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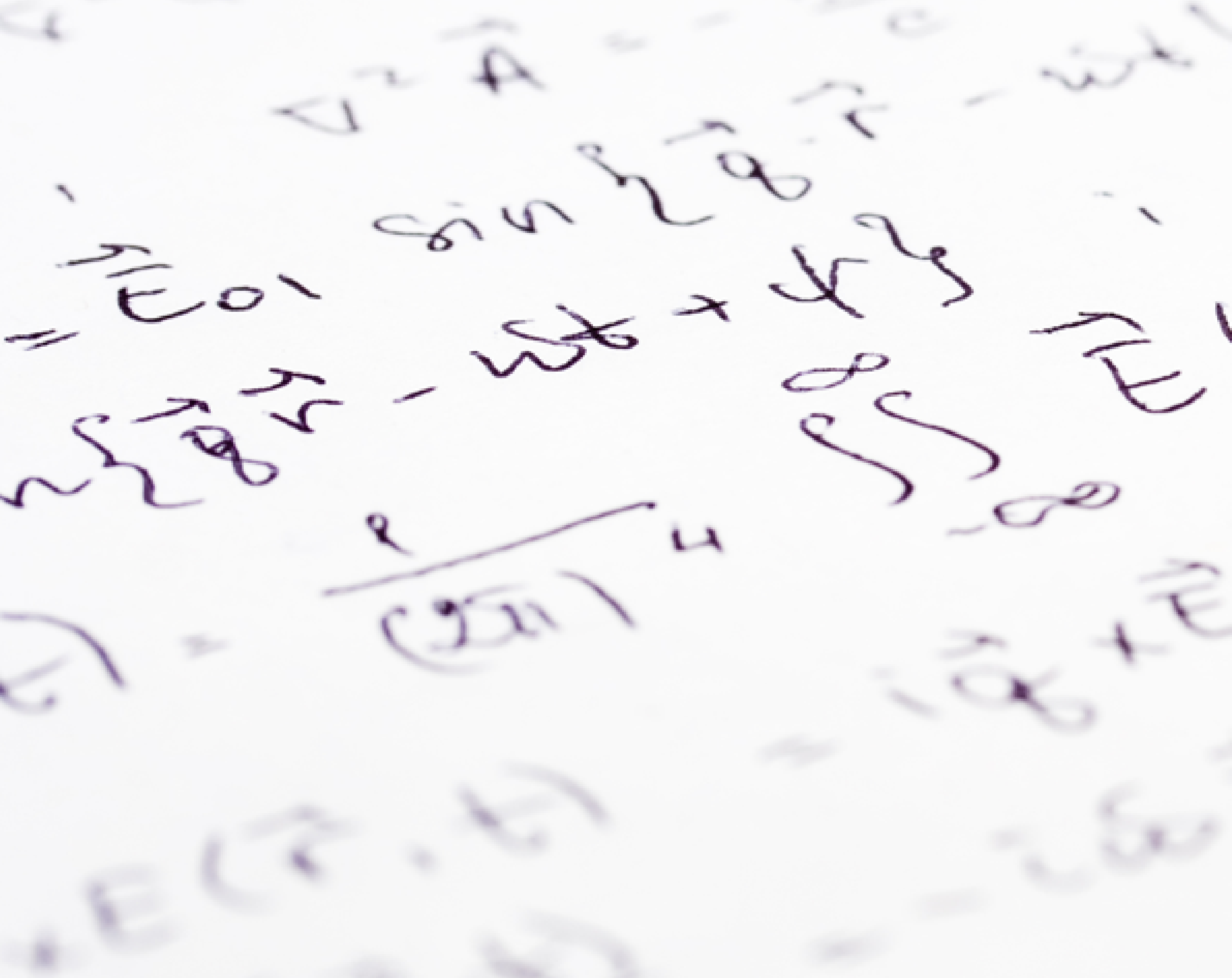

Evaluation of Mathematical Models in Finance

WITH

Pierre-Louis Lions
Olivier Guéant
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INVESTISSEMENTS D'AVENIR

#12 February 2014

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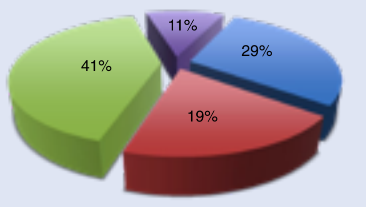

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INSTITUT EUROPLACE DE FINANCE Louis Bachelier

EDITO

The origin of the crisis

From the 1980s, the introduction of mathematics into financial practices accompanied a profound modernization of all sections of finance. This modernization has led to advances in the so-called real economy. Contemporary finance has in fact made it possible to manage economic risks (interest rate risk, currency risk, etc.), which was necessary for the growth of the global economy, investment and trade.

However, – an instance of the badly shod shoemaker – the financial industry failed to control, or even measure, its own growing internal risks. Thus after 25 years of advance, the financial sector led the world into the worst global economic crisis since 1929.

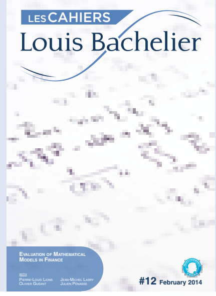
The important role of mathematical models in the pricing of credit derivatives, which were at the centre of the crisis, and more generally the obscure nature of the formulas used, made mathematicians and their models ideal scapegoats to explain the origins of crisis. All the more so since the role of finance itself was largely obscure to the general public, both with regard to its positive contributions and to the causes of the crisis and its spread to the real economy.

