Solvency II and insurers' equity holdings: A quantitative assessment

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1 Introduction

The Solvency II framework, which constitutes the body of European prudential rules for the insurance sector, entered into force on January 1, 2016, after several years of negotiations. The directive, published in 2009, provides a review clause that requires the European Commission to propose amendments to the co-legislators (Council of the EU, made up of representatives of the Member States and the European Parliament) before January 1, 2021. This exercise has already started with the Commission's request for a technical opinion to the European Insurance Supervisory Authority (EIOPA) in February 2019 on various subjects, provided for by the review clause or that had been deemed a priority. EIOPA published on December 17, 2020, its technical opinion to the European Commission on the Solvency 2 prudential regime review. The European Commission must now work on drafting its legislative proposals expected for the third quarter of 2021.

This request for an opinion includes, in particular, a point on the prudential treatment of long-term investments by insurers, in connection with the European Commission's Capital Markets Union (CMU) project, one of the objectives of which is to reduce the dependence of businesses on the banking system, chiefly concerning long-term investments.

This note proposes to quantify the possible effect of the prudential treatment of equities on the share that equities represent in the balance sheet of insurers. Available figures suggest that the entry into force of Solvency II could have encouraged insurers to reduce their exposure to equities due to the prudential treatment of this type of asset. Our study sheds light on the calibration of the prudential model—without prejudging the positive or negative effect of equity financing on the European economy.

To quantify the hypothetical impact of Solvency 2 on life insurers' investment policies, we study the optimal portfolio choice problem of a long-term investor. Our econometric approach uses historical asset returns to calibrate a portfolio choice model to the observed investment policies of life insurers. Our model includes an approximation of the regulatory constraints, which lets us measure the effect of these constraints by calculating a counterfactual allocation in the absence of regulatory constraints.

This work is a continuation of the Long Term Asset Allocation (AALT) research program. As a reminder, the AALT program was interested in the choice of a long-term investor's portfolio, who determines his allocation not only based on the expected performance of equities and other asset classes but also on their ability to hedge liabilities (Penasse and Poignard, 2018). This econometric approach makes it possible to take into account the long-term performance of equities, particularly the fact that the risk of equities decreases with the investment horizon (Campbell and Viceira, 2002; Hoevenaars et al., 2008; van Binsbergen and Brandt, 2015; Penasse, 2015), while accounting for, in a simplified manner, the regulatory cost of capital resulting from the Solvency 2 regulatory framework.

Our work differs from the previous literature by taking a more detailed account of insurance companies' investment environment, the asset-liability interactions resulting from profit sharing, and the constraint exerted by the Solvency II prudential framework. First, we offer a simpler yet more realistic model of insurance liabilities, focusing on life insurers. Our approach takes into account not only the remuneration profile of liabilities imposed by the French insurance law but also its amortization profile. Second, our approach to capital requirements approximates the relevant factors driving regulatory capital. These factors include the rules that determine the Solvency Capital Ratio (SCR), which is based on a shock amortized by the capacity to absorb losses through deferred taxes and technical provisions. However, our modeling remains very simplified with regard to the existing regulation. Still, it presents the advantage of agreeing with the stylized facts characterizing the returns on long-term assets.

Our results show a marked effect of regulatory constraints, which increases with the horizon. Over a 10-year horizon, the unconstrained equity allocation is approximately 27%, while the constrained allocation is around 12%, a drop of 15 points, which is more than half of the unconstrained equity allocation.

2 Long-term asset allocation

2.1 Optimal allocation of an unregulated insurer

We take the perspective of an investor seeking to maximize the utility associated with her long-term asset-liability ratio. The investor potentially has access to several assets and maximizes at time t her expected utility over a $t+H$ horizon. We assume preferences that are common in the literature, namely a CRRA (constant relative risk aversion) utility function:

$$\max_{\alpha_t} \mathrm{E}_t \frac{F_{t+H}^{1-\gamma}}{1-\gamma}, \tag{1}$$

where $\gamma > 1$ is the risk aversion coefficient and $F_{t+H} = \frac{A_{t+H}}{L_{t+H}}$ the asset-liability ratio (Leibowitz et al., 1994). The asset at horizon $t+H$ is given by

$$A_{t+H} = 1 + R_{A,t+H},$$

with $R_{A,t+H} = R'_{t+H}\alpha_t$; R_{t+H} is a vector $(K+1)\times 1$ of returns and α_t a vector $(K+1)\times 1$ of weight. Note that we focus on a static allocation problem, in the sense that the weights are fixed over the periods between $t+1$ and $t+H$, as in Hoevenaars et al. (2008). In this note, we assume that the investor has access to the following assets: money market, stocks, real estate, reinvested long bonds, and a risk-free asset that corresponds to a bond with a maturity of $t + H$. Note also that we obtain the classic portfolio choice problem when $L_{t+H} = 1$.

