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A BETTER UNDERSTANDING OF FINANCIAL RESEARCH

Special Edition

Chaire Risques Financiers FdR

With

Aurélien Alfonsi
Nicole El Karoui
Emmanuel Gobet
Julien Guyon
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editorial

Financial risk: a challenge for mathematicians

For some 40 years now mathematicians have been working in partnership with the finance industry on models of stock price fluctuations, in order to develop methods of valuing assets and controlling risks. However, the recent subprime crisis has shown that we have a long way to go to fully understand the functioning of markets and associated risks. The models used still fall well short of reality, particularly in regard to liquidity risk and interactions between agents.

The “Risques financiers” Chair of La Fondation du Risque was created shortly before the crisis to promote better use of mathematics in financial institutions. Currently held by Nicole El Karoui, an internationally recognized expert in financial markets and the founder of the highly reputed Master’s "Probability and Finance", this Chair brings together research teams in financial mathematics from the Ecole Polytechnique and the Ecole des Ponts et Chaussées with the Société Générale quantitative research team, headed by Lorenzo Bergomi. These teams are at the forefront of financial research in the Paris region and in some fields are world leaders. The objective of the Chair is on the one hand to “move up a gear” in terms of research, in particular by further developing international collaborations to make the Paris market more attractive for researchers, and on the other hand to promote collaborations with practitioners with a view to identifying relevant research topics and aligning academic research more closely with the concerns of the financial industry.

Research topics on which we work include:

- Effective numerical methods for the problems of valuation and risk management.
- Modeling liquidity risk, i.e. the risk associated with the impossibility of quickly finding a buyer or a seller for a large amount of assets.
- Nonlinear problems, including questions related to the interaction between agents.
- Modeling market microstructure and order books; processing extremely high frequency financial data.
- Mathematical methods for solving optimization problems in finance, in particular through the use of “backward” stochastic differential equations.

The “Modeling and Managing Financial Risks” conference was an opportunity for us to make a preliminary assessment of the first three years of the Chair, an assessment which proved overwhelmingly positive. Research teams have grown, because the existence of the Chair on the one hand attracts support from institutions and on the other hand results in top quality recruitment. We have been able to invite a large number of leading foreign researchers and to initiate ongoing collaborations with some of them. Collaborations between academics and Société Générale researchers are also under way. Much progress has been made in the Chair’s research fields. This compilation presents the “Modeling and Managing Financial Risks” conference, focusing in particular on the contributions of members of the Chair, and provides an overview of what has been achieved.

Nicole El Karoui and Nizar Touzi
Scientific Directors of the Financial Risks Chair

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Chaire Risques Financiers
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Optimal order execution

AND PRICE MANIPULATION

By Aurélien Alfonsi
Ecole des Ponts ParisTech - Université Paris-Est Marne-la-Vallée

KEY POINTS

- In their study [AS], Aurélien Alfonsi and Alexander Schied offer a simple and intuitive model of the dynamics of order books in financial markets.
- This allows them to obtain an explicit optimal strategy for placing a purchase order.

In general, the value of an asset quoted on a market is indicated by a price. This price is the average between the highest bid and lowest sell offer pending on that asset. It does not contain all the information given by the market. Such information is r epresented by the order book that contains all pending purchase and sell orders on the asset.

BIOGRAPHY

Aurélien Alfonsi

Aurélien Alfonsi is a researcher in probability and finance at the Ecole Nationale des Ponts et Chaussées, at CERMICS (Centre for Teaching and Research in Mathematics and Scientific Computing). His research focuses on modeling financial markets and numerical methods.
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Introduction

These orders, known as limit orders, can only be placed on a price grid determined by the market. When a participant wishes to acquire the asset, he may either place a limit order to buy and wait until it finds buyers or buy at the best offer price by acquiring the cheapest sell orders. The first solution has the advantage of lower cost, but the implementation and the time of the transaction are uncertain. The second solution is more expensive but immediate. In our study [AS], for greater simplicity we consider only the latter possibility.

When an agent buys a small quantity of assets on the market, he will acquire only the least expensive offers. The cost of the transaction will then approximate to the product “quantity” × “quoted price”, and consequently the information given by the order book is superfluous. The situation is different if the agent buys a large quantity of assets. More precisely, we here mean that the quantity purchased is of an order of magnitude comparable to the number of sell limit orders pending in the order book. In this case, the agent will acquire limit orders to sell at a much higher price than the quoted price. Knowledge of the order book is then necessary to determine the cost of the transaction, which will anyway be higher than the product '”quantity” × “quoted price”. In fact in this case, the agent has no incentive to buy all the assets in a single transaction. It is better for him to divide up his purchase, so that new, less expensive limit sell orders appear between his purchase orders. This brings us to the optimal execution

> The problem of optimal execution of orders is closely related to the question of market viability.

problem we have studied: given a time horizon (a few hours or days), how does one optimally divide up the purchase order so as to minimize the expected total cost of the transaction?

Methodology

To answer this question, we have proposed in [AS] a very simple model of the order book and its dynamics. The order book is represented by a shape function that describes the number of sell orders pending at a given price. When the agent purchases assets, he acquires the least expensive offers pending in the order book. This has the effect of increasing the price of the best sale offer. When the agent is inactive, new sale offers appear in the order book at a rate that we assume is proportional either to the volume or to the price movement caused by the agent. In both cases, we obtain a similar structure for the optimal strategy when transactions take place on a homogeneous time grid. To minimize cost, the agent should begin by making an initial purchase of significant size. Then, at intermediate times, he should make small purchases that exactly consume all new orders that have appeared in the order book. At the final date, he should buy the outstanding quantity needed to meet his target. The heuristics of this finding is very clear. The greater the size of the first purchase order, the more it will attract new limit orders into the order book. Conversely, the greater the size of orders, the higher the marginal cost of the asset. Thus the optimal strategy is a compromise between reducing the cost of orders and attracting new sales offers into the order book. In addition, our study gives an explicit expression of the optimal strategy. This reveals from a quantitative standpoint the impact of the shape of the order book and the resilience of the market on the optimal strategy and its cost. We find in particular that the optimal strategy is relatively insensitive to the shape of the order book, which is a good indicator of its robustness.