Clearly the reality of the situation is more complex, but it is undeniable that the lack of perspective on the part of quants regarding the models and mathematics they used played a major part in the failure to control the financial risks taken by financial institutions. Understanding what went wrong mathematically was therefore crucial, and lay behind the creation, in partnership with the Crédit Agricole, of the "Evaluation des Modèles Mathématiques en Finance" Research Initiative.

The effort by scientific teams to address this question has aimed to be complementary with the efforts of regulators, by focusing on the mathematical models used and more generally on the role of mathematics in finance. The specific nature of mathematical modelling – very different from modelling in physics, fluid mechanics or electromagnetism, for example – has often been little understood in trading rooms, and the Research Initiative has therefore conducted a study on the training of quants and its necessary evolution (see interview on page 4). Critical literature reviews have also been carried out on topics related to the crisis: calculation of Value at Risk in order to better understand extreme risks (see page 6), measurement of correlations so as to improve the modelling of multidimensional risks (see page 10), and the pricing of credit derivatives with a view to highlighting one of the major failures associated with the crisis (see page 12).

Lastly, the crisis has reminded us of the crucial importance of liquidity. Yet this is an issue completely ignored in the classic paradigm of financial mathematics. It is for this reason that the Research Initiative has launched an extensive research programme on liquidity. The first results from this work are discussed in this edition of the Cahiers (see page 14).

Pierre-Louis Lions



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BIOGRAPHIE

Pierre-Louis Lions

Pierre-Louis Lions, a French mathematician, has worked on partial differential equations and their applications. He was awarded the Fields Medal in 1994 while teaching at Paris-Dauphine University. Among other achievements, Pierre-Louis Lions was the first mathematician to give a complete solution to the Boltzmann equation and introduced, in collaboration with Michael Crandall, the notion of viscosity solutions, for which there are numerous areas of application. In recent years, in collaboration with Jean-Michel Lasry, Pierre-Louis Lions introduced and developed the theory of mean field games and its applications in economics and finance. Pierre-Louis Lions has received several other awards including the IBM Prize in 1987 and the Ampere Prize from the Academy of Sciences of Paris in 1992. He is Doctor Honoris Causa at Heriot-Watt University Edinburgh, City University of Hong Kong, the Ecole Polytechnique Fédérale de Lausanne, the University of Bucharest and the University of Santiago de Chile. He is currently professor at the prestigious Collège de France, where he holds the Partial Differential Equations and Applications Chair and he also teaches applied mathematics at the Ecole Polytechnique.

Quant training: a post-crisis issue

The Scientific Committee of the "Evaluation of Mathematical Models in Finance" Research Initiative has addressed the thorny issue of the training of quantitative analysts (or quants) and made important recommendations in order to develop current training (Graduate Programmes) and guide the creation of new training. Pierre-Louis Lions, who chaired the Scientific Council during this work, answers questions from the Louis Bachelier's Cahiers.

Can you tell us about the reasons that led the Research Initiative's Scientific Council to address the issue of quant training?

Since the early 1980s, financial mathematics has continuously transformed professional practice in trading rooms. Financial engineers and other quantitative analysts – known collectively as quants – became familiar and important actors in trading rooms. Then suddenly, within a few months, with the subprime crisis, there arose a deep mistrust of mathematical models and the people using them. In some countries, such as France, the reactions even became violent. Although it is apparent today that modelling was not necessarily responsible for all the problems, and was primarily a scapegoat, we are convinced that advances must be made in the use of mathematics by financial institutions. These advances mainly entail better training for quants, who often were unable to take a critical look at the models they were using. We therefore decided to analyse a large number of training programmes, especially training curricula that were most cited internationally, and from them we learned lessons for the future.

What are the main objections you have with regard to existing training?

All training courses have their specific characteristics and we did not want to award good and bad points. In general, there are three criticisms we made with regard to most courses: the overemphasis in the curriculum on the pricing of derivatives, teaching that is often overly theoretical, and insufficient interaction between mathematics, economics, finance, statistics and computer science.

Have quants not been generally hired to develop pricers?

Indeed they have. And in any case the most well-known model is Black-Scholes, which is an option pricing model. It is therefore natural that the pricing of derivatives plays an important role in quant training. But mathematics also has had considerable importance for portfolio management, asset allocation, risk management, execution issues, the energy markets, and so on, and there are many courses that only present the (linear) mathematical tools required for pricing, thus relegating convex optimization or stochastic optimal control to the background. Moreover, by presenting only those tools that are useful for pricing, most courses, paradoxically, do not provide the perspective needed for future quants to understand the very special character of pricing models, which are phenomenological models. The recurrent invocation of the Black-Scholes theory should not make us forget that in practice the Black-Scholes model serves only to transform prices into implicit volatilities – and not to provide prices.