Adding a liability creates a hedging demand, which depends on the correlation between assets and liabilities. The liability is given by $L_{t+H} = 1 + R_{L,t+H} \times Z_{t+H}$ with $R_{L,t+H}$ the return on the liability. The term Z_{t+H} allows for the liability to amortize; this is the amortization rate at maturity. When $Z_{t+H} = 1$, the liability is repaid at maturity. Instead, we assume that Z_{t+H} is a random variable of distribution,

$$Z_{t+H} = \exp\left(-\sum_{i=t}^{t+H} |z_t|\right),$$

where the annual amortization rate $|z_t|$ follows a folded normal distribution with parameters $\mu_z, \sigma_z : z_t \sim N(\mu_z, \sigma_z)$. The distribution is approximately normal for $\mu_z \gg 0$, and guarantees that the amortization has support $]0, 1[$.

We assume that the return on the liability is 85% of the positive return on the asset: $R_{L,t+H} = \max(0; 0.85 \times R_{A,t+H})$. This assumption reflects the redistribution rule, which in French law requires insurers to pay back to life insurance subscribers in euros 85% of the financial profits made thanks to their savings. Regarding the liability amortization distribution, we assume that $\mu_z = 4\%$ and $\sigma_z = 1\%$. This assumption reflects the relative stability of the liabilities of life insurers. Available estimates indicate that the annual depreciation is of the order of 2% in normal times and 4% in stressed times.

In the absence of liability amortization, it is possible under certain assumptions to approximate the solution of the long-term investor problem (1) (Campbell and Viceira, 2002; Hoevenaars et al., 2008). In our case, however, we solve the problem numerically by Monte Carlo. We discuss the dynamics of the returns $R_{A,t}$ below, but note already that we simulate 100,000 trajectories of the log-returns $r_{A,t}$, which allows us to evaluate the expected utility for each vector of weight α_t. For the sake of realism, we limit positions to 100% (20% for real estate) and prohibit short selling.

2.2 Regulatory constraints

Solvency II rules require the insurer to dedicate capital to limit the market risk of its portfolio. To assess the impact of Solvency 2, we assume the investor earns a return on her portfolio net of its regulatory charge. We assume this regulatory charge equals 6%. More specifically, we assume the log portfolio return, corrected for the cost of regulatory capital is equal to:

$$\tilde{r}_{A,t} = r_{A,t} - K^{reg}(\alpha_t)\log(1 + 6\%). \tag{2}$$

The regulatory capital asociated to the portfolio consists of two terms: $K^{reg}(\alpha_t) = SCR_t(\alpha_t) + VIF_t(\alpha_t)$.

The first term $SCR_t(\alpha_t)$ corresponds to the regulatory capital itself, calculated from given shocks for each asset class. This dedicated capital depends on each asset class as well as the portfolio's, through a correlation matrix between the asset classes. Let SCR_i denote the capital weight required for asset class i. The capital charge depends on the weight of the portfolio allocation. For example, if equities are subject to a 40% shock, the equity SCR will equal 20% $\times$ 40% if the portfolio weight of equities is 20%. The portfolio's capital charge then equal to

$$SCR(\alpha) = \sqrt{\sum_{(i,j)} \rho_{i,j}^{reg} \times SCR_i \times SCR_j}, \tag{3}$$

where $\rho_{i,j}^{reg}$ is the "regulatory" correlation between the active classes i and j.

We have retained the following shocks:

- Equities: 44% $\times$ 0.87 $\times$ 0.43;
- Real Estate: 25% $\times$ 0.87 $\times$ 0.43;
- Bonds: 3% $\times$ 0.87 $\times$ 0.43;
- Money markets: 0%.

The equity shock assumes a position composed of half of listed shares and half of unlisted shares. The above calculation takes into account the effect of deferred taxes (13 %) and the capacity to absorb losses through technical provisions (57 %) (source: ACPR calculations).

Regulatory correlations are given by Table 1.