The problem of optimal execution of orders is closely related to the question of market viability. According to Huberman and Stanzl [HS], price manipulation is a strategy in which one has the same number of risky assets at the final time as at the initial time, but the expected cost of these is negative. A market is viable if there is no such strategy. The idea is that if such a strategy existed, one could certainly make money by repeating it indefinitely, which would constitute a standard arbitrage. From our standpoint, price manipulation is a special case of the optimal execution problem in which one wants to buy zero assets. If it were possible to purchase no assets with an average negative cost, there would be price manipulation. In our order book model, we have been able to show that this is not feasible, although the market that it represents is viable.

Applications
for the actors concerned

- Solving the optimal execution problem is not only interesting for practitioners seeking to reduce their transaction costs but may also allow regulators to identify how listing mechanisms influence optimal strategies and the behaviour of market agents.

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Chaire Risques Financiers FdR

BIOGRAPHY

Emmanuel Gobet

Emmanuel Gobet is professor of applied mathematics at the Ecole Polytechnique. After studying at X, he defended his thesis in 1998 and obtained his accreditation to direct research in 2003. He has published some forty scientific papers in international journals. His work mainly focuses on probabilistic numerical methods (Monte Carlo simulations, stochastic approximation), processes statistics and financial mathematics (calibration methods, options valuation and hedging tools). He is also involved in many professional collaborations with financial institutions, insurance companies and energy providers.

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Eric Benhamou

Eric Benhamou is CEO of Pricing Partners. He has extensive experience in the trading room, with stints at Goldman Sachs and Natixis before founding Pricing Partners, an independent company that provides the financial community with development and risk management software based on latest generation mathematical models, thus allowing greater transparency in financial markets.

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Fast formulas for option valuation

AND REAL-TIME CALIBRATION

By Emmanuel Gobet, Ecole Polytechnique ; Eric Benhamou, Pricing Partners and Mohammed Miri, Pricing Partners

KEY POINTS

- The authors' presentations (E. Gobet's plenary lecture, M. Miri's mini symposium contribution) at the "Modelling and Managing Financial Risks" conference last January provide new and effective solutions to certain real-time calculation problems in regard to option prices and their hedging.
- In fact, in many cases, an approximate formula of the price can be quite sufficient for objectives of calibration (determination of the model's parameters) or large portfolio valuation.
- If, in addition, the formula is analytical and thus very rapid to estimate (as the famous Black-Scholes formula can be), then the resulting process is carried out in real time and can respond effectively to large-scale calculation problems involving thousands of pricing options.

Since the opening of the options market in Chicago in 1973 and the ground-breaking work of Black-Scholes and Merton, financial engineering has undergone dramatic growth, not only in terms of the types of markets (equities, foreign exchange, interest rates, credit, commodities, life insurance, etc.) and the types of financial products available (so-called vanilla options, exotic options, swing options, etc.) but also in terms of problematics (product valuation and hedging, portfolio management, risk identification and management, etc.).

The financial market needs real-time calculations

This expansion has been accompanied and made possible by many advances in computer technology and the simultaneous development of the tools of financial mathematics, especially in mathematical analysis (stochastic calculus, Malliavin calculus, etc.) and numerical calculus (Monte Carlo methods, solution methods for partial differential equations, etc.)

On the operational side, the industrialization of market finance has led to much higher requirement levels in terms of response time to the development of products, calculation of their hedging or the calculation of various risk indicators. These must be computable in real time, or at least be very rapidly computable, so that professionals are able to adjust their banking risk management in response to fast-changing financial markets. From this standpoint, calculation speed is a very powerful operational constraint. If a relevant model it is not computable or capable of being simulated in a matter of a second or two (or a minute or so, for more global banking management or portfolio calculation problems), it has to be rejected, even if it is highly suited to risk modelling. One consequence of this

> "The models tend to be constantly improved."

real-time computing constraint is that, in practice, simple models which can provide risk indicators in real time are preferable to more comprehensive models requiring more computing time. It is a choice made necessary by the constraints of high market volatility and very fast risk management, which may be at the expense of more fine-tuned risk management but with a delay effect that would obviously be harmful. Obviously this practice may have adverse consequences.

Although it might be assumed that technological progress (through the celebrated Moore's law) now allows financial products and their associated risks to be assessed ever more quickly, we are still far from the real-time calculations wanted by all trading room operatives. There are many reasons for this. On the one hand, models tend to be constantly improved to better capture the physical phenomena of financial markets and thus to better reproduce the data or information available. This refinement of models inevitably results in an inflation of computing time. On the other hand, risk management is approached more comprehensively, particularly in the context of regulatory capital calculations through Value at Risk, or that of counterparty risk known as Credit Valuation Adjustment. This calls for making calculations on very large portfolios and aggregating infinitesimal risks of each option or financial product at the level successively of the trader's portfolio, the trading floor, and the bank. It is of course essential to have a consistent approach at all levels, and one as discerning as possible, taking into account the interdependencies among risk factors.

Methodology

The idea presented at the conference is that of choosing an approximate or so-called proxy model or financial contract, in which explicit calculations are possible. This proxy is then used to approximate the quantities concerned, possibly by proposing correction terms to this approximation. The choice of proxy is on a case by case basis. There follow various situations where it has been possible to obtain fast and accurate approximate evaluation formulas.

- In Heston's stochastic volatility model (with possibly time-dependent parameters), one possible proxy model may be the Black-Scholes model by taking volatility to be 0.
- In local volatility models, the Black-Scholes model can be a proxy by setting the function of the local volatility either at spot or strike prices. One can also incorporate Hull and White-type Gaussian interest rates to obtain a Black-type proxy.
- When Gaussian jumps are added to local volatility, the same type of approximation leads to the Merton model.