In your report, you insist on the difference between modelling in physics and modelling in finance…

This difference is fundamental. Most derivatives pricing models endeavour to "explain" the price of a product by giving a virtual dynamic to the underlying price. This is very different from mechanics or thermodynamics, even though the partial differential equations used in finance are often similar to those used in physics. The difference between the two is often misunderstood, and this has serious consequences: it is absolutely essential for future quants to take a critical look at the models they will need to use. From this standpoint, too many training programmes simply provide reference models in each area, due to lack of time. To enable students to gain the necessary perspective on the models, they must be trained in numerical and statistical methods to backtest models, calibrate them, understand their validity limits, etc. Furthermore, risk management should play a much more important role in training, with a particular focus on model risk, which is often absent from curricula.

How do you reconcile the teaching of a greater number of theories with the need you have mentioned to make the teaching less theoretical?

First, we firmly believe that training should be provided over two years, not one year as is the generally the case today. This would allow time to both deepen the mathematical aspects and not be limited to the theoretical aspects. Case studies on the construction of models and their calibration/validation should be envisaged, while not omitting to analyse the limitations and shortcomings of modelling, which are just as important to be aware of, or even more so, than their successes. In order to build bridges between theory and practice, conducting collaborative projects with practitioners should also be more widespread, as should internships, which always ought to be an integral part of curricula.

Your third point concerns the need to introduce disciplines other than mathematics…

Yes, this is vital. We can mention some mathematical topics that would be worth including in the programmes – for example, Lévy processes, large deviations, backward stochastic differential equations – but the priority for training in financial mathematics should be to expand the provision of courses by linking up with departments of statistics, economics or computer science, even though the provision of computer science courses is usually already satisfactory. Introducing more econometrics, statistics and data analysis is essential. Further away from mathematics, giving future quants the basics in microeconomics and macroeconomics is important, as too is enabling them to acquire basic knowledge related to the professional world – accounting, regulation, corporate finance, etc. ¬¬– they will be entering. This much-needed disciplinary decompartmentalization is a difficult exercise for universities, and even more so in that the programmes cannot be set in stone. The needs of the industry have evolved over the last 20 years, and a balance is not immediately going to be reached, due to new problems arising from the crisis and to changes in regulation. The programmes need to be continually revised by having the members of various academic departments sit together around the table.

Your report goes further than simply training quants. It also discusses their role in financial institutions. What are your recommendations?

Quants are often assigned to specific desks. This is sometimes necessary and is not a problem in itself. However, to secure a degree of independence and to ensure the professional integrity of quants, it would be better if they did not depend hierarchically on specific desks but rather reported to a higher hierarchical level. Similarly, it is desirable that their compensation is not linked solely to the P & L of the desks they are assigned to. From this standpoint, quant assessment could be done in collaboration with independent experts. In the report we emphasize the potential role these experts could play, in particular that of intermediary between quants and the bank's management. It is impossible for general management to understand in detail what quants do, but external, independent experts could help provide an overview on the work of quants, something that is at times missing.

Reference

The Scientific Council's report: "On the training of quants".
http://www.idr-emma.org

Key points

- Training extending over two years should be favoured.
- Training programmes should expand their course offerings to disciplines other than mathematics, such as statistics, economics, computer science, etc.
- In relation to mathematics, quant training programmes are overly focused on the pricing of derivatives.

Value at Risk: theory vs practice

Value at Risk (VaR), which is essential in risk management and for regulatory reasons, is not calculated in the same way in every financial institution. A review of the academic literature on the methods for calculating VaR was carried out. A series of interviews with members of the risk departments of several major banking groups and with the regulatory authority allowed academic knowledge to be compared with banking practices and experience of the sector's actors.

Value at Risk, often referred to as VaR, is one of the most commonly used risk measures in the financial industry. Value at Risk is a very simple concept that, for a portfolio and a given confidence level p, gives a threshold of loss that is expected to be exceeded only (100-p)% of the time, over a given time horizon. Notwithstanding its well-known weaknesses (for instance, it is known not to be sub-additive), VaR is central to risk management and it plays an important role in the determination of banks' regulatory capital.

Although Value at Risk is a simple and widely used concept, its definition is, however, by no means constructive. Indeed the theoretical distribution of the PnL of a portfolio is not observable and must be estimated. Moreover, the portfolios concerned are often large and composed of complex financial assets: computing the Value at Risk of a portfolio is therefore both a modelling and a statistical problem.

Academics and practitioners usually divide the methods used to compute the Value at Risk of a portfolio into three groups: historical approaches, parametric methods – also called analytical methods – and Monte-Carlo methods. All these methods have in common a first step that consists in choosing a certain number of relevant risk factors, depending on the portfolio. The treatment then depends on the approach.

Historical approaches replay data from the past behaviour of the risk factors so as to evaluate what would have been the evolution of the price of the portfolio. VaR is then usually estimated using a quantile of the resulting empirical loss distribution, or an interpolation between two points of this empirical distribution. Historical simulations are widely used in the industry, using 1 or 2 years of past daily data. The main reason for their extensive use is that the methodology is well understood and the figures it produces are easy to communicate within the bank. A better reason for using historical simulations is that they allow the dependence between risk factors to be modelled in a non-parametric way, using the empirical distribution of the risk factors. However, basic historical approaches also have many serious drawbacks. Among these, the most important are the i.i.d.[1] assumption and asymmetry bias.