Table 1: Solvency 2 regulatory correlations

	Rates	Equities	Real Estate	Bonds
Rate	1	0.5	0.5	0.5
Action	0.5	1	0.75	0.75
Real Estate	0.5	0.75	1	0.5
Bond	0.5	0.75	0.5	1

The second term corresponds to the *Value of in-force* (VIF).

$$VIF(\alpha) = \sum_i \alpha_i \times VIF_i, \tag{4}$$

where VIF_i is equal to

- Actions: 19 %;
- Real estate: 15 %;
- Bonds: -2.5 %;
- Short roll rate: 0%.

(Source: CA Assurances).

This expression approximates the marginal effect on a life insurer's VIF of an allocation shift in favor of one of the above asset classes, with a negative sign. The VIF is an element of Solvency 2 equity that finances the capital requirement (SCR). As equity holdings negatively impact equity liabilities by 19% of the outstanding amount held compared to the holdings of monetary instruments, we treat this equity shortfall as a capital requirement.

3 Econometric model

3.1 VAR model

We now present our econometric model of returns. A large literature documents the ability of several variables to predict future returns. Our model allows us to model the returns as well as these predictor variables jointly. As we illustrate below, a vector autoregressive of order 1–VAR (1)–captures predictive relationships between future returns and past returns and other predictors. This model follows the literature closely, see, for example, Campbell and Viceira (2002):

$$X_t - \bar{X} = \Phi(X_{t-1} - \bar{X}) + \epsilon_t, \;\; \epsilon_t \sim \mathcal{N}(0, \Sigma), \tag{5}$$

where (X_t) is the vector of asset returns (money market, equities, real estate, reinvested long bonds) as well as several predictor variables (dividend-price and rent-price ratios and the 10-year rate). The vector (ϵ_t) is the error term, the distribution of which is assumed to be Gaussian, centered, covariance matrix Σ. Thus, returns in t are predicted by a linear relationship of returns and other past predictors. The vector $\bar{X}$, and the matrices Φ and Σ are the key parameters of this model.

As simple as it is, this model allows us to take into account the dynamics of reversion to the average of assets. In order to illustrate how short-term relationships allow us to calculate long-term relationships, consider this simpler case of our VAR model given by

$$\begin{aligned} r_{t+1} &= \alpha + \beta z_t + u_{t+1}, & (6) \\ z_{t+1} &= \phi z_t + v_t, & (7) \end{aligned}$$

where r is an asset return, for example an index corresponding to an investment in equities, z is a predictor, ϕ is the predictor's autocorrelation coefficient, $\beta < 1$ the predictive coefficient. The predictor follows a stationary AR (1) process, i.e. $0 < \phi < 1$. We let the errors u and v be correlated, usually with a negative correlation, $\sigma_{r,z} < 0$. The variance for two periods is written as

$$\frac{1}{2}\mathrm{Var}_t(r_{t+1} + r_{t+2}) = \frac{1}{2}\mathrm{Var}_t(r_{t+1}) + \frac{1}{2}\mathrm{Var}_t(r_{t+2}) + \mathrm{Cov}_t(r_{t+1}, r_{t+2}) \tag{8}$$

$$= \underbrace{\sigma_r}_{\text{Variance 1 period}} + \overbrace{\frac{1}{2}\beta^2\sigma_z^2}^{>0} + \overbrace{\beta\sigma_{r,z}}^{\text{Mean reversion?}} \tag{9}$$

The variance is made up of three terms. The first is the volatility associated with a single period σ_r. In the absence of predictability, $\beta = 0$ and the volatility is equal to σ_r whatever the investment horizon. The second term corresponds to the impact of the predictor's volatility on the asset's volatility. Its impact is always positive but tends to decrease rapidly with the horizon (the coefficients ϕ and β being less than 1). Finally, the last term depends on the correlation between the innovations u and v and is strongly negative: this is the mean reversion effect.

3.2 Data and econometric approach

The estimation of long-term econometric relationships is not straightforward. By definition, we are interested in estimating low-frequency price movements, which ideally requires very long and reliable data. Estimating low-frequency movements also requires the underlying econometric relationships to be stable over the long term. In practice, these conditions are rarely met. A partial solution to this problem is to use data for many countries, limiting estimation risk and producing coefficients that are more stable over time than coefficients estimated on a country-by-country basis. This is the approach we adopted for estimating the Φ and Σ matrices, which determine the persistence and the variance-covariances of the returns. Long-term returns (the vector $\bar{X}$) are notoriously difficult to estimate Merton (1980). We have decided to calibrate these parameters, in particular, to preserve the hierarchy of returns between asset classes. The remainder of this section provides details of our approach and presents our estimation results and calibration.

