LONG RUN RISK IN STATIONARY STRUCTURAL VECTOR AUTOREGRESSIVE MODEL

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Risk Forum, March, 2022

1. INTRODUCTION

Macroeconomic and financial models provide reliable predictions at short horizon : 1 to 5 years,
but there is a growing interest in long run prediction at horizons of 10 to 50 years ;

- transition to low carbon economies,
- climate risk,
- management of pension funds.

Long run predictions are difficult from series observed over periods shorter or equal to the prediction horizon.
Then the long run predictions remain mainly model based.
But the current model based approaches could be improved.

- The estimation inference has to account for the long run component even if it is difficult to detect.
- The long run predictions have to produce reasonable results compatible with the admissible values of the variable to be predicted.
- The long run risk does not have to be disregarded.

The current models often lead to surprising results when the ULR is considered.

i) Term Structure Models

Some leads to infinite ($+$ or $-$) long run interest rates as the Merton model, Ho and Lee model, arbitrage free Nelson, Siegel model.

Other models to a long run interest rate, which is deterministic constant in time : Cox, Ingersoll, Ross model, Vasicek model (deterministic $\simeq$ no risk on ULR).

ii) Macromodels

Unit root models that lead to explosive evolutions that implies infinite ULR values of the variables.

This is not compatible with the behaviour of variables as the growth rate of per capita real GDP,

the interest rates,

the real exchange rates,

some commodity prices,

the spot-forward spreads.

For recent papers trying to solve this "standard" unit root puzzle :
Beeler, Campbell (2012) : "The Long Run Risks Model and Aggregate Asset Prices : An Empirical Assessment", CFR
for the mean reverting feature of consumption.

Gospodinov, Maynard, Pesavento (2021) : "Conditional Inference in Nearly Cointegrated Vector Error Correction Models and Small Signal-to-Noise Ratio", Adv. in Econometrics,
for the analysis of spot-forward spreads.

Gourieroux, Monfort, Renne (2022) : "Ultra Long Run Term Structure Models",
for arbitrage free sequence of models leading to stochastic ULR rates

This paper considers a "simple" modelling that disentangle two time scales :

the standard one (the day, say)

an ultra long one (the century, or much more)

and the effects of latent components written in these SR and LR time scales.

2. THE DYNAMIC MODEL

For expository purpose, a linear additive vector autoregressive framework.

$$y_T(t) = y_s(t) + Ay_l(t/T), t = 1, \ldots, T.$$

where $y_s(t) = \Phi y_s(t-1) + \Omega^{1/2}\varepsilon_t$,
is independent of the continuous time Ornstein-Uhlenbeck process :

$$dy_l(\tau) = -\Theta y_l(\tau)d\tau + SdW_\tau$$

$\rightarrow$ SR parameters : Φ, Ω, LR parameters : Θ, S, A the "betas" of the LR components,
$\rightarrow$ two time scales,
$\rightarrow$ stationary processes (different from the local-to-unity models)