Such basic approaches are not model-free but rather assume i.i.d. realizations of the risk factors, despite well-documented effects such as volatility clustering of returns. To go beyond the i.i.d. hypothesis, the Hull and White approach or the Filtered Historical Simulations of Barone-Adesi and co-authors have been developed, that rescale risk factor returns to current volatility levels. Although more rigorous, these methods are often regarded by practitioners and regulators as being too procyclical.

The second drawback with using a quantile of the loss distribution obtained through historical simulation is that only a small proportion of the information available is used. In particular, when one computes the 99% VaR of a portfolio using 1 year of daily data (i.e. around 250 points), the Value at Risk will be determined by the value of the second or third largest loss. From one day to the next, assuming no change in the portfolio, a new realization will rarely change the VaR if it falls below the VaR, whereas the VaR figure will be changed (sometimes dramatically) when the new realization exceeds the previous VaR figure (i.e. asymmetry bias). To correct this bias, extreme value theory is useful. The method consists of estimating a quantile not too far in the tail (say 90%) and from this estimation deducing an estimation of the quantile one is interested in (here 99%) using a tail estimator (such as the Hill estimator). Unfortunately, extreme value theory seems to be very rarely used in practice, even though it allows more stable and accurate estimates for Value at Risk to be obtained.

The second way of computing VaR figures uses parametric methods. These approximate the portfolio PnL using Greeks, and assume a specific parametric distribution (usually Gaussian) for the risk factors. Value at Risk approximations are then obtained in closed form or using basic numerical methods. The main advantage of parametric methods is that they are not based on full repricing and work very fast. However, they may produce absurd VaR figures in the case of complex nonlinear portfolios. With the increase of computation speed and easy access to parallelization techniques, parametric methods should no longer be used, except possibly to have VaR approximations for designing variance reduction techniques for Monte-Carlo simulations (see below).

Monte-Carlo techniques constitute the third class of methods for computing VaR figures. They are widely used by practitioners, who regard them as being more forward-looking than historical methods. Monte-Carlo methods assume a parametric distribution for risk factors (often calibrated to past data) and draw values of the risk factors using this distribution before full repricing of the portfolio. In addition to being forward-looking, the main advantage of Monte-Carlo simulations is that the number of draws is not limited to one or two years of daily data as it is with historical simulations. Hence, the resulting quantile estimate is likely to be more accurate. However, although in theory any distribution for the risk factors can be used, the dependence between risk factors is often modelled with a Gaussian distribution, thus ignoring tail dependences that may impact VaR figures.

It is noteworthy that Monte-Carlo techniques are time-consuming. In fact, banks that decided to use Monte-Carlo methods often invested in hardware for parallel computing so as to deliver VaR figures on a daily basis. To accelerate computation, one can use variance reduction techniques such as importance sampling or stratification, as described in the work of Glasserman, Heidelberger and Shahabuddin. Surprisingly, we found that variance reduction techniques are almost never used in practice.

To conclude, the choice between historical simulations and Monte-Carlo simulations is based on a trade-off between the number of scenarios and the accuracy of the model for the dependence structure of the risk factors. We shall not decide which method is better, and in any case, the answer depends on the nature of the portfolio. However, for the two classes of methods, we noticed that academic advances coming from extreme value theory or variance reduction techniques could be used more often.

1. Independent and identically distributed

Reference

Literature review conducted: "Computing the Value at Risk of a Portfolio: Academic literature and practitioners' response."
http://www.idr-emma.org

BIOGRAPHIE

Olivier Guéant

Olivier Guéant is a former student of the Ecole Normale Supérieure (rue d'Ulm) and a graduate of the ENSAE, and has a PhD in applied mathematics from the University Paris-Dauphine. He currently holds the post of Associate Professor at the University Paris-Diderot, is a member of the Laboratoire Jacques-Louis Lions and teaches, among other courses, high frequency trading and optimal execution in the M2MO. His research interests include mean field game theory, market microstructure and optimal execution. Olivier Guéant's recent work has involved deducing liquidity premia from the market microstructure, based on the theory of indifference pricing.

Further reading...

- G. Barone-Adesi and K. Giannopoulos. Non parametric VaR techniques. Myths and realities. Economic Notes, 30(2):167–181, 2001.
- J. Boudoukh, M. Richardson, and R. Whitelaw. The best of both worlds. Risk, 11(5):64–67, 1998.
- L. De Haan and L. Peng. Comparison of tail index estimators. Statistica Neerlandica,52(1):60–70, 1998.
- P. Glasserman, P. Heidelberger, and P. Shahabuddin. Importance sampling and stratification for Value-at-Risk. Citeseer, 1999.
- P. Glasserman, P. Heidelberger, and P. Shahabuddin. Portfolio Value-at-Risk with heavy-tailed risk factors. Mathematical Finance, 12(3):239–269, 2002.
- B.M. Hill. A simple general approach to inference about the tail of a distribution. The Annals of Statistics, 3(5):1163–1174, 1975.
- J. Hull and A. White. Incorporating volatility updating into the historical simulation method for Value-at-Risk. Journal of Risk, 1(1):5–19, 1998.
- F. Jamshidian and Y. Zhu. Scenario simulation: Theory and methodology. Finance and Stochastics, 1(1):43–67, 1996.