Our financial series consist of short-term interest rates, equity returns, real estate returns, and bond returns for sixteen developed countries, as well as their predictor variables (dividend-price and rent-to-price ratios and the 10-year rate). These data are provided by Jordà et al. (2019) and have been available since 1870, but we choose to focus on post-war data. We present in Table 2 the average returns, volatilities, and Sharpe ratios for each of the elements of our VAR as well as the product of our calibration.

Except for real estate, we recognize the usual hierarchy of average returns between equities, bonds, and monetary instruments. Real estate investments' long-term performance comes as a surprise, especially as real estate returns appear to be significantly less volatile than equity returns. However, calculating any long-term returns is an uncertain exercise, especially when it comes to real estate returns. Long-term real estate returns are indeed still poorly documented. Jordà et al. (2019) claims that these returns are very attractive over the long term, but these estimates are new and remain controversial. Besides, property returns are sensitive to the assumptions about depreciation rates and the geographic area used. Note also that these returns concern retail, residential real estate and may not represent the investable returns from an institutional investor viewpoint. Finally, another concern is that real estate indices tend to be too smooth (e.g., Brounen and Eichholtz 2003). Real estate returns are also too persistent, with autocorrelations likely overstating reality. This overestimation of persistence also leads to an underestimation of volatility. Rather than correcting the persistence of our real estate series, we prefer to adjust its long-term trend.

Table 2: Yields: statistics and calibration

	Données (1945-2016)			Calibration		
	μ	σ	Sharpe	μ	σ	Sharpe
$r_{ct,t}$	5.10	3.61	–	0.50	3.61	–
$r_{e,t}$	12.00	21.56	0.32	4.50	21.56	0.19
$r_{h,t}$	11.54	8.42	0.77	3.50	8.42	0.36
$r_{b,t}$	6.83	8.47	0.20	1.50	6.78	0.15

This table shows the average returns (μ), volatility (σ) as well as the Sharpe ratio for money returns ($r_{ct,t}$), stocks ($r_{e,t}$), real estate ($r_{h,t}$) and bonds ($r_{b,t}$).

Our calibration assumes a low long-term inflation rate and, therefore, a lower short-term rate than over the available period. We adjust average returns on stocks, real estate, and bonds to reflect the common risk hierarchy between asset classes. As the volatility of bond yields is relatively high, we have reduced it by 20% to reflect a duration profile that we assume to be more conservative, which allows us to preserve a bond Sharpe ratio that we believe is reasonable. The real estate Sharpe ratio remains relatively high, which can be motivated by a liquidity premium associated with real estate investments.

We also calibrate the long-term yield curve, which determines the risk-free rate associated with each investment horizon. We assume that the 1-year rate is 0.5 % and that the rates increase linearly to 1 % for the 20-year term.

Finally, the matrices Φ and Σ are estimated using ordinary least squares by imposing the homogeneity assumption that the matrices are the same for the sixteen countries considered. We present the results of the estimation of the VAR model in Table 3. These coefficients are consistent with the literature but are difficult to interpret individually. On the other hand, we can use the Φ and Σ matrices to calculate the volatilities according to the investment horizon, which are easily interpretable.