In these three examples, one can approximately calculate vanilla option prices as the sum of the price in the proxy model plus correction terms equal to a combination of sensitivities (Greeks) calculated in the proxy model. The numerical tests carried out generally show an impressive gain of about a factor of 100 in the computing time over the best previously available numerical method (FFT in the Heston model, resolution of dual EDP for the local volatility model with jumps). A complementary procedure can also deduce changes in implied volatilities, further facilitating calibration.
The case of exotic options is also possible. Studies currently being finalized process average (Asian) or basket options. Incorporating linear dividends can obtain new approximations that are very accurate and fast. This study with affine dividends has similarities with the valuation of guaranteed life insurance policies, commonly known as variable annuities, in particular in the case of GMABs.

Although approximation formulas are simple and precise, their mathematical derivation is quite complex and requires a detour via stochastic analysis (Malliavin calculus) and stochastic disturbances, combined with ingenious parameterizations. In addition to the formulas, fine error estimates are established, clearly showing the role of the parameters of the model or of the payoff in the accuracy of the formulas, and thus defining the framework for reasonably applying these approximations in real time.

Far from being closed, the subject is expanding rapidly and is thus promises major advances.

Mohammed Miri

Mohammed Miri is Research and Development team leader at Pricing Partners. After studying at Ensimag, in 2009 he defended his doctoral thesis on approximation methods for obtaining real-time calculations.

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Applications for the actors concerned

- The tools developed by E. Gobet and his co-authors allow prices and hedge indices of options to be calculated in real time in relatively sophisticated models. Financial establishments will thus be able to take into account risk factors that were previously inaccessible because of computational time constraints.
- Furthermore, with their better grasp of non-linear effects in options portfolios, these new algorithms improve the calculation of indicators for estimating regulatory capital or adjustments for counterparty risk and contribute to a clearer view of risk at bank level.

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Chaire Risques Financiers FdR

The smile

IN STOCHASTIC VOLATILITY MODELS

By Julien Guyon
Société Générale, Global Markets Quantitative Research

KEY POINTS

- Julien Guyon and Lorenzo Bergomi are concerned with “stochastic volatility” models. In a completely general way, they calculate an exact limited development of the implied volatility smile at order 2 in volatility of volatility.

Louis Bachelier in 1905, followed by Francis Black and Myron Scholes in 1973, put forward the first two theories of derivative products, also called options. In these two models, uncertainty is fully quantified by a single number, σ, the volatility of asset prices. For example, in the Black-Scholes model, which is the market benchmark, the price of call options is given by the well-known Black-Scholes formula, and at each observed price in the market, for a given maturity T and strike price K, there is one and only one constant volatility $\sigma_{BS}(T,K)$, called implied volatility , obtained by reversing the formula.

Introduction

Although the Black-Scholes model can explain the prices observed in the market, all the implied volatilities $\sigma_{BS}(T,K)$ are equal, irrespective of the maturity and the strike price of the option. But this is not what occurs. Typically, we find that implied volatilities depend on maturity T, and for a fixed maturity, there is a volatility “smile” i.e. the implied volatility depends on the strike price K. The term “smile” is used because the graph $K \rightarrow \sigma_{BS}(T,K)$ often looks like a smile, i.e. it is often convex, first decreasing, then increasing. Figure 1 shows such a volatility smile.

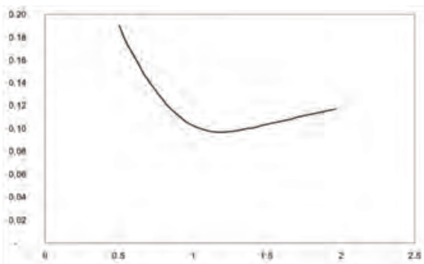

Figure 1: A smile observed in the 5-10 year swaptions market

In the case of equity markets, the smile significantly decreases in the low strike price zone. In this zone, traders correct prices produced by the Black-Scholes model by increasing implied volatility, mainly for two reasons: first, to take account of the possibility of a crash or a major downward price movement, events that in the Black-Scholes model are underweighted (for example, we observe that the historical volatility of a share is typically greater when its price is lower); and second, to take account of the lack of liquidity options whose strike price diverges considerably from

BIOGRAPHY

Julien Guyon

Julien Guyon works in Paris in the Société Générale quantitative research team. He has a doctorate in applied mathematics from the Ecole des Ponts (Paris), with particular expertise in probability and statistics. He holds degrees from the Ecole Polytechnique (Paris), Université Paris 6 and ENPC. He is also a professor at the Ecole des Ponts.

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the current value of the asset. This second reason explains why we as often see the smile "accentuate" for very high strike prices.

Methodology

To account for these observations, it is therefore necessary to modify the Black-Scholes model in order to make it produce a smile. One solution often used is to replace constant volatility σ by a process that itself has random fluctuations. It is these "stochastic volatility" models that we, together with Lorenzo Bergomi, head of quantitative research at Société Générale, are interested in. Using perturbation expansion techniques, we have shown that in fact there is a completely general formula for the smile generated by stochastic volatility models. We presented our findings at the "Modelling and Managing Financial Risks" conference, organized by the Risques Financiers Chair in Paris from 10 to 13 January 2011.

Our main finding is as follows. Just as price fluctuations are measured by volatility σ, volatility fluctuations can be measured by the "volatility of volatility" ω. In an entirely general way, in stochastic volatility models, we can calculate an exact limited smile expansion at order 2 in volatility of volatility ω. At order 2, the smile is described by a simple functional form:

$$\sigma_{BS}(T,K) \approx I_T^{ATM} + s_T \ln\left(\frac{K}{F_T}\right) + k_T \ln^2\left(\frac{K}{F_T}\right)$$

where F_T is the initial value of the forward and the coefficients I_T^{ATM}, S_T and K_T, which represent respectively the implied spot probability (i.e. for the value of the strike price equal to the price of the underlying), its slope and its curvature, are expressed simply using the model's parameters. Whichever stochastic volatility model is used, we can calculate these three numbers and deduce the smile from them. The expression of the smile and the expressions of the coefficients I_T^{ATM}, S_T and K_T show that the smile of stochastic volatility models is structurally constrained.