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INVESTISSEMENTS D'AVENIR

Correlation measures: large-scale, high frequency and representation

Portfolio management, risk management, the pricing of options on a number of underlying securities ... in finance, there are correlations everywhere. But are they well estimated? Are the figures obtained correctly analysed? Are the concepts used by financiers the best available? To answer these questions, a review of the literature on measures of dependence was conducted over several months, and many practitioners were interviewed. interviewés.

BIOGRAPHIE

Jean-Michel Lasry

Jean-Michel Lasry is Emeritus Professor at Université Paris Dauphine and Chairman of the Steering Committee of the Finance and Sustainable Development Chair. Prior to his retirement in 2013, he was Senior Scientific Advisor at Crédit Agricole CIB (previously CALYON). He was also a member of the Executive Committee of CALYON Markets Activities for four years as well as the Global Head of Research & Capital Management. Before that, he was Deputy CEO of CPR Bank in Paris for four years. From 1994 to 1996, Jean-Michel Lasry was the CEO of the Caisse Autonome de Refinancement. From 1990 to 1993, he was a member of the Executive Committee of CDC Banking Divisions and a Board Member of CDC Gestion. He worked as a Professor at Université Paris-Dauphine and École Polytechnique for 17 years, and has had more than 100 papers published in mathematics and economics journals.

Whether in finance, statistics and genomics, estimating the correlation, or more generally the dependence structure, among several random variables is essential. But to measure the dependence structure, it must first be defined. For in talking about correlation, most people will think of linear correlation, otherwise known as Pearson correlation, which is defined in the case of two random variables as the covariance of the variables divided by their respective standard deviation.

In general, the dependence structure between two variables is characterized by a so-called copula function. But this copula is not a simple measure, because it is a function, not a number. From this copula, we therefore define simpler statistics, such as Spearman's ρ (i.e. the rank correlation), Kendall's τ, Goodman and Kruskal's coefficients, and tail dependency measures used for understanding extreme risks. Although these statistics are used less frequently than linear correlation, they have the merit of being based only on the dependence structure between variables and not on their respective marginal laws.

This is not the case for linear correlation, and this fact has consequences that are often ignored by practitioners. Thus the (linear) correlation between two variables is often compared to -1 or 1, even though the maximum and minimum limits are dependent on the respective laws of the two variables and may be closer to 0 than to -1 or 1. These habits clearly stem from a Gaussian framework, where the correlation alone captures the dependence structure and varies between -1 and 1.

If Pearson correlation, despite its capacity to measure linear dependencies only, is found everywhere in finance, it is because of the omnipresence of variance as a measure of risk. Hence accurate estimation of a correlation coefficient or correlation matrix is essential.

In the case of a coefficient, we know that the standard estimator is usually biased, and we know from Fisher that bias, like asymmetry in the distribution of the estimator, can be partly corrected. But in finance, the main problem is often estimating correlation or covariance matrices between a large number N of assets, since it entails estimating a large number, i.e. N (N-1)/2, of parameters (pairwise correlations), which is usually more or less the same order of magnitude as the number NT of data used to estimate them (where T is the time horizon taken for this calculation, which is limited for many reasons, including the likelihood of stationarity hypotheses regarding the correlations). This problem is particularly significant in portfolio management, where, in Markowitz's theory, the inverse of the covariance matrix plays a central role. Various methods have been proposed to improve the estimation of covariance matrices in a context of asset allocation. First of all, factor methods, which give a particular structure to the covariance matrix. Used in most commercial software, these methods contribute nothing to estimating the covariance matrix, but studies have shown that they are useful for estimating its inverse. Secondly, shrinkage, the best known method for "cleaning" a correlation matrix, involves considering a convex combination of the empirical correlation matrix and of a previously chosen correlation matrix. Finally, the theory of random matrices has allowed spectral methods for cleaning correlation matrices to be developed, by trying to distinguish, in the spectrum of an empirical correlation matrix, what is information and what is noise. This approach, developed by physicists (particularly the work of J.-P. Bouchaud and his co-authors – see below) and not by statisticians, gives very satisfactory results. The theory of random matrices has also helped develop methods to select the optimal time horizon for estimating correlation matrices.

While portfolio management has benefitted from the contributions of scientists (statisticians and econophysicists) on correlation methods for improving allocation methods and so-called out-of-sample risk measurement, other areas of quantitative finance have not stood still, even if the advances are less well known. For instance, measurement of correlation at the level of high frequency trading has profited from new approaches by econometricians and statisticians.

Because of the asynchronous nature of tick-by-tick data, empirical covariance is characterized by a bias towards 0 that becomes progressively marked as the observation frequency increases: the so-called Epps effect. To overcome this problem, several approaches have been suggested, including a Fourier approach proposed by Malliavin and Mancino. But the most common approach used to estimate correlation of asynchronous data is the one developed by Hayashi and Yoshida, after whom the Hayashi-Yoshida estimator is named. It should be noted that although these estimators allow a synchronization of series not to be used, they are nevertheless not robust for taking account of another effect related to high-frequency trading, namely the presence of microstructure noise which, though not generated by bias as in the case of volatility (signature plot), makes the standard deviation of the above-mentioned estimators soar. To mitigate microstructure noise, recent studies have proposed subsampling, double sampling or pre-averaging methods with the help of kernel estimators. This area of research is new and rapidly developing.