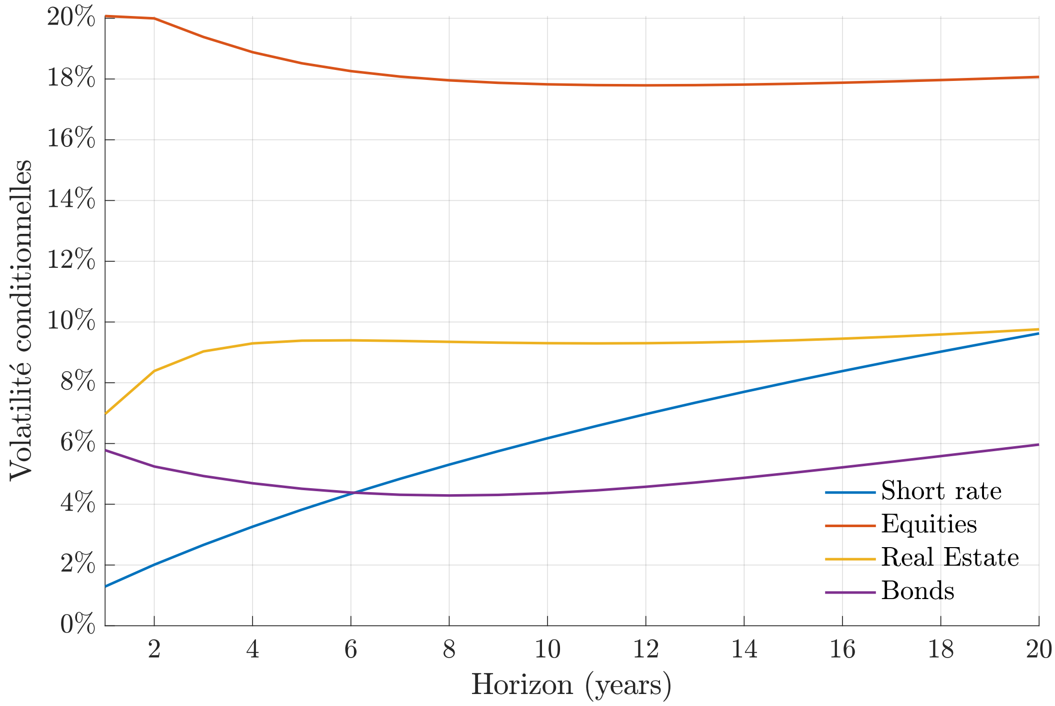
Table 3: VAR estimate

	$r_{ct,t+1}$	$r_{e,t+1}$	$r_{h,t+1}$	$r_{b,t+1}$	$y_{10,t+1}$	dp_{t+1}	rp_{t+1}
$r_{ct,t}$	0.81^{***}	-0.69	-0.23	-0.25	0.11^{**}	-0.28	-0.05
(s.e.)	(0.04)	(0.60)	(0.17)	(0.22)	(0.05)	(0.54)	(0.20)
$r_{e,t}$	0.02^{***}	0.00	0.05^{***}	-0.04	0.01^{***}	-0.23^{***}	-0.05^{***}
(s.e.)	(0.00)	(0.06)	(0.01)	(0.02)	(0.00)	(0.06)	(0.01)
$r_{h,t}$	0.03^{***}	-0.28^{***}	0.41^{***}	-0.19^{***}	0.03^{***}	0.43^{***}	-0.25^{***}
(s.e.)	(0.01)	(0.10)	(0.06)	(0.05)	(0.01)	(0.14)	(0.06)
$r_{b,t}$	-0.05^{***}	0.54^{***}	-0.04	-0.18^{**}	-0.04^{***}	-0.44^{***}	-0.09^{**}
(s.e.)	(0.01)	(0.16)	(0.04)	(0.08)	(0.01)	(0.14)	(0.03)
$y_{10,t}$	0.14^{***}	1.07	0.35^{*}	1.60^{***}	0.88^{***}	0.35	0.45^{**}
(s.e.)	(0.04)	(0.66)	(0.19)	(0.21)	(0.06)	(0.55)	(0.19)
dp_t	0.00	0.10^{***}	-0.01	-0.01	0.00	0.75^{***}	0.01^{**}
(s.e.)	(0.00)	(0.03)	(0.01)	(0.01)	(0.00)	(0.03)	(0.01)
rp_t	0.00	0.02	0.07^{***}	-0.02	0.00	-0.01	0.94^{***}
(s.e.)	(0.00)	(0.05)	(0.01)	(0.01)	(0.00)	(0.04)	(0.01)
R^2	0.872	0.133	0.314	0.273	0.932	0.739	0.955
N	976	976	976	976	976	976	976

$^{***}\ p < 0.01$, $^{**}\ p < 0.05$, $^{*}\ p < 0.1$.

Figure 1 shows the term structure of volatility for our four asset classes. We observe, in agreement with the literature, that equity volatility decreases with the horizon. This decrease is, in absolute terms, relatively modest. For a one-year horizon, the annualized volatility is 20%, while for a 20-year horizon, it is approximately 18%. On the contrary, we observe that the monetary and real estate volatilities increase sharply with the horizon, while bond volatility describes a slight U-shaped curve.

Figure 1: Term structure of volatilities



4 Results

We present the asset allocation results with and without Solvency 2 constraints in Figures 2 and 3. To compute these allocations, we initiate the predictive variables in our VAR model to their long-term values for simplicity. We assume a risk aversion coefficient $\gamma = 7$, which allows us to approximate the weight of obligations (approximately 70 %) that we typically observe among life insurers.

In the absence of regulatory constraints, we observe a relatively equal and stable distribution between bonds and equities and real estate (Graph 2). Depending on the horizon, the investor substitutes bonds held to maturity (the risk-free asset, in green) for risky reinvested bonds (purple). In contrast, allocations between bonds and equities and real estate are stable for horizons exceeding five years.