For example:

- The spot slope is generated by the covariance between the price of the underlying asset and its volatility.
- The spot smile is convex if the price of the underlying asset and its volatility are uncorrelated, but this is not necessarily true in the general case.
- We can show, for short maturities, how the slope and curvature of the smile depend on the implied spot volatility.

In particular, our work allow us to find the limited development of the smile at order 2 in volatility of volatility carried out by Alan Lewis [5] in the case of the Heston model – more precisely, in the case of a parameterized family of Heston-type models. Our result is of course much more powerful, since we do not assume any particular form for the stochastic volatility model. We get the development at order 2 in volatility of volatility of the Bergomi model [1]. The exact calculation of the smile by Monte Carlo allowed us to verify that the development at order 2 is very accurate for a wide spectrum of volatilities of volatility ω and for a large range of maturities and strike prices.

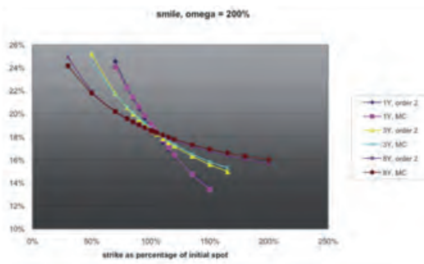

Figure 2: Exact smile (MC) and our (order2) smile formula in the Bergomi model

Figure 2 shows smiles obtained for typical values of volatility of volatility (ω = 200%) and of correlations between spot and volatilities for three maturities (1, 3 and 8 years):

- first, the exact smile obtained using a Monte-Carlo simulation (MC),
- second, the smile obtained from our new (order 2) formula.

The approximation is highly satisfactory.

In conclusion, although our results can be used for parameter calibration purposes, we do not think it is always advisable to calibrate the parameters of a stochastic volatility model on the smile it produces. In particular, if the model is used to evaluate exotic options whose risks are orthogonal to those of conventional purchase options, such a calibration would only offer unfounded psychological well-being and could have undesirable consequences (mispricing). However, it frequently happens that some well-chosen purchase options can significantly reduce the risk of an exotic option. In this case, our formula may be used to choose the model's parameters so that it correctly assesses the price of these hedging instruments. Finally let us insist on the fact that our work not only allows the smile produced by stochastic volatility models to be precisely quantified, but also and especially allows the structural constraints of that smile to be specified. It thus has a twofold value, both quantitative and qualitative.

Applications

for the actors concerned

- The results have a twofold value, both quantitative and qualitative. On the one hand, they allow the structural constraints on implied volatility to be specified.
- On the other, they can be used for parameter calibration purposes and can significantly reduce the model risk associa ted with the development of an exotic option.

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MODELING AND MANAGING FINANCIAL RISKS

a Financial Risk Chair Conference

Peter Tankov
CMAP-Ecole Polytechnique

The "Modeling and Managing Financial Risks" international conference has been the main scientific event of the first five years of the Financial Risk Chair. This four-day event was organized in January 2011 by the CMAP financial mathematics team. The conference presentations on various aspects of financial risk took place in the amphitheatres of the Campus de Cordeliers. Located on the site of the old Cordeliers monastery, these historical buildings are now owned by the University of Paris VI and are used for organizing prestigious events.

The plenary lectures were given by distinguished speakers invited by the scientific committee, and presentations from the parallel sessions were selected from the many contributions received by the organizing committee. The call for papers was circulated widely in France and abroad and a special effort was made to attract contributions from young researchers.

Some figures:

66 presentations in total, including 12 by practitioners, 25 by speakers from abroad, 9 by members of the Financial Risk Chair, and 21 in the plenary session. 219 participants registered, including 53 practitioners and 87 from abroad.

Among the plenary session speakers during the first three days:

■ **Ioannis Karatzas** talked about models for large-scale equity markets. He is interested in particular in the relationship between the size of a listed company, i.e. the proportion of its stock in the overall volume, and its performance, and the implications of this relationship on long-term portfolio optimization.

■ **Emmanuel Gobet** presented a method for calculating option prices in the presence of dividends. It develops highly accurate approximations for real time evaluation.

■ **Nicole El Karoui** gave a three-session short course on different aspects of the historical interaction between mathematics and financial markets. The two main topics were on the one hand portfolio management under the constraint of drawdown and its link with the theory of martingales and the other hand backward stochastic differential equations and their applications for risk measurement.

■ **Xunyu Zhou** spoke about optimal timing for buying or selling an asset, in the case where the buyer/seller is not completely rational but underestimates or overestimates certain probabilities.

■ **Paul Embrechts** presented his thinking on the role of financial engineering in financial crises and their prevention. Emphasis was placed in particular on the social role of financial mathematics in managing pensions.

■ **Jean-Pierre Fouque** presented new approximations for stochastic volatility models, i.e. models in which the variability of the assets may itself change randomly from one day to another.

■ **Walter Schachermayer** introduced innovative work on multidimensional measures of risk. The aim is to be able aggregate various types of risk into a single measure.

■ **Nizar Touzi** presentation focussed on price bounds in the presence of calibration constraints. This involves calculating all possible values of an asset without making any assumptions about its behaviour, but using only the prices of other assets available on the market.

The last day was devoted specifically to the interaction between academics and practitioners, with presentations by **Bruno Dupire, Julien Guyon, Lorenzo Bergomi, Salah Amraoui, Alex Lipton, Jean-Philippe Bouchaud, Charles-Albert Lehalle, Rama Cont and Frédéric Abergel.** There was also extensive participation by practitioners throughout the four days of the conference.

The afternoon of the second day of the conference was an opportunity to celebrate the 20th anniversary of the "Probability and Finance" Master's. Many former Master's students attended this event with presentations by Helyette Geman, Nicole El Karoui, Shige Peng, Gilles Pagès, Jean Jacod and Bruno Dupire. It was followed by cocktails.