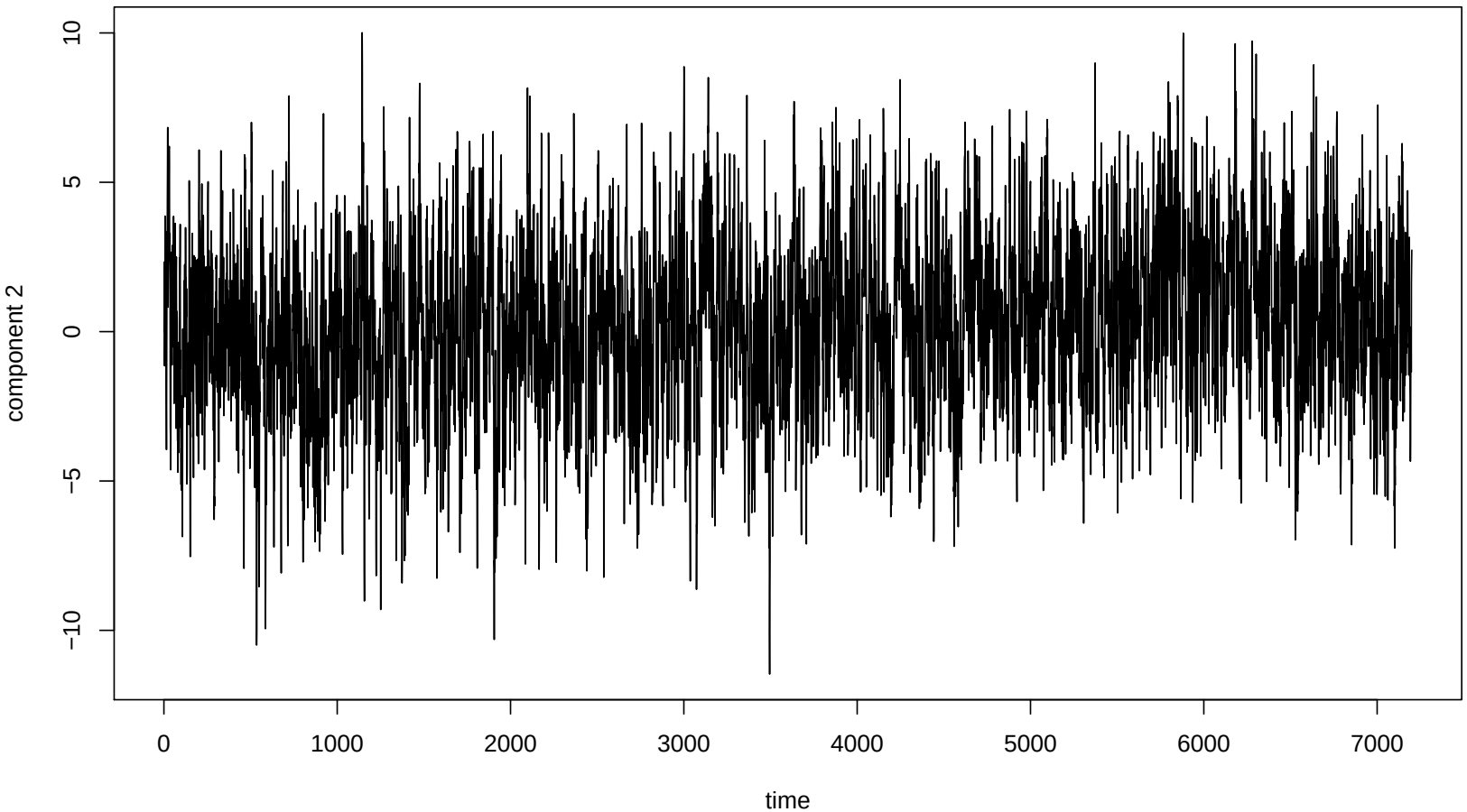
It t is fixed, $t/T \sim 0$, and $T \to \infty$,