This first graph gives us an idea, through the prism of our model, of what an insurer's allocation would be in the absence of regulatory constraints. Figure 3 shows the allocations when the Solvency 2 constraints are in place. The graph 4 shows the difference between the two allocations, asset class by asset class. Allocations to non-bond assets fall significantly, which is unsurprising since Solvency 2 penalizes risky assets more. We observe a relative steepening of equity and real estate allocations. For relatively short horizons, we also notice an increase in the monetary allocation, to the detriment of risk-free bonds, a logical consequence of the non-penalization of monetary instruments by the regulator.

Figure 2: Asset allocation (without Solvency 2 constraint)

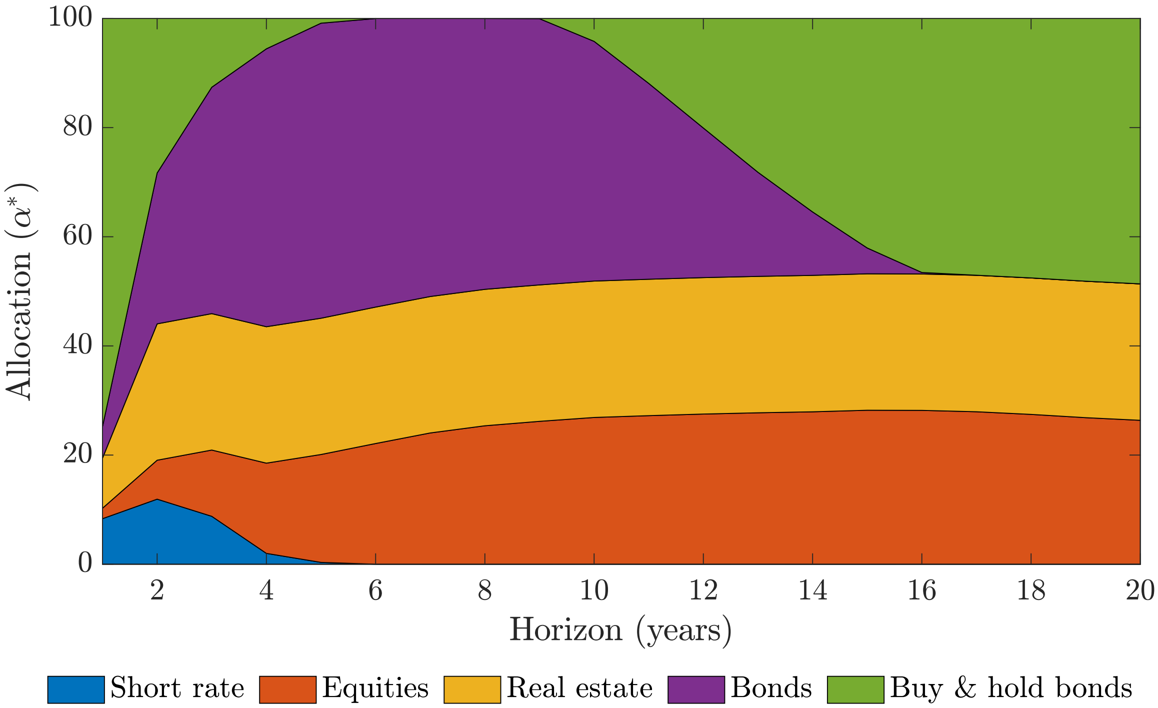


Figure 3: Asset allocation (with Solvency 2 constraint)

