

LES CAHIERS

# Louis Bachelier



## SUSTAINABLE DEVELOPMENT, QUANTITATIVES APPROACHES

WITH

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#11 October 2013

## “Promoting, sharing and disseminating financial research”

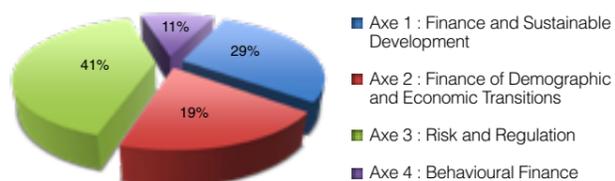
Created in September 2008, the Institut Louis Bachelier (ILB) is an internationally networked research centre with the mission of promoting, sharing and disseminating French financial research and teaching.

### The creation of scientific teams of excellence

The Institut Louis Bachelier is a unique organization that brings together, around industrial partnerships, the best research teams in economics and mathematics, as attested by the LABEX (Laboratoire d'Excellence) certification awarded to the ILB within the framework of its Finance and Sustainable Development project..

- Creation of research programmes directly linked to the financial industry : 30 Chairs and research initiatives have been created under the aegis of the Institut Europlace de Finance (EIF) and the Fondation du Risque (FDR) since 2007, involving more than 200 researchers.
- Management and organization of innovative R&D projects in collaboration with the Pôle Finance Innovation.
- Contribution to and support for the emergence of new training at undergraduate, masters and doctoral level in phase with the requirements of the Paris Stock Exchange.
- Cooperation with French, European, American and Asian universities and research centres.

Breakdown of 30 Chairs and research initiatives in terms of the four strategic axes of LABEX Finance and Sustainable Growth



### Enhancing the impact of research

The Institut Louis Bachelier disseminates the widest and most effective results from its research programs, particularly to French and European regulatory authorities.

- The quarterly review “Les Cahiers Louis Bachelier” presents research work from its Chairs and research initiatives in language accessible to a wide public.
- Publication of discussion papers aiming to clearly inform the public authorities and finance professionals on current topics.
- The “Recherche en Finance” portal in partnership with AGEFI.
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### Reflection and discussion at a European level

The Institut Louis Bachelier is a veritable crossroads for encounters and contacts with a view to encouraging interaction between the world of research and economic actors.

- Financial Risks International Forum : this annual event aims to present the best international research work and, by means of exchanges, discussions and round-tables, to address the concerns of economic actors.
- Thematic Semesters : organized in the form of lectures, seminars and courses, these thematic semesters aim to encourage exchanges between academics and professionals on shared problem areas.
- Chairs Day : held annually, this event aims to present and compare the work carried out in the context of the Chairs and research initiatives of the Institut Louis Bachelier.
- Scientific Mornings : occasions for reviewing the latest developments in financial research through the research projects supported by the Institut Europlace de Finance.

## INDEX

4 Is individual rationality an obstacle to sustainable development?  
By Ivar Ekeland

6 Should the installation of non-emitting power generation equipment be subsidized?  
By Nizar Touzi

10 Understanding the interactions between commodity derivatives markets and physical markets  
By Delphine Lautier and Bertrand Villeneuve

12 Introducing the scarcity function to anticipate the market price of electricity  
By Luciano Campi

14 How should the optimal trading dates in illiquid markets be determined?  
By Emmanuel Gobet

### Video



Find the Jean-Michel Lasry's video interview on [www.louisbachelier.org](http://www.louisbachelier.org)

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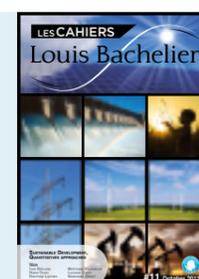
Developing new tools to better address sustainable development issues in the models used in economics and finance has been the aim of the Finance and Sustainable Development Chair since its creation in 2006. Created in conjunction with the Chair, the Finance of Energy Markets Research Center, a Research Initiative of the Chair, brings together academics and finance industry professionals and enables them to share their practical experience of these issues, to their mutual benefit.

To illustrate the diversity of research undertaken, we want in this issue of the Cahier to present a sample of the work currently being carried out through the Chair and the FiME Research Initiative. Thus Ivar Ekeland's contribution introduces the free rider problem as a major obstacle to global and sustainable management of natural resources. In the next article, Luciano Campi presents an innovative method for modelling futures contracts in electricity markets. Then Emmanuel Gobet considers the optimisation of numerical methods applied to hedging strategies. The research presented by Nizar Touzi aims to better understand the dynamics of investment in new non-carbon power generation technologies. Finally, Delphine Lautier and Bertrand Villeneuve summarize their modelling of commodity market equilibria (spot markets and futures markets).

We take the opportunity of this publication to thank EDF, Credit Agricole, the Europlace Institute of Finance Foundation and the Louis Bachelier Institute for their support and continued commitment over the past seven years. We also thank the scientific institutions – Université Paris-Dauphine, the Ecole Polytechnique and the CREST – which have hosted and supported this project. Finally we would like to thank the researchers who have devoted their skills, talents and energies to this project, making numerous scientific advances, of which this Cahier, for lack of space, can present only a small portion.