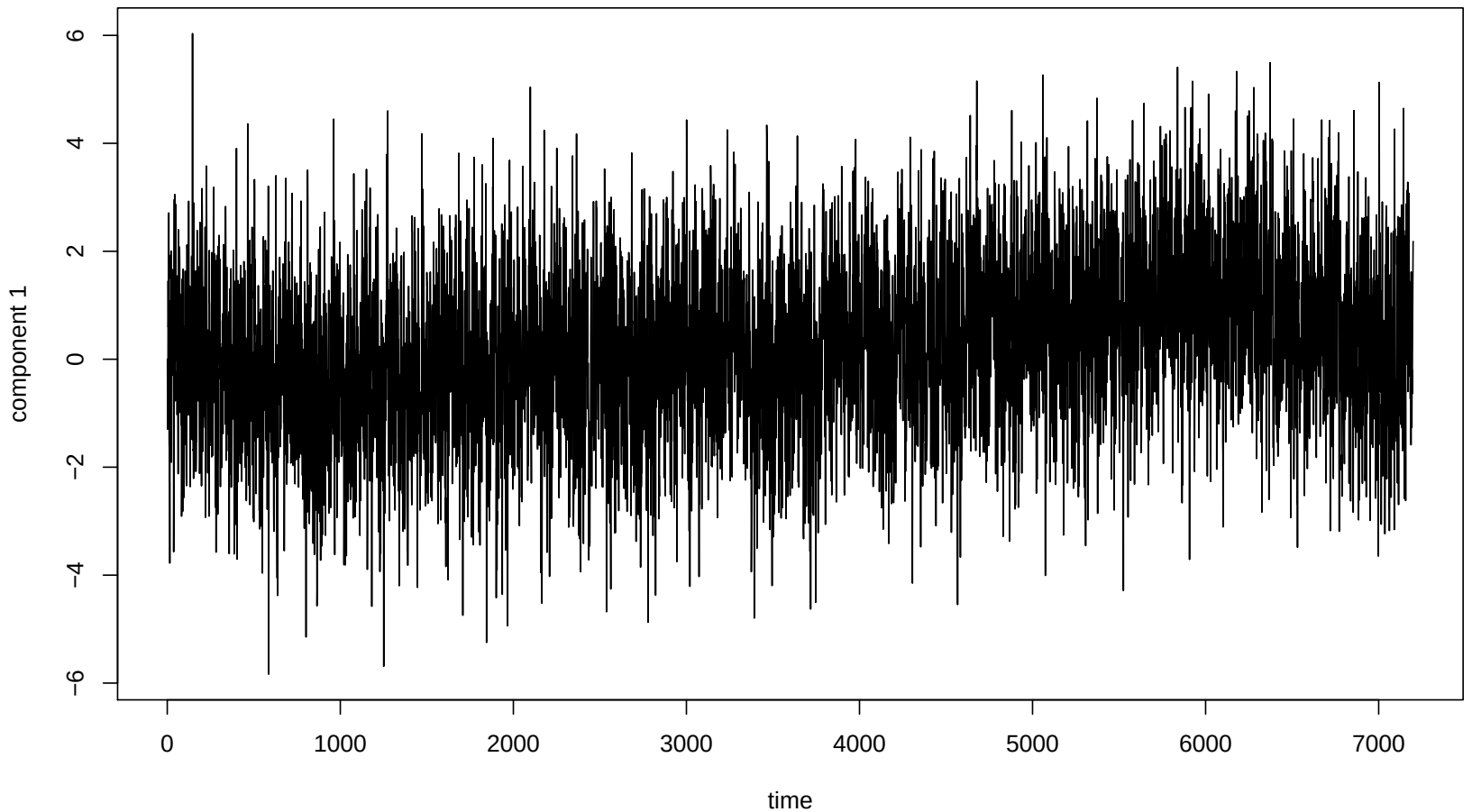
the LR component is a smooth stochastically time varying level.

It is difficult to detect by eye, or by the standard statistical approaches, which are based on "averages" over the whole period $(1, \ldots, T)$, that will eliminate the ULR component (for large T).

The likelihood principal does not apply.

Figure 1 : Simulated Trajectory of Bivariate Process $y(t)$




component 1
6
4
2
0
−2
−4
−6
0
1000
2000
3000
4000
5000
6000
7000
time

3. THE RESULTS

i) **The standard ACF are misleading. They have to be replaced by "localized" ACF $\rightarrow$ SR ACF and LR ACF.**

SR ACF : computed on short episodes at distant dates,
LR ACF : computed on averaged values at distant times.

