Enhancing Customer Experience of Savings products through optimal product design

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1 Introduction

The world's older population is growing rapidly. According to the United Nations, there was a substantial increase of 48% (from 607 to 901 million) of people aged 60 or over between 2000 and 2015, which might reach nearly 2.1 billion by 2050. Moreover, the "oldest-old" (aged 80 or over) population accounted for 14% of old population (aged 60 or over) in 2015 and is expected to triple 2015's value by 2050.

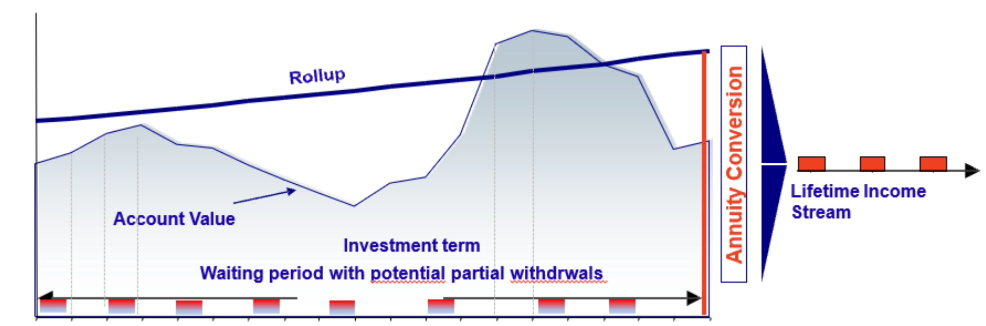
As a result of this demographic shifts and longer life expectancy, increasing lifestyle and health care costs, the idea that individuals and households need to plan for their own retirement is gaining a lot of attention. On the other hand, low interest rates are putting pressure, pushing consumers alike to look for ways to make the most of their assets (€1800 billion in France and $6000 billion in the US for the Individual Savings) and optimize their behaviour. As a result, savings behaviour has undergone significant changes towards a more efficient use over the last decade.

As illustrated by the emergence of increasingly differentiated retirement savings patterns, in line with new needs and to the subsequent life phases. Meeting the expectations of the insured then requires an optimization of the product design, through product features tailored to the savings profiles, for an enhanced Customer Experience.

Savings contracts entitle the policyholder to receive an income stream at maturity for the rest of her life, post a waiting period during which savings are invested in financial assets ("Account Value"), and benefit from an increasing guaranteed level at a minimum "rollup" rate. The insured is also allowed to withdraw a certain amount on a yearly basis during the waiting period. Besides, if the contract contains a death benefit, then a certain amount is paid to the beneficiaries in case the policyholder dies during the term of the contract.

Figure 1

Retirement Savings mechanism

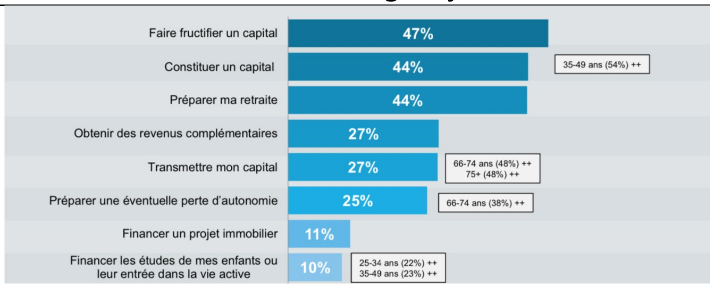


2 Customer Experience is significantly different across ages, as illustrated by strongly differentiated Savings behaviour patterns

Savings behaviours have undergone significant changes over the past decade. Admittedly, the objectives of building up savings are diverse, as illustrated below:

