{ 0.6, 0.9 }	0.07358	0.00007	0.17576	0.24941
{ 1, 6 }	{ 184, -46 }	{ 21, 26 }	{ 0.6, 1.9 }	0.0756	0.00255	0.18252	0.26067
{ 1, 9 }	{ 184, -20 }	{ 21, 29 }	{ 0.6, 1.6 }	0.08394	0.00405	0.18827	0.27626

Table 6.4: FTPs for different pairs of member takers on defaulted portfolio 0 and 8 for the case of 1 CCP and 20 members where information in { , } relates to the first taking member on defaulted portfolio 0 and to the second taking member on defaulted portfolio 8.

A Cholesky decomposition for the credit-market model

In terms of the random variables

$$\begin{cases} \widetilde{X}_i = \sqrt{\rho^{cr}}\mathcal{T} - \sqrt{\rho_i^{wwr}}\sqrt[4]{\dfrac{1-\rho^{cr}}{1-\rho^{mkt}}}\mathcal{X}_i + \sqrt{1-\rho^{cr}}\sqrt{1 - \dfrac{\rho_i^{wwr}}{\sqrt{1-\rho^{cr}}\sqrt{1-\rho^{mkt}}}}\mathcal{T}_i \\ \\ \widetilde{Y}_i = \sqrt{\rho^{mkt}}\mathcal{E} + \sqrt{\rho_i^{wwr}}\sqrt[4]{\dfrac{1-\rho^{mkt}}{1-\rho^{cr}}}\mathcal{X}_i + \sqrt{1-\rho^{mkt}}\sqrt{1 - \dfrac{\rho_i^{wwr}}{\sqrt{1-\rho^{cr}}\sqrt{1-\rho^{mkt}}}}\mathcal{E}_i, \end{cases} \tag{26}$$

our market model (17) is shortly written as

$$\begin{cases} {S_\nu}^{-1}\Big(F_i\big(\tau_i\big)\Big) = \sqrt{\dfrac{\nu}{\mathcal{W}_i^c}}\widetilde{X}_i \\ \\ \dfrac{\Delta\mathcal{P}_i}{\mathrm{nom}_i\sigma_i\sqrt{\Delta_\ell}} = \sqrt{\dfrac{\nu}{\mathcal{W}_i^m}}\widetilde{Y}_i. \end{cases} \tag{27}$$

Proposition A.1 *The model* $\big(\mathcal{T}, \mathcal{X}_i, \widetilde{X}_i, \mathcal{E}, \widetilde{Y}_i, \mathcal{X}_j, \widetilde{X}_j, \widetilde{Y}_j\big)$ *admits the Cholesky decomposition*

$$\big(\mathcal{T}, \mathcal{X}_i, \widetilde{X}_i, \mathcal{E}, \widetilde{Y}_i, \mathcal{X}_j, \widetilde{X}_j, \widetilde{Y}_j\big)^\top = \Sigma\big(\mathcal{T}, \mathcal{X}_i, \mathcal{T}_i, \mathcal{E}, \mathcal{E}_i, \mathcal{X}_j, \mathcal{T}_j, \mathcal{E}_j\big)^\top, \tag{28}$$

where

$$\Sigma = \begin{pmatrix} \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 \\ \boldsymbol{\alpha} & \boldsymbol{\beta_i} & \boldsymbol{\gamma_i} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 \\ 0 & \boldsymbol{\mu_i} & 0 & \boldsymbol{\delta} & \boldsymbol{\nu_i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 \\ \boldsymbol{\alpha} & 0 & 0 & 0 & 0 & \boldsymbol{\beta_j} & \boldsymbol{\gamma_j} & 0 \\ 0 & 0 & 0 & \boldsymbol{\delta} & 0 & \boldsymbol{\mu_j} & 0 & \boldsymbol{\nu_j} \end{pmatrix}. \tag{29}$$

Proof. Denoting

$$\begin{cases} \alpha = \sqrt{\rho^{cr}} \\ \delta = \sqrt{\rho^{mkt}} \\ \beta_k = -\sqrt{\rho_k^{wwr}} \sqrt[4]{\dfrac{1-\rho^{cr}}{1-\rho^{mkt}}} \\ \gamma_k = \sqrt{1-\rho^{cr}} \sqrt{1 - \dfrac{\rho_k^{wwr}}{\sqrt{1-\rho^{cr}}\sqrt{1-\rho^{mkt}}}} \\ \mu_k = \sqrt{\rho_i^{wwr}} \sqrt[4]{\dfrac{1-\rho^{mkt}}{1-\rho^{cr}}} \\ \nu_k = \sqrt{1-\rho^{mkt}} \sqrt{1 - \dfrac{\rho_i^{wwr}}{\sqrt{1-\rho^{cr}}\sqrt{1-\rho^{mkt}}}} \\ \text{for } k \in \{i, j\}, \end{cases}$$

the (sparse[44]) covariance matrix of $\left(\mathcal{T}, \mathcal{X}_i, \widetilde{X}_i, \mathcal{E}, \widetilde{Y}_i, \mathcal{X}_j, \widetilde{X}_j, \widetilde{Y}_i\right)$ is

$$\Gamma = \begin{pmatrix} \mathbf{1} & 0 & \boldsymbol{\alpha} & 0 & 0 & 0 & \boldsymbol{\alpha} & 0 \\ 0 & \mathbf{1} & \boldsymbol{\beta_i} & 0 & \boldsymbol{\mu_i} & 0 & 0 & 0 \\ \boldsymbol{\alpha} & \boldsymbol{\beta_i} & \mathbf{1} & 0 & -\boldsymbol{\rho_i^{wwr}} & 0 & \boldsymbol{\alpha^2} & 0 \\ 0 & 0 & 0 & \mathbf{1} & \boldsymbol{\delta} & 0 & 0 & \boldsymbol{\delta} \\ 0 & \boldsymbol{\mu_i} & -\boldsymbol{\rho_i^{wwr}} & \boldsymbol{\delta} & \mathbf{1} & 0 & 0 & \boldsymbol{\delta^2} \\ 0 & 0 & 0 & 0 & 0 & \mathbf{1} & \boldsymbol{\beta_j} & \boldsymbol{\mu_j} \\ \boldsymbol{\alpha} & 0 & \boldsymbol{\alpha^2} & 0 & 0 & \boldsymbol{\beta_j} & \mathbf{1} & -\boldsymbol{\rho_j^{wwr}} \\ 0 & 0 & 0 & \boldsymbol{\delta} & \boldsymbol{\delta^2} & \boldsymbol{\mu_j} & -\boldsymbol{\rho_j^{wwr}} & \mathbf{1} \end{pmatrix}, \tag{30}$$

with Cholesky decomposition lower triangular matrix Σ (s.t. $\Sigma\Sigma^\top = \Gamma$) given by (29). ∎

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[44]The non-zero coefficients are bold to ease reading

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