By way of conclusion, work on measures of dependence has recently taken a new turn. The work of statisticians in other disciplines – biology, linguistics, image processing, and now "big data" (to use the popular term designating new data arising from the growth of the internet, especially Web 2.0 and social networks) – confronted by the general problem of dependence structure, is opening up new paths that will certainly lead financial professional to renew their quantitative methodologies, especially in relation to questions of risk and asset management based on the analysis of dependencies. For example, graph theory techniques (spanning tree, clustering and graph representations, etc.) are developing and will become increasingly important. The sparse data problems predominant in "big data" are fast giving rise to new methodologies, whose impact will very probably be considerable in all financial areas concerned with dependencies (of which there are many, as we have seen above). It is therefore likely that the field of correlations and dependencies will change dramatically within the next three or five years.

Reference

Literature review conducted: "Correlation and dependence measures: Academic literature and selected financial applications".
http://www.idr-emma.org

Further reading...

- J.P. Bouchaud and M. Potters. Financial applications of random matrix theory: a short review. Arxiv preprint arXiv:0910.1205, 2009.
- K. Christensen, S. Kinnebrock, and M. Podolskij. Pre-averaging estimators of the ex-post covariance matrix in noisy diffusion models with non-synchronous data. Journal of Econometrics, 159(1):116{133, 2010.
- L. Zhang. Estimating covariation: Epps effect, microstructure noise. Journal of Econometrics, 160(1):33-47, 2011.

Pricing CDO's: a return to reality

With the subprime crisis, the financial modelling of credit derivatives emerged as one of the major failures of financial mathematics in these early years of the century. Despite the disappearance of the market, a "post-mortem" analysis of models for credit derivatives portfolio management and in particular for the pricing of synthetic CDOs has been conducted so as to learn from the mistakes made ... for the future.

With the emergence of multiname credit derivatives (such as CDOs) and their ever greater presence in the books of banks in the late 1990s and early 2000s, many "pricing" models appeared in the academic literature. Historically, the models initially proposed in the literature were inspired by those used for corporate bonds These were either models based on the dynamics of default intensities, or so-called structural models – inspired by Merton and his real options. Early approaches to default intensity (intensity-based models) were quickly criticized because they did not produce a sufficiently high correlation between defaults of the different underlying assets[1]. With regard to (inherently dynamic) structural approaches, they led to the famous Li model, based on the copula models[2] used by practitioners... until the disappearance of the market.

Li's model (or the one-factor Gaussian copula model) involves modelling the occurrence of a default through the crossing of a threshold by a Gaussian random (or latent) variable, the latent variables of different underlying assets being correlated pairwise at the same level of ρ correlation. The success of Li's model stems primarily from the fact that, since it is a copula model, we can first calibrate the parameters specific to each underlying asset, then the parameter ρ (rho), incorrectly termed correlation, partly capturing the dependence structure between defaults. Its success is also due to the parallel drawn with the Black-Scholes model for options – an erroneous parallel, to which we return.

The way the one-factor Gaussian copula model is used has evolved over time. The calibration of the correlation parameter ρ was first done, in the case of synthetic CDOs on iTraxx or CDX, tranche by tranche (compound correlation). However, the impossibility of calibrating the model to market spreads or, conversely, the existence of several correlation parameters consistent with market spreads caused practices to change (not to mention arbitrage and problems of addressing non-standard tranches in this framework). The calibration of the correlation parameter ρ was indeed subsequently done only on equity-type tranches, a mezzanine tranche thus being viewed as the difference between two equity tranches. This so-called base correlation approach ensures the uniqueness of the rho parameter. However, it was soon criticized when there was no correlation parameter compatible with market spreads. Moreover, like the volatility smile for options, the existence of a correlation skew stimulated the search for new models. Various papers have shown that the use of a double t copula or a NIG (Normal Inverse Gaussian) copula could reduce correlation skew. Other copulas have been suggested, such as Student's copula, Archimedean copulas and Marshall-Olkin copulas. Stochastic correlation approaches and local correlation have also been proposed. Finally, particularly to obtain models applicable to senior and super-senior tranches, the hypothesis of a constant recovery rate (arbitrarily assumed to be 40%) has been abandoned and variable or stochastic recovery rate models have been proposed.

The one-factor Gaussian copula model has remained one of the main references in trading rooms and practitioners readily speak of a model achieving consensus. Like the Black-Scholes model, with which one can compare very different option prices through the implicit volatilities arising from price, practitioners have appreciated being able to quote very diverse CDO tranches through a unique parameter, namely the rho correlation parameter of the Gaussian copula. However, it is well known that the Gaussian copula does not allow tail dependence. Moreover, using only one factor to model more than 100 underlying assets is a very questionable approximation. Finally, and most importantly, a parallel with the Black-Scholes model has been drawn in terms of portfolio and hedging management even though the credit derivatives market is inherently very incomplete3.