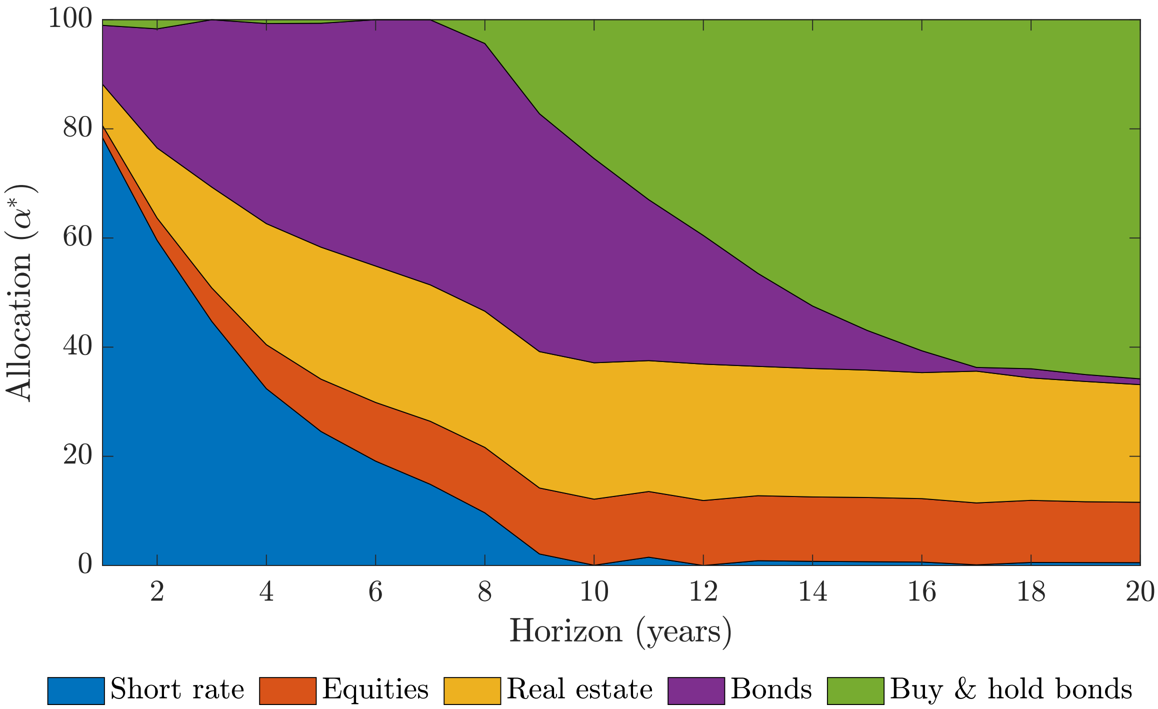
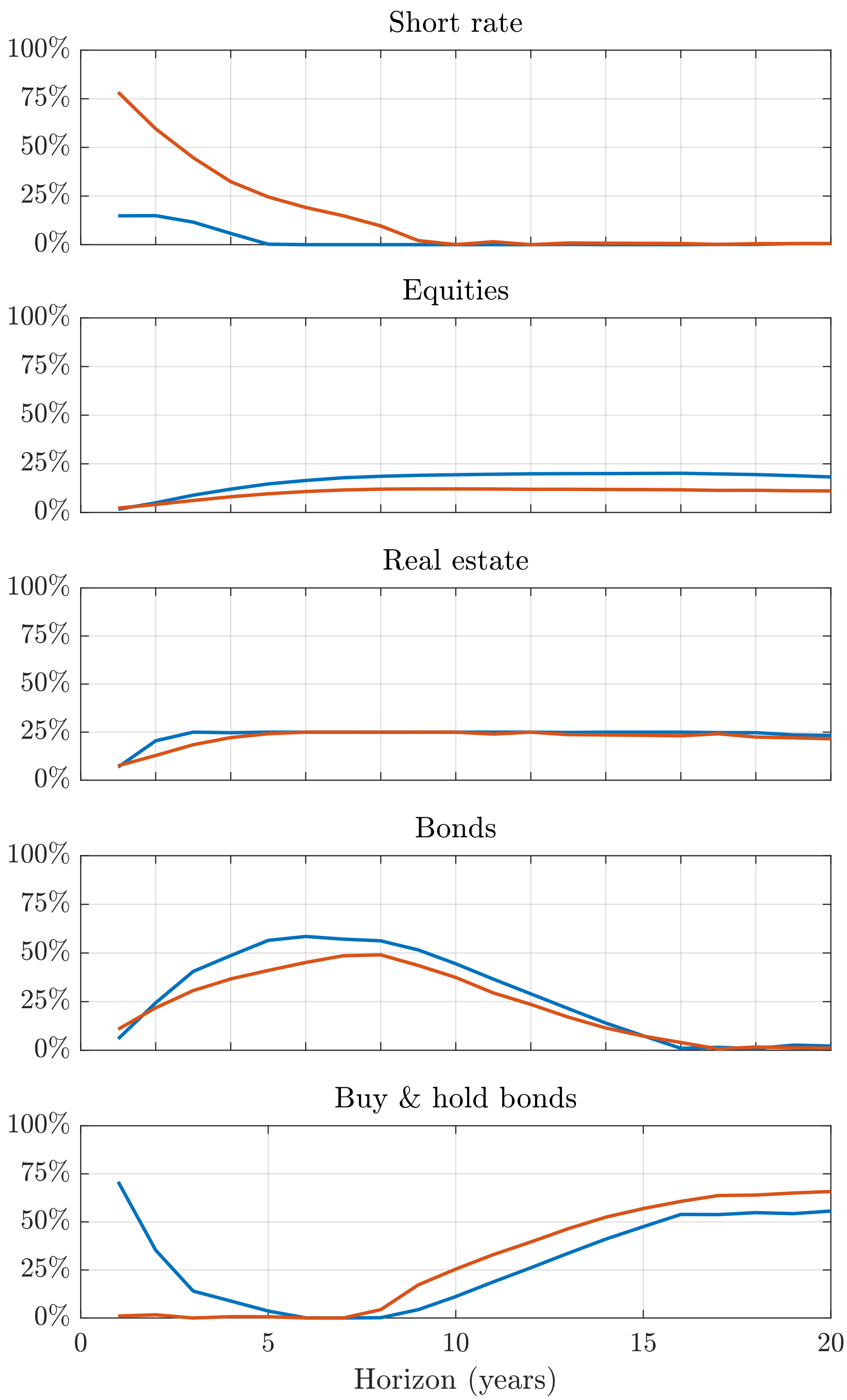


