Discussion of: Required Capital for Long-run Risks

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Long-run risks

Most econometric models are inadequate for long-run predictions

1. **Lag orders**: limited
2. **History**: limited

- climate change;
- long-term maturity assets;

- Prudential regulation typically based on yearly VaR (ES);
- Estimation of long-run ”not yet seen” risk often missing.

Ultra-Long-Run as reduced form to the computation of the *long-run required capital* (e.g. balance sheet of insurance companies).

- stochastic process driven by shocks with long lasting impact and small magnitude.

This paper - dynamic model

Reminder, cf. previous discussion of Gourieroux and Jasiak 2021:

$$\begin{cases} y_T(t) = y_s(t) + Ay_l(t/T), \quad t = 1, \dots, T \\ dy_l(\tau) = -\Theta y_l(\tau) d\tau + S dW_\tau \end{cases} \tag{1}$$

Suppose the observations is T times smaller:

$$dy(t) = -\frac{\Theta}{T} y(t) dt + \frac{S}{\sqrt{T}} dW_t \tag{2}$$

This paper - Required capital design

$$RC_{t+h} = \sum_{k=1}^{h} RCC_{t+k} \tag{3}$$

with:

- $t + H$ date at which long run risk completely realizes;
- $t + h < t + H$ intermediate dates;
- RCC_{t+k} required capital call at $t + h$.

If the "in fine" (or cumulated) loss X_{t+H} is known:

$$RC_{t+H} = X_{t+H} \tag{4}$$

then for the rate $\delta > 0$ and $\delta \neq 1$

$$RC_{t+h} = X_{t+H} \frac{1 - \delta^h}{1 - \delta^H} \tag{5}$$

This paper - Stochastic loss at maturity and regulatory discounting

$$RC_{t+h} - RC_{t+h-1} = \frac{1-\delta}{1-\delta^{H-h+1}} \left[\frac{X^*_{t+h,t+H}}{(1+r)^{H-h}} - RC_{t+h-1} \right] \tag{6}$$

with:

- $X^*_{t+h,t+H}$ a valuation at $t+h$ of X_{t+H};
- r regulatory long-run discount rate.

Different valuation methods:

- Mean-variance;
- Certainty equivalent;
- Risk-Neutral.

This paper - Mean-variance, ULR and cumulated loss

- Cumulated loss: $X_{t+H} = \sum_{h=1}^{H} P_{t+h}$;
- $P_t = g\left[y_s(t), y_l(t)\right]$, where $y_l(t)$ follows an ULR process;
- Mean-variance scheme $X^*_{t+h,t+H} = E_{t+h}X_{t+H} + \frac{A}{2}V_{t+H}X_{t+H}$

Then the conditional distribution of X_{t+H} at $t+h$:

$$X_{t+h} + H\int_0^1 G\left[\tilde{y}(u)\right] du \tag{7}$$

$\Longrightarrow$ stochastic via the long-run component only.

Conclusion and suggestion

- Long-run risk (climate change) is a major concern for the regulation of financial institutions;
- In your motivations, you mentioned "**risk not observed in the past**", not clear how to introduce such risks in an ULR model?
- The incremental required capital you model is:

$$RC_{t+h} - RC_{t+h-1} = \frac{1-\delta}{1-\delta^{H-h+1}} \left[\frac{X^*_{t+h,t+H}}{(1+r)^{H-h}} - RC_{t+h-1} \right] \tag{8}$$

- And you introduce the ULR model with a Mean-variance to simplify:

$$X_{t+h} + H \int_0^1 G\left[\tilde{y}(u)\right] du \tag{9}$$

- More illustration of ULR modeling, potential forms of G applicable to insurance would be welcome to compare with current practices and potential gaps with the current capital regulation.