Figure 2

Retirement Savings objectives

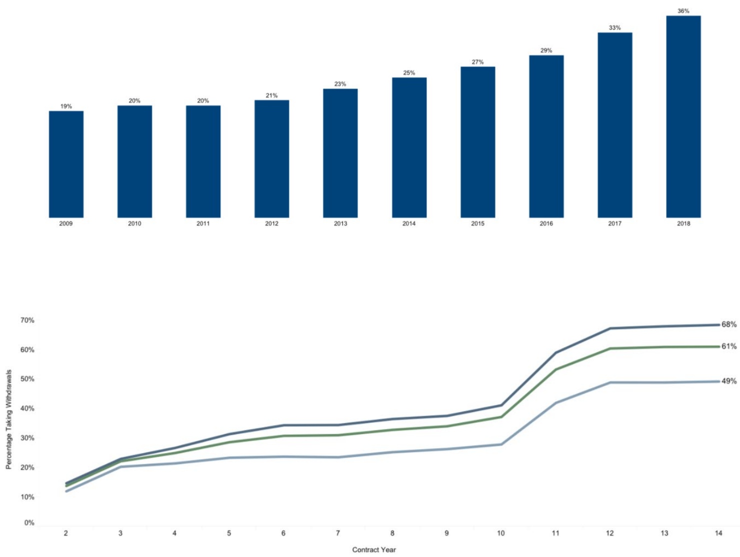


Source : AMF (France)

Still policyholders have made increasing use of their accumulated savings, and this in an accelerated way with age.

Figure 3

Use of Savings



Source LIMRA (USA)

To this end, there are three types of Savings use behaviour patterns, illustrated by the frequency and withdrawals amounts, strongly influenced by age:

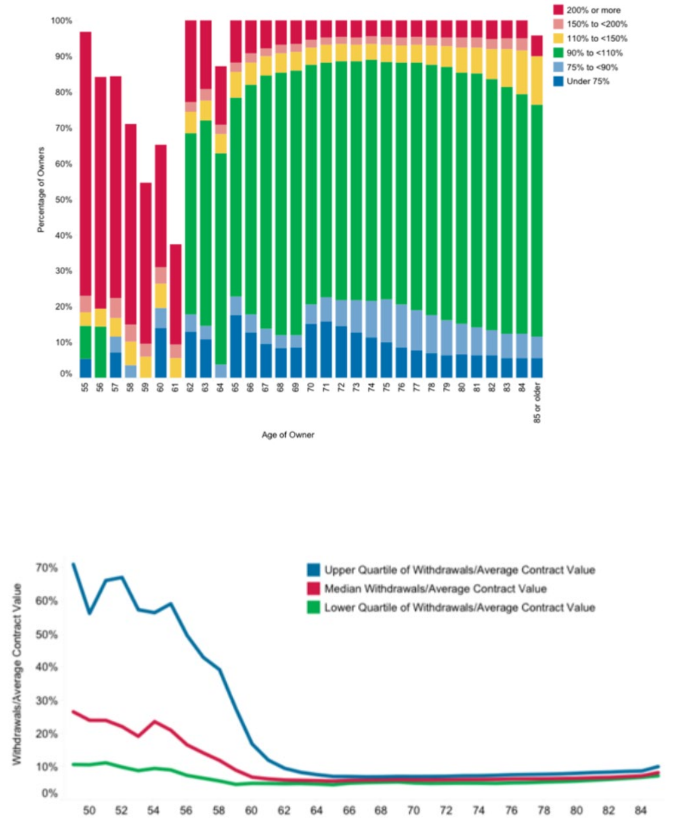
- Before the age of 60: one-off withdrawals of heterogeneous sizes, most often massive (>200% of the annual guaranteed withdrawal) illustrative of the consumption of expensive durable goods, real estate purchases, financing education and vocational trainings, children's studies, management of periods of unemployment

- From 60- to 70-year-old: regular withdrawals, of moderate and homogeneous sizes during retirement, illustrative of the entry into annuity.

- Post 70-year-old: mostly the entry into annuity with regular moderate and homogeneous withdrawals; but also excess withdrawals (for tax reasons or donation).

Figure 4

Withdrawals patterns



Source LIMRA (USA)

This segmentation of customer profiles reflects a strong differentiation of savings behaviours, which calls for an optimization of the product design tailored to its uses for an enriched Customer Experience.