Figure 5 focuses on the impact of SCR and VIF policies on equity allocation. To construct the figure in Panel A, we start from the unconstrained allocation and impose only the cost of regulatory capital imposed by the SCRs. For the figure in Panel B, we take into account the effect on Solvency 2 capital (VIF), in addition to the effect of the SCR. Finally, the figure in Panel C reflects the cumulative Solvency 2 effects linked to the capital requirement and equity. It, therefore, corresponds to the differences between the equity allocations represented on the graph 4.

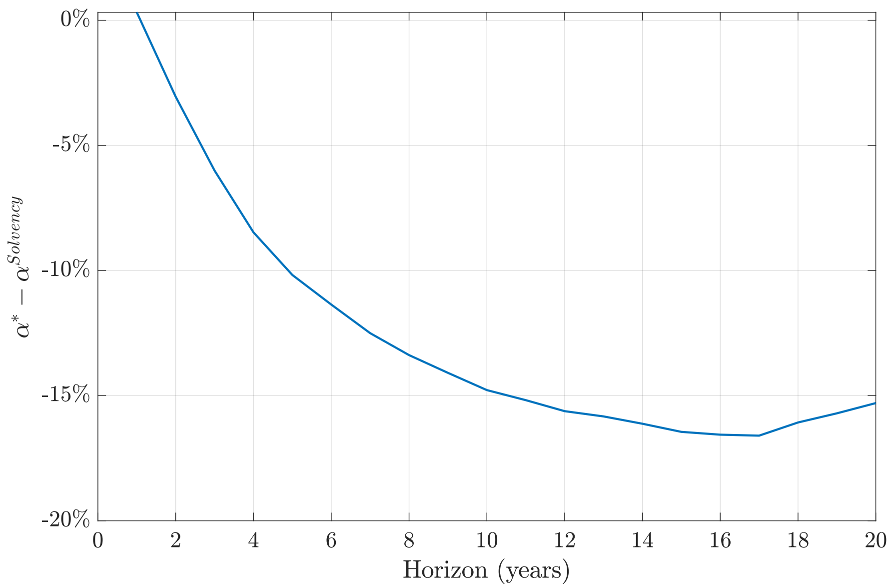
The effect of regulation is relatively similar for the SCR and VIF policies. The effect first increases with the horizon and shows a maximum for horizons of 10-12 years, and then reaches a plateau. The cumulative impact is quite significant, in particular for relatively longer horizons. For example, over a 10-year horizon, the unconstrained equity allocation is approximately 27 %, while the constrained allocation is around 12 %, a 15 percentage points drop, which is more than half of the unconstrained allocation.

Figure 4: Allocation by asset class, with and without SCR



Note: asset allocation without Solvency 2 constraint (in blue) and with Solvency 2 constraint (in red).

Figure 5: Impact on equity allocation



5 Conclusion

In this note, we have looked at the optimal portfolio choice of a long-term investor. Our approach is based on an econometric model of asset returns taking into account assets risk profiles, which varies depending on the investment horizon. We estimated these dynamics by exploiting post-war historical data in a panel of 16 developed countries. Our approach also uses calibration to reproduce an asset allocation in line with observed insurers' positions while obtaining reasonable long-term returns.

Our results show a marked effect of regulatory constraints, which increases with the investment horizon. For a 10-year horizon, the unconstrained equity allocation is approximately 27 %, while the constrained allocation is around 12 %, a drop of 15 percentage points, which is more than half of the unconstrained allocation.

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