Figure 2 : Trajectory of $\hat{m}(c_k)$

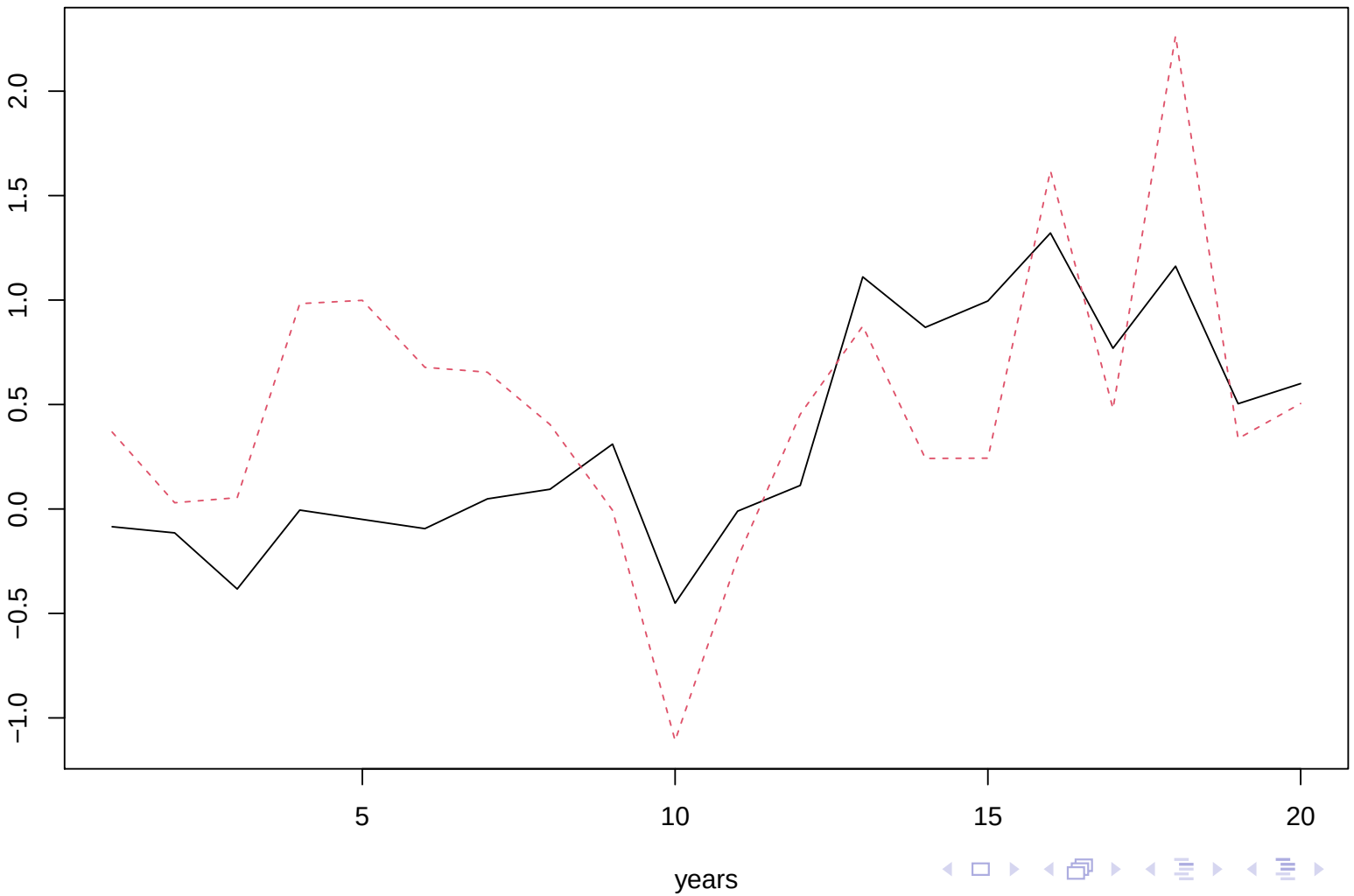