3 Explicit modelling of efficient savings behaviour enhances Customer Experience by better meeting customers' expectations

3.1. Explicit modelling of efficient savings behaviour

3.1.1. Formulation and basic notations

We consider an x-year old policyholder possessing a savings and retirement contract. At inception, an initial endowment is invested in a risky asset S_t. The specifications of the contract include a set of dates 0 = $t_0 < t_1 < t_n < t_N = T$, where $t_0 = 0$ is the contact inception and $t_N = T$ its maturity. These so-called contract anniversaries are the dates in which events can take place, *i.e.* withdrawals, payments, etc...

An analysis of the design of general equity-indexed annuities from the investor's perspective and a generalization of the conventional design is proposed in Boyle and Tian (2008) [2].

3.1.1.1. The contract assumptions

The financial market

The pricing is based on the common pricing literature which assumes the existence of a risk neutral measure $\mathbb{Q}$ under which future cash flows can be valued as their expected discounted values. The existence of such measure implies an arbitrage free financial market.

We assume that the risky asset S_t, which serves as an underlying mutual fund for the variable annuity, follows a Geometric Brownian motion with constant coefficients under $\mathbb{Q}$:

$$\frac{dS_t}{S_t} = rS_t dt + \sigma_t dW_t$$

where σ is the volatility or the risky asset, r the risk-free rate and Wa standard Brownian motion under $\mathbb{Q}$. The money market evolves with risk-free interest rate, and the numeraire process Bt is given by:

$$dB_t = rB_t dt$$

Under the risk neutral probability measure, the discounted asset process $B_t^{-1}\, St$ is a martingale.

The mortality assumption

It is common practice among insurers to use deterministic mortality rate to evaluate and replicate their policy pool. We also make the common assumption that financial markets and biometric events are independent.

Let us introduce the mortality notations as:

- x_0 : the policyholder's age at the contract inception;
- q_n: the probability that the policyholder, aged x_0 at inception, dies between time t_{n-1} and t_n;
- p_n : the probability that the policyholder, aged x_0 at inception, is alive at time t_n;
- ω: the limiting age beyond which survival is impossible. According to the definition, we have $p_n = (1 - q_n)p_{n-1}$, where $n \in \{1,2, \ldots, N\}$.

From the insurer's perspective, the percentage of active contracts in a large policy pool of policyholders aged $x = x_0 + t_n$ at a given time t_n is thus given by p_n

3.1.1.2. The contract state variables

At a given anniversary date t_n, the value of the contract, purchased by an x_0 year old policyholder at inception, is determined by three main state variables: the account value, the benefit base, and the two-states variable determining if he is alive or dead at time t_n.

- Account value A_t: the value of the investment account, which is indexed on the asset value S_t, and reduced by withdrawals and fees.

- Benefit base G_t: also referred as the guaranteed account, is an "imaginary" wealth upon which annuities, guaranteed withdrawals and benefits are calculated. However, if the insured wants to lapse the contract, he will not be able to get this wealth.

- Death process I_n: a two-states variable in {0,1} informing if the policyholder died during (t_{n-1} , t_n], or is still alive at t_n. The death probability in the interval (t_{n-1}, t_n] is given by $q_n = \mathbb{P}(I_n = 0 \mid I_{n-1} = 1)$, which depends on the policyholder's age at inception.

We restrict our analysis to single premium contracts $A_0 = G_0$, i.e., one premium at inception with no additional contributions. The policyholder can either withdraw money or exercise the income benefit. Withdrawals include "zero" withdrawals and completely surrender the contract, *i.e.* lapse.

