Discussion of: Required Capital for Long-run Risks

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Long-run risks

*Most econometric models are inadequate for long-run predictions*

1. **Lag orders**: limited
2. **History**: limited
   - climate change;
   - long-term maturity assets;
   - Prudential regulation typically based on yearly VaR (ES);
   - Estimation of long-run ”not yet seen” risk often missing.

Ultra-Long-Run as reduced form to the computation of the *long-run required capital* (e.g. balance sheet of insurance companies).

   - stochastic process driven by shocks with long lasting impact and small magnitude.
This paper - dynamic model

Reminder, cf. previous discussion of Gourieroux and Jasiak 2021:

\[
\begin{align*}
    y_T(t) &= y_s(t) + Ay_l(t/T), \quad t = 1, \ldots, T \\
    dy_l(\tau) &= -\Theta y_l(\tau)d\tau + SdW_\tau
\end{align*}
\]  

Suppose the observations is $T$ times smaller:

\[
d y(t) = \frac{-\Theta}{T} y(t)dt + \frac{S}{\sqrt{T}}dW_t
\]
This paper - Required capital design

\[ RC_{t+h} = \sum_{k=1}^{h} RCC_{t+k} \]  \hspace{1cm} (3)

with:
- \( t + H \) date at which long run risk completely realizes;
- \( t + h < t + H \) intermediate dates;
- \( RCC_{t+k} \) required capital call at \( t + h \).

If the ”in fine” (or cumulated) loss \( X_{t+H} \) is known:

\[ RC_{t+H} = X_{t+H} \]  \hspace{1cm} (4)

then for the rate \( \delta > 0 \) and \( \delta \neq 1 \)

\[ RC_{t+h} = X_{t+H} \frac{1 - \delta^h}{1 - \delta^H} \]  \hspace{1cm} (5)
This paper - Stochastic loss at maturity and regulatory discounting

\[ RC_{t+h} - RC_{t+h-1} = \frac{1 - \delta}{1 - \delta^{H-h+1}} \left[ \frac{X^*_{t+h,t+H}}{(1 + r)^{H-h}} - RC_{t+h-1} \right] \]  

(6)

with:

- \( X^*_{t+h,t+H} \) a valuation at \( t + h \) of \( X_{t+H} \);
- \( r \) regulatory long-run discount rate.

Different valuation methods:

- Mean-variance;
- Certainty equivalent;
- Risk-Neutral.
This paper - Mean-variance, ULR and cumulated loss

- Cumulated loss: \( X_{t+H} = \sum_{h=1}^{H} P_{t+h} \);
- \( P_t = g[y_s(t), y_l(t)] \), where \( y_l(t) \) follows an ULR process;
- Mean-variance scheme \( X_{t+h, t+H}^* = E_{t+h}X_{t+H} + \frac{A}{2}V_{t+H}X_{t+H} \)

Then the conditional distribution of \( X_{t+H} \) at \( t+h \):

\[
X_{t+h} + H \int_0^1 G[\tilde{y}(u)] \, du
\]

\( \Rightarrow \) stochastic via the long-run component only.
Conclusion and suggestion

- Long-run risk (climate change) is a major concern for the regulation of financial institutions;
- In your motivations, you mentioned ”risk not observed in the past”, not clear how to introduce such risks in an ULR model?
- The incremental required capital you model is:

\[ RC_{t+h} - RC_{t+h-1} = \frac{1 - \delta}{1 - \delta^{H-h+1}} \left[ \frac{X^*_t h, t+H}{(1 + r)^{H-h}} - RC_{t+h-1} \right] \]  

(8)

- And you introduce the ULR model with a Mean-variance to simplify:

\[ X_{t+h} + H \int_0^1 G[\bar{y}(u)] \, du \]  

(9)

- More illustration of ULR modeling, potential forms of \( G \) applicable to insurance would be welcome to compare with current practices and potential gaps with the current capital regulation.