Figure 3 : The Standard ACF

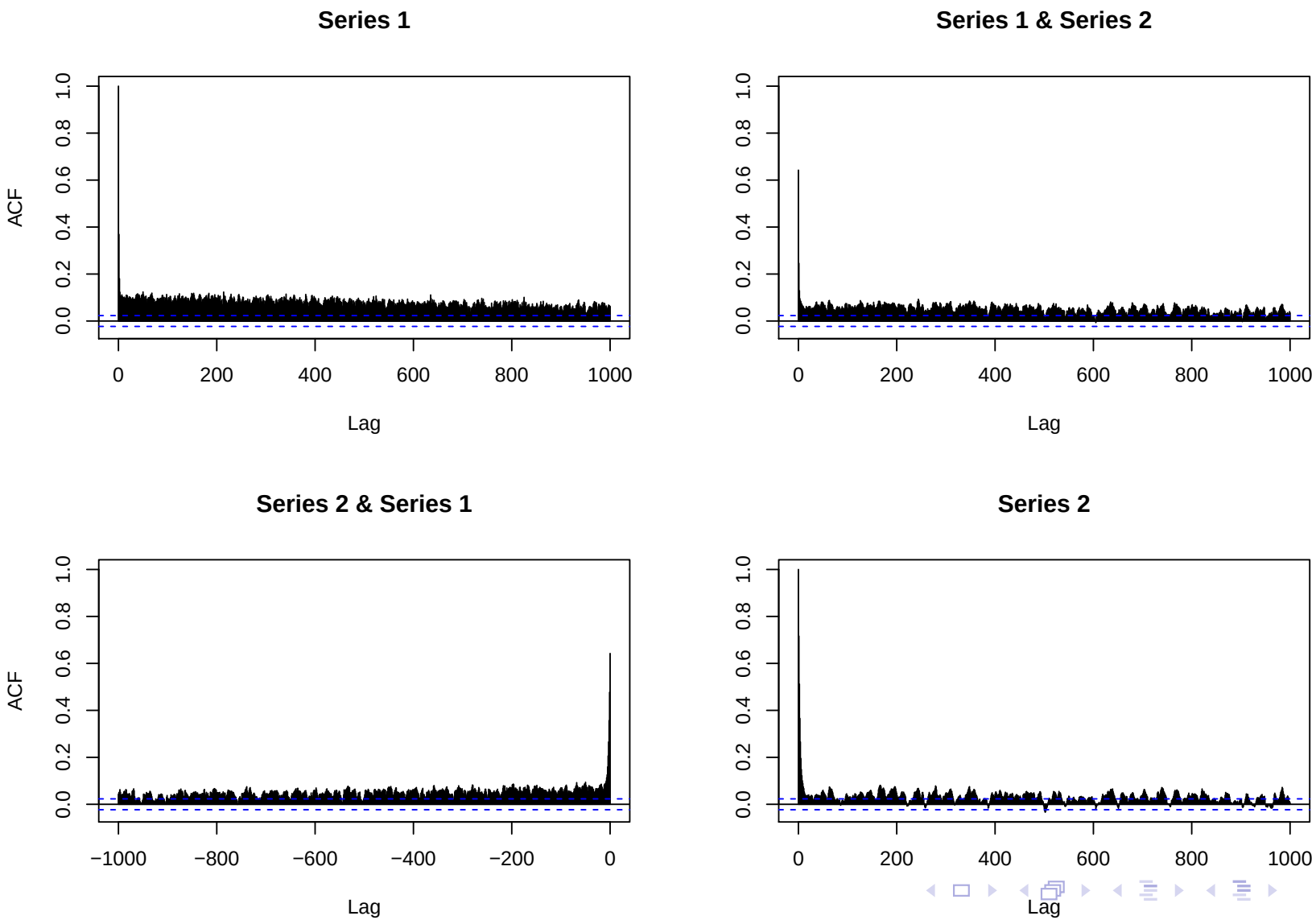
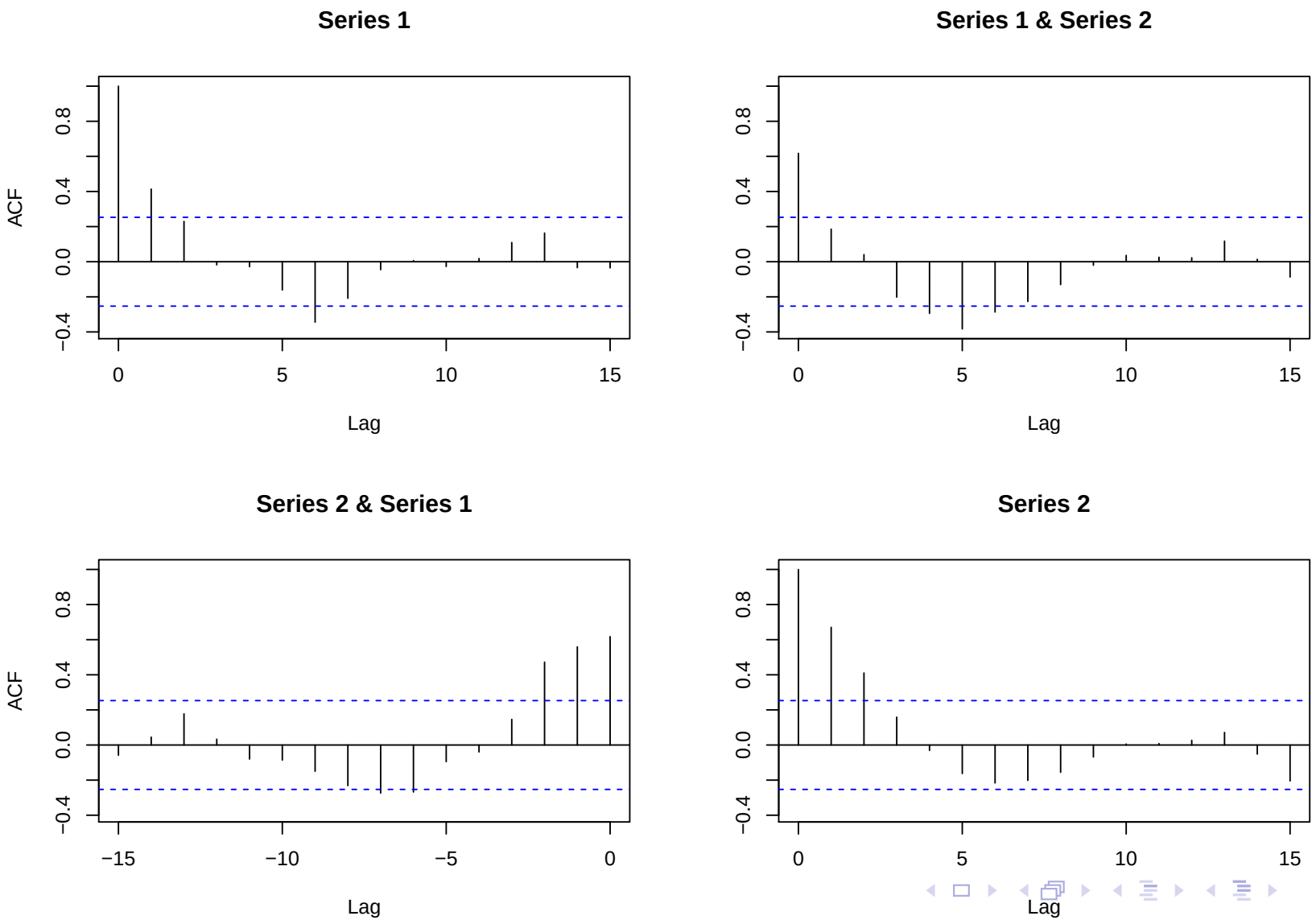