Overall, the conference was a great success, both in terms of the quality of presentations and in terms of the interaction between academic communities and practitioners. The conference programme together with the presentations is available on the conference website at ***www.cmap.polytechnique.fr/financialrisks/conference2011/***

BIOGRAPHY

Peter Tankov is a lecturer and researcher in financial mathematics at the Ecole Polytechnique. The author of "Financial Modeling with Jump Processes", his research focuses on modelling jump risk in finance and on stochastic processes with jumps. Together with C. Hillairet and J. Guyon, he was one of the organizers of the "Modeling and Managing Financial Risks" conference.

Quantitative trading

RATIONALIZATION OF NEGOTIATION IN THE MARKETS

By Charles-Albert Lehalle, Crédit Agricole Cheuvreux

KEY POINTS

- Optimal trading is a recent field which establishes a link between an investment strategy in the broad sense and the state of supply and demand in an electronic market. In this way it provides a better understanding of one type of hitherto little explored risk by combining the findings of applied mathematics and quantitative finance with cexpertise in modelling discrete processes on large samples.
- C.-A. Lehalle and his colleagues are developing powerful digital tools for optimal trading and for geographical optimization in placing orders.

Quantitative trading brings together techniques allowing the implementation of market investment decisions to be streamlined. These techniques were created around the purchase and sale of large quantities of shares in electronic markets. If one buys or sells too quickly , the price will be affected (this effect is known as "market impact"), whereas if one buys or sells very slowly to avoid disrupting the price formation process, one runs the risk that the price changes during the implementation of the decision, thus making it obsolete.

In order to optimize this balance – trading slowly so as to avoid too much market impact, trading quickly so as to avoid excessive exposure to market risk –, it is first necessary to model the nature of market impact, and hence to study the microstructure of markets. It is also important to correctly measure the risk involved in waiting (the role of randomness in the price formation process) and therefore to make use of the findings of processes statistics. Once these two stages are complete, one can clearly articulate an optimization problem corresponding to the negotiation problematic and then solve it.

The microstructure of markets

The microstructure of markets is commonly understood as the set of rules and behaviour that structure the bidding procedures established for price formation. In terms of markets driven by orders, there are two main types of bidding procedure: call auctions, which usually begin and close trading days, and continuous auctions, which apply during the rest of the trading day.

Call auctions are very similar to regular auctions of works of art, except that the sellers participate as well as the buyers. This is made possible through the fungibility of the shares of a listed company.

BIOGRAPHY

Charles-Albert Lehalle

Charles-Albert Lehalle heads the Credit Agricole Cheuvreux quantitative research team responsible for optimization of negotiation in equity markets. He has published numerous articles on the optimal control of trading, estimation of market impact and measurement of the effectiveness of a negotiation process. He teaches the Quantitative Trading course in the Masters in Probability and Finance at the University of Paris VI and ENSAE.
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During the “auctioning” stage, buyers and sellers declare their interests simultaneously in the form of quantities and prices. The sum of buy and sell offers form two cumulated order books. At the time of the “call” an equilibrium price is calculated which corresponds to the intersection of the two order books.

Continuous auctions are conducted according to a different principle: the cumulated order books do not overlap and the matching of buy and sell orders are implemented continuously. As soon as a buy order is declared at a higher price than a sell order already present in the order book (or a seller declares a price lower than that of an already present buyer), a trade is generated which “cleans” the book in real time. For continuous auctions, it is useful to distinguish between liquidity providers, who are present in the order book, and liquidity consumers, whose orders generate immediate trades by meeting the orders of liquidity providers. Liquidity providers are patient, willing to wait before engaging in a transaction; liquidity consumers are impatient and are willing to “pay for” the supply-demand spread in return for the certainty of a transaction.

The price formation process therefore comes from the concatenation of call and continuous auctions, and arises from the dynamics of order books. The recent increase (uninterrupted in Europe since 2007) in the frequency of order changes by the various participants is currently leading to most types of negotiation being conducted in electronic high frequency trading markets, in contrast to the situation a few years ago when orders were submitted by phone. In 2011, a CAC 40-listed tradable asset generates about 50,000 transactions a day, conveyed by some 600,000 movements in the ten top prices in the order book.

The modeling of the arrival processes of orders according to their price, quantity, state of order books and the recent past has been the subject of many studies, though without any definitive conclusion emerging. Because the rules of the various order books available and the behaviour of the different actors are constantly changing, it is difficult to draw conclusions as to a “best” model of the price formation process. A number of promising proposals have nonetheless emerged, including point processes (Bacry et al. 2011), models of price movements immediately after a transaction followed by an order book relaxation period (Bouchaud, Mezard and Potters, 2002 and Gatheral, 2010), and a joint modelling of order volumes and quoted prices (Cont and De Larrard 2011).

The negotiation problematic

Once the problem is properly stated in terms of bidding procedures, the problematic of the purchase or sale of a large number of shares clearly emerges.

The first part of the rationalization of trading involves controlling risk. Several equivalent formal frameworks were put forward in the late 1990s by various academics (Almgren, Chriss 2000). The principle is that of specifying a criterion to be minimized: this was initially a mean-variance equilibrium that can reflect the desire to act slowly (in order to minimize the market impact cost, which is on the side of the mean) and the desire to act quickly (in order to minimize the variance of prices during the trading period). Today more sophisticated techniques are commonly used (Bouchard, Dang, Lehalle 2011). These enable one to determine, though the numerical optimization of “optimal trading curves” (see Figure 1) that govern the number of shares, whether it is wise to buy or sell in order not to be too heavily exposed to various risks.

It is then a matter of actually buying or selling shares by participating in the auction procedure. This is done by means of trading robots, also called “liquidity seekers”. In this context one should not neglect the importance of optimizing the choice of order book with which the trading robot will interact. Indeed for the same listed share, auction procedures take place in parallel in traditional markets (NYSE or Euronext for French shares) and alternative exchanges such as BATS, Chi-X or Turquoise, as well as in “Dark Pools” (anonymous order books). Geographical optimization of the placing of orders is supported by what is generally called a “Smart Order Router” (SOR). Proposals to formalize these problematics have recently been made by Pagès, Laruelle, Lehalle (2009).