3.1.1.3. The income and death benefits

At maturity, the holder of the contract can select to take a lump sum of the account value A_{t_n}, or annuitize the benefit base at pre-specified guaranteed annuitization rate. Annuity factors, which give the annuitization rates, denoted by $ä_{tN}^{act}$ for the actual and $ä_{tN}^{gua}$ for the guaranteed, are defined as the price of an annuity paying one dollar each year with either at the market's rates curve, or an internal guaranteed rates defined by the insurer. The calculations of the annuity factors take into account the probability that the insurer is alive in the future with probability p. They are given by:

$$ä_{t_N}^{(.)} = \sum_{t_i=t_N}^{\omega-x_0} p_i \, e^{-r_{t_i}^{(.)}(t_i-t_N)}$$

These factors are increasing, since an older insurer will likely have less annuities than a younger one. Therefore, annuitizing the account value is equivalent to a lump sum, and annuitizing a benefit base G is equivalent to the amount I_{t_n} equal to

$$G\,\frac{ä_{t_N}^{gua}}{ä_{t_N}^{act}}.$$

If the contract also contains a death guarantee, then the D amount is paid to the beneficiaries in the event of the death of the insured during the term of the contract and the cash flow ends. With $\delta=(\delta_1, \delta_2, \dots, \delta_N)$ the death indicator variable and the probability of death $q_n = P(\,\delta_n = 1) = (1 - p_n)$.

Cash flows are recognized at discrete annual interim dates t_n between the signing and expiry of the contract: $0 = t_0 < t_1 < \cdots < t_n < \cdots < t_N = T$.

For the sake of simplicity, we assume that the policyholder can take withdrawals each policy anniversary t_n and denote by γ_n the withdrawals amount. The income benefit also starts at anniversary dates, and, in case of a death benefit, the latter is paid out at these dates as well. Thus, the state variables described above may have discontinuities at times $t_1, \dots t_n$. Therefore, for a state variable Y, we distinguish between its value "Yt_n–" before and "Yt_n+" after events take place at the anniversary date.

3.1.2. Contract state variables dynamics

Development between two policy years (t_{n-1}, t_n]

Assuming that an annual guarantee fee α is continuously charged by the issuer, the value of the account value A_{tn} evolves as:

$$A_{t_n^-} = A_{t_{n-1}^+} \times \frac{S_{t_n}}{S_{t_{n-1}}} exp(-\alpha \Delta t), \quad n = 1,2,\dots,N,$$

where $\Delta t = t_n - t_{n-1}$

In practice, the guaranteed fee is charged discretely and proportional to the account value that can easily be incorporated into the wealth process. Denoting the discretely charged fee with the annual basis as $\bar{\alpha}$, the wealth process becomes:

$$A_{t_n^-} = A_{t_{n-1}^+} \frac{S_{t_{n-1}}}{S_{t_n}} (1 - \bar{\alpha}) \Delta t.$$

The guarantee offered by the contract is denoted G, increasing at a guaranteed annual rate η during the waiting period, and serving as the basis for converting the contract into a life annuity I, associated with pre-defined annuity factors at the signing of the contract.

$$G_{\mathrm{t}+1} = (1 + \eta) G_{\mathrm{t}}$$

The benefit base remains constant between two policy years, *i.e:*

$$G_{t_n^-} = G_{t_{n-1}^+}$$

Transition at a policy year t_n

The contract events take place at the discrete policy years.

On each date t_i the future cash flow f_n can therefore take 3 forms:

- with probability $q_n = (1 - p_n)$: death insurance amount D_n
- with probability p_n : withdrawal $\gamma_n \in [0; A_{t_n}]$ (which reduce A_{t_n} and the guarantee G_{t_n}, in an equal or even way); or exercise the income benefit I_{t_n}:

$$f_n(t_n, A_{t_n}, \mathrm{I}_{t_n}, \mathrm{D}_{t_n}) = \begin{cases} * \ \mathrm{D}_n \ with\ probability\ (1 - p_n) \\ \quad * Max(\gamma_n \in [0; A_{t_N}]; \mathrm{I}_n) \\ \quad\quad with\ probability\ p_n \end{cases}$$

3.1.3. Contract valuation

Traditionally the customers' behaviour has been modelled by historical or backward-looking statistical regressions which have empirically been not enough to anticipate dynamically extrapolate the observed shifts in behaviour and meet the customers risk appetites. In contrast, an "efficient" behaviour strategy valuation is a prudent forward-looking approach where policyholders behave in a way that maximizes the net present value of the future cash-flows, depending on key drivers such as market conditions and product design features. See B. Bouchard, A. Kalife, S. Mouti, X. Tan, and L. Wang (2015) [1] and A. Kalife, S. Mouti, G. López Ruiz, X. Tan (2018) [6].