The Black-Scholes model is mistakenly viewed as a pricing model, but let us not forget that it is in fact a cost model, giving the replication cost (i.e. the production cost) of a product with a given payoff. The fact that this cost is also the price of the option then follows, for example, from the assumption that there is no arbitrage opportunity. In the Black-Scholes model, as in all complete market models, hedging and pricing are essentially the two faces of the same coin: hedging against price risk of the underlying asset(s) costs the price of the product. This logic would clearly not apply for multiname credit derivatives because the number of risks that could not be covered is high. Hence the models proposed in the literature are models calibrated on prices (in practice, on spreads) and not pricing models. Above all, their use for hedging is unfounded and turns out in practice to be particularly risky, as has been shown by the few papers dealing with hedging in addition to pricing. A multiname credit derivatives portfolio hedged against correlation risk in a one-factor copula model, and potentially against other risks, remains particularly exposed to… model risk! A risk that is too often ignored.

Quantitative analysts' main failure with regard to CDOs and the distrust of models that followed from this does not, however, stem from the models' lack of sophistication[4]. Rather we should blame the mistakes made in interpreting models, learn the lesson of the unfounded parallels made with the Black-Scholes model, and return to economic basics: when an asset is not replicable, its payoff is not intrinsically linked to a price and hedging can only be partial. In this respect, the case of options (even though the Black-Scholes model is clearly an approximation) is the exception and not the rule. Training of quantitative analysts, focussed worldwide on options pricing, has certainly played a major role in the mistakes made.

So-called pricing models calibrated on market prices, though complex, are unable to successfully manage complex product portfolios in an incomplete market. A financial portfolio management and therefore risk management approach proves to be indispensable.

1. *More recent models with affine jump-diffusion processes have shown better results.*
2. *In probability theory, the joint law of a set of n random variables can be described by the n marginal laws of the underlying random variables, to which is added a function, known as a copula function, characterizing the dependence between these random variables*
3. *cf. the inability in practice to hedge risks such as jump to default or the value of the recovery ratet*
4. *It should be noted that the academic literature has proposed many models other than those mentioned above, such as contagion models and top-down models*

BIOGRAPHIE

Olivier Guéant

Olivier Guéant is a former student of the Ecole Normale Supérieure (rue d'Ulm) and a graduate of the ENSAE, and has a PhD in applied mathematics from the University Paris-Dauphine. He currently holds the post of Associate Professor at the University Paris-Diderot, is a member of the Laboratoire Jacques-Louis Lions and teaches, among other courses, high frequency trading and optimal execution in the M2MO. His research interests include mean field game theory, market microstructure and optimal execution. Olivier Guéant's recent work has involved deducing liquidity premia from the market microstructure, based on the theory of indifference pricing.

Reference

Literature review conducted: "Copula methods and CDO pricing models".
http://www.idr-emma.org

Further reading...

- R. Cont and Y. Kan. Dynamic hedging of portfolio credit derivatives. 2008.
- D. Li. On default correlation: a copula function approach. Journal of Fixed Income,2000.
- A. Lipton and A. Rennie. Credit correlation: life after copulas. World Scientific, 2007
- G. Meissner. The Definitive Guide to CDOs. Risk Books, 2008.
- A. Mortensen. Semi-analytical valuation of basket credit derivatives in intensity-based models. 2005.

Liquidity: the big forgotten question

The mathematical models commonly used by banks were developed in a period of abundant liquidity and thus constitute a simplistic representation of reality. The purpose of this study is to draw up a critical inventory of our understanding of the concept of liquidity, by comparing economic theories to the accumulated experience of practitioners. Are our models pertinent for explaining the chronic instability of liquidity?

An essential feature of bank liquidity is its profoundly unstable nature. Liquidity undergoes a succession of states, even though most of the time it is in a state of relative equilibrium.

It thus has a cost, that of giving up an asset for a given period, a cost measured by the difference in price, at comparable quality of risk, between highly liquid and less liquid assets. This price of liquidity determines the quantities traded. In other words, liquidity is traded in the same way as any other economic good.

It is also subject to extreme conditions, of abundance and scarcity, which have succeeded one another over the past fifteen years. Until the 2007 crisis arose, an impression of ample and almost free liquidity predominated. Up to the early 2000s, liquidity – measured, for example, by the Euribor-Eonia swap spread – was virtually free up to one to two years and still inexpensive for longer periods (up 30 bp over 10 years, for example). It also had very low volatility and was traded among all actors, with no significant discrimination.

This state of abundance lasted more than twenty years and had a number of consequences, both for the management of banks and the organization of the market. Firstly, it gave a distinct role to large systemic banks. Their size enabled them to circulate liquidity from one point to another around the globe without friction costs. Currencies were thus fully fungible. It was possible, through the use of derivatives – which were also liquid – to borrow in the U.S. dollar market for 18 months and convert the amount into euros overnight. These big banks were, moreover, almost the only financial institutions to borrow from the ECB, although they were liquid. While they had surplus liquidity, the systemic banks would redistribute it.

Another significant consequence of the abundance of liquidity was that large banks could cheaply finance various market activities, particularly arbitrage on their own behalf. Assets that would have required long-term funding were thus financed in the short term. We can speak of a liquidity arbitrage business model that was not hampered by regulation or by the price of liquidity. In addition, there was the development of securitization activities, which allowed banks to increase their leverage, via the off-balance sheet.