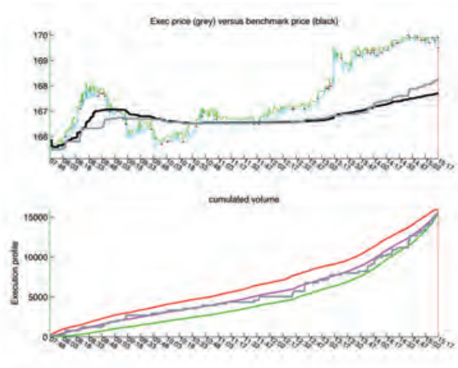

Figure 1: A real example of the implementation of a trading algorithm. At the top, market prices (values of the “best bid” and “best ask” in green and blue, coloured dots for transactions obtained by the algorithm) on which are superimposed volume weighted average prices (VWAP) of the market (black) and the algorithm (grey). Below, low volumes traded 100% rebased in late trading: black for the market, grey for the algorithm, and colour for the different limits calculated by a trading optimizer.

Applications
for the actors concerned

- The optimization of interactions with the order books of electronic markets, even though infinitesimal in terms of implementation time (deciding to intervene in seconds), is a genuine source of economies for much of the business of financial market actors such as market makers and fund managers.

Further reading

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No-arbitrage bounds
FOR LOOKBACK OPTIONS

By Pierre Henry-Labordère, Société Générale, and Nizar Touzi, Ecole Polytechnique, CMAP

BIOGRAPHY

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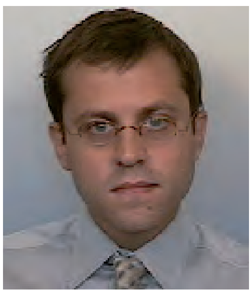

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KEY POINTS

- We provide a general method for seeking bounds on derivatives prices, given the market information on a specific date.
- These bounds should be robust in the sense that they do not depend on a particular modelling choice, but are based solely on the single basic law of market finance, namely the concept of no-arbitrage.
- However, we make an additional assumption of linearity of the functional evaluation and of the continuity of the price process of the underlying asset.

We will focus on the evaluation at the date of origin of a derivative written on a single underlying of a pay-off, payable at maturity T, depending on the overall trajectory of the underlying asset.

Methodology

We assume that the underlying asset is available for exchange in continuous time with no constraints on trading volumes. We also assume a zero interest rate context. It is well known that the absence of arbitrage implies that the price process of the underlying asset is a martingale under a certain measure of probability, commonly called risk neutral probability.

The investor also has access to a European options market and can trade European options with maturity T for all positive prices during the fiscal year. The no-arbitrage assumption together with the linearity of the functional assessment then allows us to identify the marginal distribution of the price of the underlying asset at time T under risk neutral probability. This possibility was noted some time ago by Breeden and Litzenberger.

With the two preceding assumptions – the linearity of the evaluation function and the absence of arbitrage in the market for the underlying asset and in the European call options market – we thus have two conditions applying to the underlying asset: under risk neutral probability, it is a marginal distribution martingale at a given T. The question addressed in our study is to find the bounds

> The question addressed in our study is to find the bounds on the price of an exotic option for this underlying asset without any other assumptions in the model.

on the price of an exotic option for this underlying asset without any other assumptions in the model.

We express these bounds as robust hedging problems, i.e. the hedge should ensure risk protection, whatever the underlying model. We show that these hedging problems allow a dual formulation which is more suited to numerical or analytical calculation methods and which treats our problem both.

- as an optimal transportation problem, of a different kind to the problemsstudied in the traditional literature because of the martingale condition imposed on us by the absence of arbitrage,
- and as the well-known Skorohod embedding problem, which has generated extensive literature in the probability community.

David Hobson had noticed the relationship with Skorohod's embedding problem back in 1998. In particular, Hobson and Klimmek (2011) have shown that for a class of Lookback options, the so-called Azéma and Yor solution to Skorohod's embedding problem gives the optimal upper bound. Out approach also allows this explicit result to be obtained.

A natural extension of our problem is to assume that the investor has access to the marginal law of the underlying asset price on several dates before maturity. In particular, the case of two marginals and one Lookback option depending only on the maximum was fully solved by Brown, Hobson and Rogers (2001). For cases with several marginals, Madan and Yor (2002) gave an explicit solution under a monotonicity condition for related barycentric functions. With our approach, we are able to obtain a solution to the problem in the general case, avoiding Madan and Yor's condition.

Conclusion

Our approach offers a general methodology for the formulation of the problem of no-arbitrage bounds. These bounds are very useful for the practice of financial markets as they can directly test whether a given price is consistent with all the prices available on the market at a given date. It is clear that in general these bounds give a sufficiently wide acceptable price interval. However, the interval size decreases with the amount of information made available to the investor.

Several extensions to our work may be envisaged. First, we should not expect to obtain explicit formulas for general examples of structured products. The important thing is to come up with effective numerical methods for approximating the solution of our problem. This task is partially addressed in Xiaolu Tan's thesis work at the Ecole Polytechnique.

Finally, not all explicit calculation procedures have been exploited, even in the context of very specific Lookback options. What happens for a general payoff that does not conform to Hobson and Klimek's conditions? What happens if all the marginals are known? This latter condition could lead to a simplified context thanks to the power of differential calculus for continuous variables.

An example.

We used our analytic results to determine the optimal upper bound of an option call written on the Eurostock maximum between two dates t1=1 year and t2=2 years for different strikes. By way of comparison, we evaluated this product in Monte Carlo with a local volatility model introduced by Dupire (1993).

Strike	Borne Haute	Vol.Locale
80	41,24	33,15
90	32,10	24,78
100	23,49	17,17
110	15,77	10,73
120	9,40	5,86
130	4,83	2,75

Recommandations for the actors concerned

- These bounds are ver y useful for actors operating in financial markets since they allow them to directly test whether a given price is consistent with all a vailable prices in the market at a given date.
- The price inter val provided by the method also allows banks to quantify the model risk associated with an exotic product.