The stochastic control problem

The customer experience is measured by the value of the retirement savings contract, *i.e.* the average consumption of accumulated savings discounted, is therefore the average of the sum of the future cash flows f_n discounted over the life of the retirement savings contract:

$$\mathrm{V}(t_0, A_0, \mathrm{I}_0) = \mathbb{E}_{t_0}^{\mathbb{Q}}\, \mathbb{E}_{t_0}^{\delta} \left[\sum_{n=1}^{N} e^{-rt_n} f_n\right]$$

Where $\mathbb{E}_{t_0}^{\mathbb{Q}}\, \mathbb{E}_{t_0}^{\delta}$ denotes the expectation is conditional on information available at time t_0, i.e with respect to both the financial risky asset process under $\mathbb{Q}$, and the mortality process under the real probability measure δ.

Noting $f = (f_1, f_2, \dots, f_N)$ the savings strategy, it is considered that the empirical profiles correspond to an optimal strategy that maximizes customer satisfaction:

$$f^* = \mathrm{argsup}_f \mathrm{V}(t_0, A_0, \mathrm{I}_0)$$

This falls within the framework of standard optimal stochastic control problems for a controlled Markov process, whose numerical resolution is carried out in a retrograde manner starting from the terminal condition of date N. Finding the contract value $V(t_n, A_n, \mathrm{I}_n)$ at time t_n is done via a backward Bellman equation.

Since the account value A evolves between two anniversary dates, whereas the benefit base is a constant piecewise function (*i.e.* changes at anniversary dates only), the value is driven by a PDE with jump conditions at each withdrawing date to link the prices at the adjacent periods: the required backward recursion is thus written between t_{n+1}^- and t_n^+ as:

$$\begin{aligned} V(t_n^+, A, G) &= \mathbb{E}^{\mathbb{I}}[B_{n,n+1}(1_{I_{n+1}=1} \times \mathbb{E}_{t_n^+}^{\mathbb{Q}}[V(t_{n+1}^-, A_{t_{n+1}^-}, G_{t_{n+1}^-}) \mid A, G] \\ &\quad + 1_{I_{n+1}=0} \times \mathbb{E}_{t_n^+}^{\mathbb{Q}}[D(t_{n+1}^-, A_{t_{n+1}^-}, G_{t_{n+1}^-}) \mid A, G])] \\ &= (1 - q_{n+1})\mathbb{E}_{t_n^+}^{\mathbb{Q}}[V(t_{n+1}^-, A_{t_{n+1}^-}, G_{t_{n+1}^-}) \mid A, G] + q_{n+1}\mathbb{E}_{t_n^+}^{\mathbb{Q}}[D(t_{n+1}^-, A_{t_{n+1}^-}, G_{t_{n+1}^-}) \mid A, G] \end{aligned}$$

With jump condition:

$$V(t_n^-, A, G) = max_{\gamma_n \in \mathcal{A}_n} \left(\bar{f}_n(A, G, \gamma_n) + V\left(t_n^+, h^A(A, G, \gamma_n), h^G(A, G, \gamma_n)\right)\right)$$

Numerical scheme

The algorithm starts from a final condition for the contract value at t_N^-. Subsequently, solving the PDE gives solution for the contract value at t_{n-1}^+.

The PDE used to calculate the expected value under the assumed risk-neutral process for the risky asset St is easily derived using Feynman-Kac theorem.