This situation is fairly well understood by economic theory, which speaks of the compromise between liquidity and leverage. Recall that the traditional role of banks is to finance risky and illiquid assets through non-risky and liquid deposits, as explained by the now classic model developed by Bryant (1980) and Diamond and Dybvig (1983). This transformation of risk is the cause of the inherent fragility of the banking system and the rationale for its regulation. Thus, for example, the concept of "too big to fail", the role of which is to avert banking panics. It is rational for banks, knowing that central banks will provide liquidity in the event of crisis, to have high leverage, at the expense of liquidity. More precisely, given an increase in leverage among its competitors, a bank's best response is to increase its own leverage too (Tirole, 2011). Consequently, through competition among banks, a spiral develops that can lead them to take on excessive risk and at the same time results in a shortage of liquidity, first for banks and then in the market.

Liquidity began slowly drying up in 2001, following the attacks on the World Trade Center. Note first that the majority of systemic banks, which amply provided market liquidity in the 1980s, then mostly become deficient in liquidity. Causes for concern included the growth of mutual funds and life insurance – which were replacing intermediate savings with banks, for example – and especially the consequent endogenous rise in property prices. On September 11, the interbank settlement systems were blocked following the collapse of the Twin Towers, preventing liquidity trading among banks. Swap agreements between central banks and the resumption of clearing then restored liquidity. But belief in the fungibility of currencies had ended.

Despite the introduction of limits on currencies and closer monitoring of their liquidity, banks only partially drew the lesson of September 11. And for good reason: any deleveraging would have been penalized in terms of market share. With the stock market crash, liquidity continued to be abundant, fuelled by accommodating central bank policies. As short-term rates remained low, banks continued their leverage policies, which logically reduced their liquidity. Thus the low cost of liquidity contributed to the disappearance of that liquidity. To this must be added the introduction of Basel 2, which increases the cost of interbank lending.

This slow decline was abruptly interrupted by the 2007 crisis, which led to a drying up of liquidity by drawing attention to the solvency of counterparties, after the Lehman bankruptcy. Although the transition from abundance to scarcity occurred with remarkable suddenness, it should be noted that this breakdown was partly anticipated. A number of warning signs – a change in the behaviour of private bank lenders, worries about the U.S. housing market – were available to large banks, which had both the size and the organization to collect and analyse information that was often "privileged".

It is this ability to analyse a large amount of information that now distinguishes systemic banks, which are often able to foresee changes in liquidity. Such hitherto secondary information is today decisive in the money market, which is now characterized by instability. Only money market funds operate in the money market, which at present is held at arm's by the central banks. Discrimination between market players and volatility have reappeared: it seems that credit risk and liquidity risk are increasingly inseparable.

Three complementary economic ideas illuminate this new liquidity regime. First, the classical concept of the bank run, i.e. the self-fulfilling "race to the bank", can be generalized to the interbank market. Brunnermeier (2009) showed that any market that advantages early withdrawals of funds could be characterized as such a race. Second, the notion of adverse selection: when it is impossible to distinguish solvent banks from the rest, good quality banks withdraw from the market. Again, any doubt about the soundness of banks may be enough to trigger the collapse of the money market[1]. Finally, the concept of precautionary saving can explain the behaviour of some, overly conservative, banks which precipitate the collapse of the market[2].

Note that these theories are able, to some extent, to account for the predictable nature of liquidity crises. However, economic theory is still in its infancy with regard to the question of the interactions between these various concepts. The integration between microeconomics and macroeconomics thus still needs to be improved. Microeconomic regulation of banks needs to be able to quantify its own macroeconomic effects, just as macroeconomic models can no longer ignore microeconomic phenomena such as adverse selection. Economic theory does, however, provide a number of concepts, from which banks – which now include the cost of liquidity in their models – could benefit.

1. *See the famous paper on "The Market for 'Lemons'" (Akerlof, 1970). Malherbe (2013) uses a mechanism of this kind to show that the asset market can dry up in a self-fulfilling way*

BIOGRAPHIE

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A former student of Cachan Ecole Normale Supérieure and an ESSEC graduate, Julien Pénasse is a doctoral student at ESSEC and Cergy-Pontoise University. Before enrolling for his thesis, he spent nearly six years in trading room as a financial analyst at Natixis. His thesis focuses on bubbles and speculative behavior, especially in the art market. He also collaborates on the "Long Term Asset Allocation" Research Initiative (Collège de France/CNP/Caisse des Dépôts).

Reference

"Liquidity, Banks, and Markets: a Decade of Experience".
http://www.idr-emma.org

Further reading...

- Allen, Franklin, Elena Carletti, and Douglas Gale. 2009. "Interbanking Market Liquidity and Central Bank Intervention." Journal of Monetary Economics 56 (5) (June): 639–652.
- Brunnermeier, Markus K. 2009. "Deciphering the Liquidity and Credit Crunch 2007–2008." Journal of Economic Perspectives 23 (1) (January): 77–100.
- Bryant, John. 1980. "A Model of Reserves, Bank Runs, and Deposit Insurance." Journal of Banking & Finance 4 (4) (December): 335–344.
- Diamond, DW, and PH Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity." The Journal of Political Economy 91 (3): 401–419.
- Diamond, DW, and RG Rajan. 2005. "Liquidity Shortages and Banking Crises." The Journal of Finance 60 (2): 615–647.
- Malherbe, Frederic. 2013. "Self-fulfilling Liquidity Dry-ups." Journal of Finance.
- Tirole, Jean. 2011. "Illiquidity and All Its Friends." Journal of Economic Literature 49 (2) (June): 287–325