Further reading

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PROBABILITIES AND FINANCE MASTER'S CELEBRATES ITS 20TH ANNIVERSARY

20 years of training, 900 students, 2 major crises

The teaching of financial mathematics is at a very high level in France and young graduates in this discipline are highly sought after in the job market. With its 900 graduates and twenty years of existence, the "Probability and Finance" Master's (UPMC/EP) has played a decisive role in this development, as attested by an article in the Wall Street Journal in 2006.

BIOGRAPHY

Nicole El Karoui is a researcher at the Ecole Polytechnique's Center of Applied Mathematics (CMAP) and a mathematics professor at the University of Paris VII's (Univ.Pierre & Marie Curie) Laboratory of Probabilities and Random Models (PMA). She is in charge of the latter institution's Master's 2 program, "Probabilities and Finance", a joint degree with the Ecole Polytechnique. She is scientific director of the Institut Louis Bachelier, director of the financial risk research chairs for the Fondation du Risque (FdR) as well as of the future derivatives research chairs co-sponsored by the Federation of French Banking.

THE MASTER

Key figures

The Master's has so far trained 900 graduates, most of them science students and half from the Grandes Ecoles. A high proportion of those taking the Master's are foreign students. "Quants" (quantitative engineers) make up the leading occupation after training, followed some considerable way behind by trading, risk-management and hedge funds. At least a third of our graduates currently work in the four major French banks, and another third in foreign banks in London or Japan. For the past five years, more than half of our graduates' first jobs have been in London.

Key dates

In the late 1980s, the first financial futures markets opened in Paris (MATIF in 1986 and MONEP in 1987). Antoine Paye set up the options products business at the Société Générale. The NYSE's Black Monday (October 1987) was on everyone's mind.

In 1990, Nicole El Karoui (Professor at Paris VI) and Helyette Geman (Professor at ESSEC) created the Probabilities and Finance option within the Probabilities DEA at Paris VI University (Director Jean Jacod). The DEA was co-accredited with the Ecole Polytechnique, IENPC and ESSEC. It was the first training of its kind in a science environment.

The aim was to train quants, who understood and used the methodologies developed in the United States over the previous fifteen years, particularly in the area of interest rates, for which there was a strong demand. The training scheme that was set up is still relevant today, as is explained below.

1990-2000

From modest beginnings – six graduates the first year – the Master's grew rapidly in response to considerable demand. There were two specific challenges. One was to draw attention to the course outside of trading rooms, especially with Human Resources departments. The other difficulty was that the Anglo-Saxon banks in London did not usually take trainees on a long-term basis, partly because of the issue of professional secrecy. Here it was a matter of waiting until a higher proportion of managers were French.

It was a very exciting time for both our students and the teams in the banks, because everything was happening at incredible speed – broadly speaking, that of the calculating power of computers. Tools that seemed impossible to use were fully integrated within three years. There was no problem finding jobs, wages were high but still reasonable, and quant bonuses were far from exorbitant.

At the same time, academic research was actively being developed, often in association with banks. Marc Yor, Professor at Paris VI and a member of the Academy of Sciences, played a key role, along with other teachers on the Master's. A new research discipline, "financial mathematics", was needed in Europe in particular. Mathematicians did not always view it in a favourable light.

A number of crisis – LTCM, the Asian crisis, Enron and Barings, among others – were occasionally reminders that these markets can lead to very heavy losses and that there is always a temptation to operate with huge leverage. 1998 saw the regulator require that a daily Value at Risk be produced on market risks, i.e. on the aggregate activity of a trading floor. Even though computers had become more powerful and decreased in cost, this was a real challenge. It was a major task for financial institutions, as well as for the educational establishments that produced the quants who would have to implement this new measure. Academic exchanges with the market were intensifying, especially around the appropriateness of VaR as a measure of risk.

Students from the Grandes Ecoles were becoming more numerous in our training programme, as well as in the programmes created at Paris VII, Dauphine, Paris I and then Marne la Vallée and Evry and in the past few years at the Grandes Ecoles themselves.

Between 1995 and 2000, Nicole El Karoui left the Ecole Polytechnique and set up a Financial Mathematics research team, though remaining very involved in the training under the Ecole Polytechnique; Helyette Geman took over as head of Paris Dauphine's DEA 203 and left the training programme; and Gilles Pagès arrived at Paris VI, where he developed numerical probabilities and participated in the management of the Master's, together with Marc Yor.

2000-2008

The coming of the millennium saw the bursting of the dot.com bubble, which disrupted our student intake for a year and a half (2001-02) but not the demand for training. Mergers between banks were becoming more frequent. The first credit derivatives were being introduced, and many courses started focussing on modelling them. The academic community had its reservations about the models put forward.

The "El Karoui DEA" proved popular and our students began to acquire international exposure, culminating in 2006 with the investigative article in the Wall Street Journal on the 30% of French quants in the financial markets, followed a little later by a piece in Le Monde. We had created another luxury product for export, with bonuses having reached astronomical levels, especially for traders. The French approach to training "engineers", with a strong background in mathematics and a great capacity for adaptation, supplemented by high-level academic training, was particularly well suited to professions of this kind.

2008-2010

The bankruptcy of Lehman Brothers in September 2008 badly affected the Master's. Financial mathematics was stigmatized and held responsible for all sorts of evils. Even though the methodological criticisms were often very superficial, the more interesting question of the role and responsibility of scientists in the real world was perspicacious, particularly as finance is far from having the supposed "neutrality" of other more traditional disciplines.

The subprime crisis had little affect on the derivatives business apart from credit. Applications for the Master's (for the 2009 academic year) reached a very high level, and despite our stringent selection we accepted a large number of candidates. 2009 was a difficult year for finding traineeships, and was hardly easier for first jobs. This proved particularly hard for foreign students, for whom the issue of "papers" was problematic yet crucial.