When the risky asset follows a geometric Brownian motion process, the governing PDE right after a withdrawal decision t_n^+ to right before the following one t_{n+1}^- for $n = N - 1, N - 2, \dots, 0$ is expressed as the follows

$$\partial_t \Phi + \frac{1}{2}\sigma^2 A^2 \partial_{AA}\Phi + (r - \alpha^A)A_t \partial_A \Phi - \alpha^G G_{t_n^+} \partial_A \Phi - r\Phi = 0$$

to which we add boundary conditions defined in the next section. It is is solved using the Crank-Nicolson finite differences methods. See (Dai, Kuen Kwok, and Zong (2008) [3], Huang, Forsyth, and Labahn (2012) [5].

Note again that the benefit base changes only at the anniversary dates and is a constant parameter between two anniversary dates. Applying the jump condition to the solution at t_{N-1}^+ , we obtain the solution at t_{N-1}^- from which further backward time stepping gives us solution at t_{n-2}^+, and so on. The numerical algorithm takes the following key steps:

1. Generate a finite grid for the account value A and benefit base G, i.e. $A_0 < A_1 < \dots < A_J$ and $0 = G_0 < G_1 < \dots < G_K$.

2. At t_N, define the final condition for each note point (A_j, G_K), j = 1, 2,..., J and k = 1, 2,...,K to get Φ (t_n^- ,A,G) and the boundary conditions for Amin and Amax for each potential $G_{k\in\{1,2,\dots,K\}}$.

3. For each potential benefit G_k, k = 1, 2,...,K, solve the PDE using the Crank-Nicolson finite differences scheme to obtain $\Phi(t_{N-1}^+$, A,G).

4. Apply the jump condition to obtain Φ (t_{N-1}^-) for all the values of γ_{N-1} and find the withdrawal amount γ_{N-1}^* that minimizes $\Phi(t_{N-1}^-$,A,G). In general, this involves a two-dimensional interpolation in (A,G).

5. Repeat (3) and (4) for t = t_{N-2},t_{N-3},...,t_1.

6. Evaluate the PDE for the backward time step t_1 to t_0 to obtain solution $\Phi(t_0$, A,G) at A_0 and G_0

Localization and boundary conditions

Within each time interval (t_{n-1}, t_n), only the account value varies since all the benefit bases, death and life, remain constant. Thus, for t ∈ ($t_n^\mp$, t_{n+1}^-], the annuity value Φ(t, A,G) solves the following linear PDE for each fixed value of the benefit base G

$$\partial_t \Phi + \mathscr{L}\Phi = 0$$

$$\mathscr{L}\Phi = \frac{1}{2}\sigma^2 A^2 \partial_{\Lambda\Lambda}\Phi + (r - \alpha^A)A\partial_\Lambda \Phi - \alpha^G G \partial_\Lambda \Phi - r\Phi$$

This equation is originally posed on the domain (t, A) ∈ [0,T] × [0,∞). For computational purposes, and because asset prices are finite and so is the account value, one needs to localize this domain to [0,T]×[0,

A_{max}] where A_{max} is large enough not to be attained by the account value during the lifetime of the annuity. Thus, we need to add complementary boundary conditions. We consider that we are between two anniversary dates $t_n^{\mp}$ and t_{n+1}^{-} backwards.

- When A = 0, the policyholder has no longer the possibility to make any withdrawal from his account. However, if the IB election is possible, then the income period begins, given the policyholder is alive, and the death benefit is activated if he is dead at t_{n+1}. Since the account value is equal to zero, then the annuitization will be indexed on the benefit base. Therefore, we have :

$$\Phi(t,0,\mathrm{G}) = e^{-r(t_{n+1}-t)}\left(p_{n+1}\frac{\ddot{a}^{gua}_{t_{n+1}}}{\ddot{a}^{act}_{t_{n+1}}}\mathrm{G} + p_n q_{n+1}\mathrm{D}(t_{n+1}^{-},0,\mathrm{G})\right)$$