Figure 4 : The Local ACF



2) **This is a state space model with more state variables than observables ("underdetermined system"), but for T large, $y_s(t), t = 1, \ldots, T$, and $y_l(c), c \in [0, 1]$, are filtered without errors !**

The LR component is filtered by local-in-time averaging.

3) **The matrix A is consistently estimated as well as the number of underlying LR factors.**

The number of LR factors is obtained by a PCA applied on $\hat{A}$.

4) **The SR parameters are consistently estimated.**

The LR parameter S is consistently estimated, but the LR parameter Θ cannot be estimated consistently :

The impossibility Theorem.

$\rightarrow$ a finite sample feature even for $T \rightarrow \infty$

There is still a ULR uncertainty even with a large number of observations.

5) **For prediction intervals ;**

- **A first effect due to the theoretical prediction interval.**
- **A second due to the plug in of estimations** $\hat{\Phi}, \hat{\Omega}, \hat{S}, \hat{\Theta}, \hat{A}$ **(standard estimation risk).**
- **Another (significant) effect by the impossibility theorem.**

SR prediction : only the SR component matters.

LR prediction : only the LR component matters.

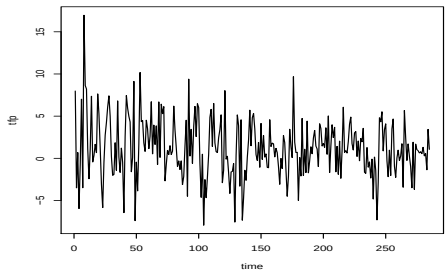
4. ILLUSTRATION

An empirical exercise to facilitate the comparison with the recent literature.

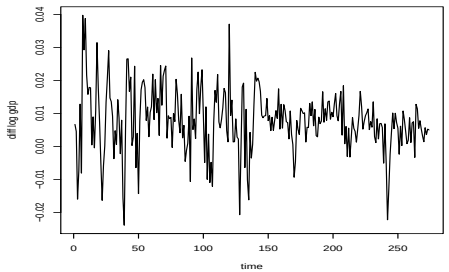
The same macrodata from the FRED database as in Muller, Watson (2016), (2020).

Muller, Watson (2016) : "Measuring Uncertainty About Long Run Predictions", Review of Economic Studies, 83, 1711-1740.

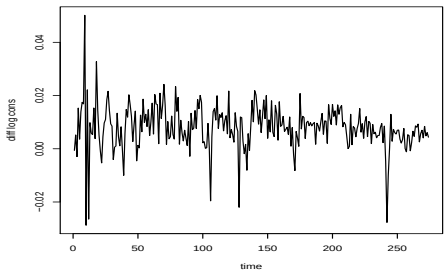
Figure a.1 : The series



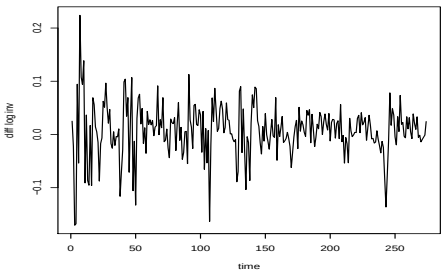
(a) TFP rate



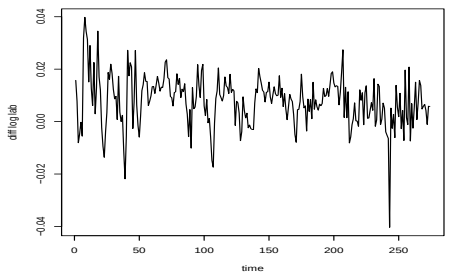
(b) GDP



(c) Consumption



(c) Investment



(e) Labour Compensation

Table 2 : Prediction Intervals at 95%

Series	Raw Data	Filtered ULR (unadjusted)	Filtered ULR (adjusted for estimation risk)
tfp	$(0.781, 1.634)$	$(0.437, 1.436)$	$(0.108, 1.763)$
gdp	$(0.0065, 0.0090)$	$(0.0054, 0.0081)$	$(0.0048, 0.0087)$
c	$(0.0069, 0.0089)$	$(0.0070, 0.0094)$	$(0.0067, 0.0096)$
inv	$(0.0065, 0.0085)$	$(-0.0017, 0.0113)$	$(-0.0136, 0.0231)$
lab	$(0.0065, 0.0091)$	$(0.0076, 0.0111)$	$(0.0069, 0.0119)$

5. CONCLUDING REMARKS

- We have developed a new modelling (and the associated statistical inference) for multivariate stationary series with both SR and LR component.
- The main message is the existence of a significant estimation risk for ULR predictions, with consequences when computing the ULR VaR, and ULR required capital.
- This analysis can be extended to nonlinear dynamic frameworks with nonlinear effects and/or cross effects of the SR and LR latent components.

Gourieroux, Jasiak (2022) : "Long Run Predictions", forthcoming Annals of Economics and Statistics.

Gourieroux, Kim, Meddahi (2022) : "Stationary Ultra Long Run Component".