The crisis gave a very negative image of financial markets, which led to questioning by a number of students. Moreover, the crisis in Greece showed that considerable uncertainty still remained. Under regulation injunctions, control measures in institutions were being strengthened, and quantitative engineers were hired for this purpose. The situation in London was very different, since many teams had been laid off as a result of the crisis. As the economy recovered, teams were re-formed and our students were in great demand. Over the past two years, the major London banks (both Anglo-Saxon and others) have recruited many more of our students than French banks.

For several years, banks and hedge funds have been developing high-frequency trading on their own account or on behalf of others. Given the scale of demand, we have oriented an optional part of our training in this direction, linking it to a wider debate around risk. Although this area represents only a part of our curriculum, it is something we devote much thought to.

Students

The number of students coming from Grandes Ecoles has greatly increased in recent years, largely through media coverage of the Master's. At the Ecole Polytechnique, since the arrival of Nicole El Karoui in 1998, training in financial mathematics has been reorganized, and the research team has been strengthened and become highly visible. As a result, Polytechnique students (including foreigners) are strongly represented in our training programme – somewhat at the expense of University students, who have been slightly put off by their strong presence. The number of women students is extremely low; and their focus is most often on control organizations. There are a large number of foreign students, more than 50% of the total, even though the teaching is always in French. Some of these students follow the joint Master's organized by Fudan University and the Ecole Polytechnique. On their return to China, and recruited by U.S. banks, they are beginning to build a future financial industry, since all the signs are that the market is shifting, faster than expected, to Asia and particularly China. The hope is that French banks will also benefit from this.

The curriculum

The core curriculum of the original Master's has changed little in terms of its principles, which have remained basically the same for the last twenty years, except for increased emphasis on the teaching of regulation and risk management. Firstly, there is an introduction to finance for all students, essential for understanding the world of economics, business, and financial markets. Above all, however, we offer specialized finance market teaching, in the field of financial derivatives. This sector has a strong applied mathematics component, both in terms of probability and in calculation methods: the problems concern modelling with a view to calculating the price of the product sold and hedging the risks it generates, resulting in real-time calculations, but

" The issue of risk is central to the training, since it is inherently the core business of the quant engineer. "

calling for little by way of financial analysis. Following the crisis, we are heightening students' awareness of risk issues within a broader framework through courses on risks given jointly by academics and professionals, as well as through a specific course on regulation.

One of the keys to the success of this master is undoubtedly the spirit in which it was conceived by its founders – Nicole El Karoui and Helyette Geman – who had previously just spent a year in a bank dissecting and explaining to practitioners the first stochastic models of interest rates. This experience clearly showed that these could be not simply a collection of current models – Black-Scholes, Vasicek, HJM, etc. – in the style of Prévert , but very much a fundamental knowledge base to provide students with the wherewithal for an autonomous future, essential for their evolution in a highly dynamic industry constantly seeking innovations and new territories. This is also indispensable for withstanding the pressure of business ("the market is always right") when necessary.

Most of the theoretical teaching is provided in a relatively traditional way (hypotheses, theorems, proofs, etc.), and the core of the training consists of courses in stochastic calculus, numerical simulations and statistics and their use in relation to derivatives. The optional courses are very flexible (e.g. credit calibration, U.S. options in commodity markets and climate derivatives, insurance products for high frequency management, etc.), and enable students to choose options to fit their personal requirements. Here, academics and professionals are involved on a roughly 50-50 basis, since we aim always to keep abreast of the market issues emerging from current models and future topics, something that is crucial for training such as ours.

In the second stage of the programme, it is important that each student works in a bank research unit (or comparable unit at a specialized software publisher, energy company, etc.) in order to apply the spectrum of tools acquired in the first semester in an operational framework – "in a combat situation", to use the martial metaphor of an equity research manager of a large bank. This is the only way to measure the real-time complexity of markets, their interactions, liquidity, operational and regulatory constraints, and many other realities ; and is a prerequisite for becoming a good quant capable of developing models that lead to appropriate strategies for hedging derivatives market risks.

The dynamism of the market is reflected in the ever higher level of training requirements, across an increasingly wide spectrum of knowledge. We also offer the opportunity for students wishing to spend a term bringing together all this knowledge by conducting case studies jointly supervised by an academic and a professional, prior to their traineeship in a bank.

The issue of risk is central to the training, since it is inherently the core business of the quant engineer, and also because it is not possible to treat risks separately. Training technically competent students, who are aware of the structural risks in financial markets as well as of their "social" responsibility, is the challenge that our Master's programme aims to meet.

MASTER'S SUPERVISORY PERSONNEL

For Pierre et Marie Curie University

- Nicole El Karoui
- Gilles Pagès
- Marc Yor

For Ecole Polytechnique

- Emmanuel Gobet
- Nizar Touzi

4th EUROPEAN SUMMER SCHOOL IN FINANCIAL MATHEMATICS

ETH Zürich & École Polytechnique

Zürich, 5-9 September 2011

An EMS Applied Mathematics school

The European Summer School in Financial Mathematics aims at bringing together the most talented young researchers in the field, looking at the very young who only just started their PhD studies. The Scientific Committee consists of European leaders and representatives of financial mathematics. We warmly thank them for their encouragement and for accepting to be part of this committee. Their first and main task is to identify the most promising candidates. The successful candidates will be sponsored for their travel and living expenses during the summer school.

The fourth European Summer School is held at ETH Zurich for the first time. We gratefully acknowledge the support of the French Federation of Banks (Fédération Bancaire Française) and the ETH Foundation.

The Summer School is centred around one or two advanced courses taught by widely recognised experts. We are particularly interested in a well-balanced combination of theoretical and practical input. We welcome suggestions for topics to cover in the courses. There will also be some students' seminars where the young participants can get teaching experience and discuss their research.

We hope that the Summer School leads to an active cooperation between the various European institutions. We also hope that the Summer School provides an opportunity to open doors for European students amongst different research centres. Our ambition is also to put the foundations for a more visible European network at the doctoral level in the field of financial mathematics. We very much count on the members of the Scientific Committee to help us make this wish a reality.

This school belongs to the series of the EMS Applied Mathematics schools.

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SMAI

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