- When A = $A_{max,}$ we consider retrieving all the cash more interesting than any other strategy if the policyholder is alive. If he dies, the death benefit will be activated. Therefore, the Dirichlet boundary condition for this case is

$$\Phi(t,\mathrm{A_{max}},\mathrm{G}) = e^{-r(t_{n+1}-t)}\left(p_{n+1}\mathrm{A_{max}} + p_n q_{n+1}\mathrm{A_{max}}\right) = e^{-r(t_{n+1}-t)}p_n\mathrm{A_{max}}.$$

Let us define the solution domains

$$\bar{\Omega}_n = [t_{n-1}^{+}, t_n^{-}] \times [0, \mathrm{A_{max}}]$$
$$\bar{\Omega} = \bigcup_{t_n}[t_{n-1}^{+}, t_n^{-}] \times [0, \mathrm{A_{max}}].$$

Construction of the scheme

Let (A_0, A_1,..., A_J) be the equally spaced grid in the direction of the account value with $A_0 = 0$ and $A_J = A_{max}$. Analogously (G_0,...,G_K) is an equally spaced grid for the benefit base with $G_0 = 0$ and $G_K = G_{max} = A_{max}$. The spacial steps for both variables are considered to be equal. That is :

$$\Delta\mathrm{A} = \Delta\mathrm{G}, \quad \text{where } \Delta\mathrm{A} = \frac{\mathrm{A_{max}}-\mathrm{A_0}}{\mathrm{J}} \text{ and } \Delta\mathrm{G} = \frac{\mathrm{G_{max}}-\mathrm{G_0}}{\mathrm{K}}$$

Hence, $A_j = j\Delta A$ and $G_k = k\Delta G$, $\forall j,k$. The discrete time steps are denoted by $n\Delta t$ for n = 1,...,N where $T = N\Delta t$. Since, in our analysis, we consider that events occur only at anniversary dates which are yearly, $\Delta t = 1$ and each time t_n coincides with the discrete time step $t_n = n$.

The numerical procedure to solve the approximation is the standard finite difference approach, using the general theta-scheme (the Crank-Nicolson scheme). We employ the two-level implicit finite difference scheme to discretize.

Recall that changes in the benefit base only occur at withdrawal dates. After withdrawing the amount γ_n at time t_n, the account value changes from At_n^{-} to At_n^{+}, and the benefit base drops from Gt_n^{-} to Gt_n^{+}.

The application of the jump condition decreases the account value and benefit base. For each G_j, a continuous solution from the PDE is associated. We can restrict the possible values for the withdrawal amount to multiples of ΔA. This implies, for a given account value A_j at time t_n^{-}, the withdrawal amount γ takes j possible values: $\gamma = A_j - A_i$, i = 1, 2,..., j. However, numerical tests showed that a finer grid is preferable for the withdrawal amount. Therefore, it is not guaranteed that the account value, nor the benefit base after the withdrawal, At_n^- and Gt_n^+, fall within their respective grid nodes. To solve this issue, a two-dimensional interpolation is required.

3.2. Enhancing Customer Experience through product features that meet customer expectations

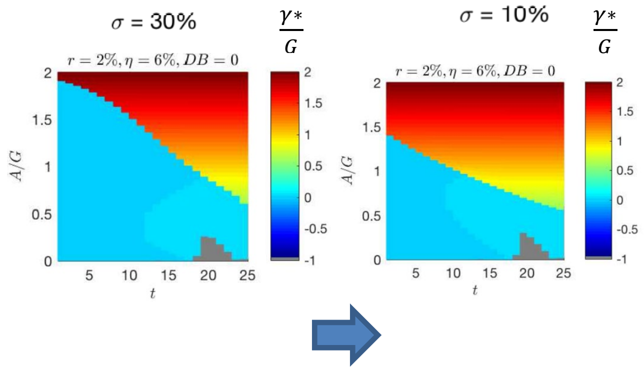
The introduction of certain product features leads to savings consumption behaviours in line with the observed patterns, for an enhanced Customer Experience.

Behaviours take the form of (i) withdrawals $\frac{\gamma*}{G} > 0$ (expressed as % of guarantee G) or (ii) activation of the life annuity ($\frac{\gamma*}{G} = -1$) in the chart below, depending on the past duration t and the ratio "A/G" (investment account A divided by guarantee G).

1/ The choice of less risky S investment assets (illustrated by a lower volatility - 10% *vs.* 30% in the chart) is in favour of earlier withdrawals, thus moving the behavior towards the empirical pattern observed for those below 60 years.

Figure 5

Impact of Asset Volatility

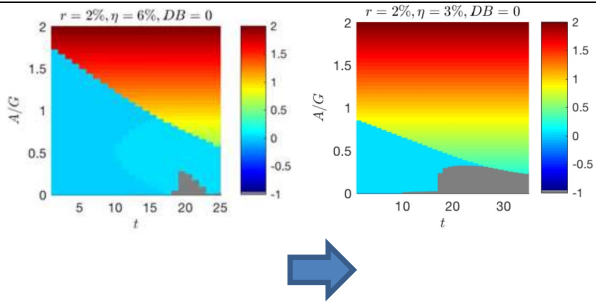


2/ The introduction of a dynamic roll-up rate η for the guarantee G (*i.e.* decreasing as interest rates fall) on the one hand reduces the disparity in withdrawal sizes, on the other hand promotes a more frequent activation of the life annuity, thus moving the behaviour towards the empirical pattern of retirees aged from 60 to 70 years.

Figure 6

Impact of a variable roll-up guarantee rate

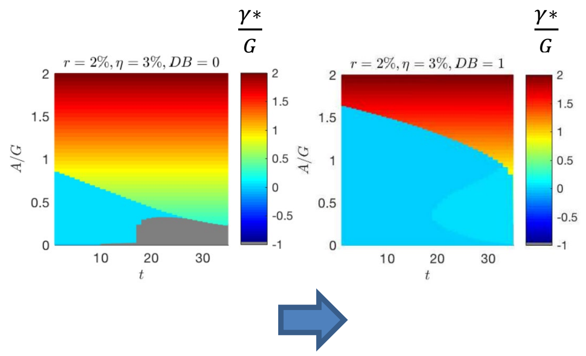
$\frac{\gamma*}{G}$ $\frac{\gamma*}{G}$



3/ The introduction of a death guarantee on the one hand promotes the smaller size of withdrawals, on the other hand postpone the withdrawals and the activation of the life annuity, thus moving the behaviour towards the empirical pattern of retirees over 70 years.

Figure 7

Impact of a death guarantee

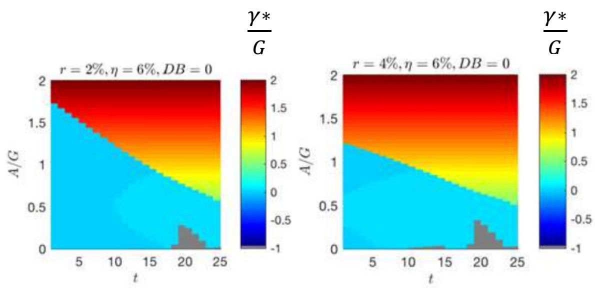


Savings behaviours depend as expected on the economic environment: a low investment account and/or a high guarantee provide an incentive to reduce the frequency and size of withdrawals, and to activate the life annuity earlier.

Similarly, the effectiveness of product design changes is influenced by the economic environment: withdrawals increase in frequency and size with interest rates and the level of investment assets.

Figure 8

Impact of a death guarantee



Conclusion

Longer life expectancy, increasing lifestyle and health care costs, and the persistence of low interest rates have contributed to the emergence of increasingly differentiated savings behaviour patterns.

The explicit quantitative modelling of efficient savings behaviour makes it possible to select the product features that meet customers ‘needs and expectations, in line with their empirical savings behaviour patterns, for an enhanced Customer Experience.

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