Jean-Michel Lasry, President of the Steering Committee of the Finance and Sustainable Development Chair.

René Aid, Director of the FiME Research Initiative



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# Is individual rationality an obstacle to sustainable development?

## A Ivar Ekeland's article

Sustainable development involves meeting the needs of the present generation without compromising the ability of future generations to meet their own needs. To comply with this criterion, mechanisms and institutions must be put in place that can overcome the free rider problem, namely that it is always in some people's interest to expect other people to do what they are not prepared to do themselves.

## BIOGRAPHY



### Ivar Ekeland

Ivar Ekeland is a former President of Université Paris-Dauphine, and held the Canada Research Chair in Mathematical Economics at the University of British Columbia until his retirement in 2011. He is a Fellow of the Royal Society of Canada, a foreign member of the Norwegian, Palestinian and Austrian Academies of Sciences. He holds honorary doctorate from the Universities of British Columbia, Saint Petersburg and Vienne. He has also written numerous books and papers in mathematics, economics and finance.

Since the industrial revolution, mankind has freed itself from Malthus's iron law: its growth is no longer limited by the food resources available. The world's population has increased from 700 million in 1700 to seven billion today. This tenfold increase gives only a partial idea of the impact of human beings on the planet: we live longer and better than our ancestors, i.e. we consume many more resources and over a longer period of time. These resources, long considered inexhaustible, are now reaching their limits: the seas are being depleted of fish, 0.5% of arable land disappears every year, the climate is becoming unstable, average temperatures are rising. The consequences, already visible today, will become much more serious in the long term: the Stern Review, for example, estimates that

if current trends continue, the effect of climate change alone will cost the world economy 10% of GDP every year.

The concept of sustainable development first appeared with the Commission on Environment and Development, known as the Brundtland Commission, set up by the United Nations in 1987. This defines sustainable development as "development that meets the needs of the present generation without compromising the ability of future generations to meet their own needs". This definition introduces two major differences in relation to economic development as it was understood and practiced in the twentieth century: the global management of resources and the introduction of the very long term.

## METHODOLOGY

Ivar Ekeland focuses primarily on analysing the free rider problem between generations. He assumes that each generation is sincerely committed to the principle of sustainable development: it were possible to engage in, and have its successors engage in, a long-term growth policy meeting this criterion, it would. Unfortunately this is not possible, because a government does not bind its successors. He shows that under these conditions, we can stop the destruction of the environment at a certain point, but we can never restore an environment that is already destroyed.

### The climate: a public good

Economists classify resources into renewables (crops, fish) and non-renewables (oil, minerals). For the former, it is a matter of ensuring the optimal renewal of the stock, whereas the latter become progressively exhausted. What is new today is that certain resources like air and water, which until now were thought of as inexhaustible, are becoming scarce, and therefore must also be managed. This management is complicated by the fact that it often concerns public goods. What economists call a public good, as opposed to

a private good, is a good that everyone can enjoy, without having to acquire or produce it: if I want to eat steak, I have to buy it, but the sun shines for everyone, good and evil alike. The quality of the air or water is a public good: everyone benefits from it or suffers because of it, whether or not one is a polluter. The problem with public goods is that they have to be financed, and because one can benefit from them without having paid for them, everyone behaves like a free rider. This is precisely what is happening at the moment in international negotiations on climate change. If we can control it, all nations will benefit, whether or not they have participated in the collective effort. They therefore try and ensure that it is the others who make the effort, and because everyone behaves in this way, nobody does anything. This is what economists call an equilibrium, and as long as it lasts the climate continues to deteriorate.

**“ we are all in agreement, but we want it to be others who do the work, and the result is that nobody does anything ”**

### From the long term to the very long term

The long term is no longer what it used to be. Sustainable development requires our governments to take into account the needs of future generations, i.e. a horizon extending to fifty or a hundred years rather than ten or twenty now. Note that future generations do not vote and have no way of making their preferences known: it is a great temptation for today's policy-makers and voters to say, like Louis XV of France, "Après moi le déluge" (roughly, "It's someone else's problem"). Even if they resist this

temptation, they may succumb to another, which is to postpone the efforts required, i.e. pass them onto the next generation. This is what we are currently seeing: the fight against climate change has taken a back seat, because we believe that getting out of the 2008 banking crisis is more urgent. The free rider problem re-arises: we are all in agreement, but we want it to be others who do the work, and the result is that nobody does anything.

Sustainable development is the final stage of globalization: we can no longer think of the economy outside of the – henceforth limited – framework of the planet Earth, and can no longer neglect the impact on it of human activity. For economic theory (and mankind), sustainable development raises the most difficult problems it has faced and, moreover, it is not clear that it will succeed in surmounting them. One thing is certain at this point: the market alone will not solve them.

## Key points

- The concept of sustainable development entails two key differences compared to the classical concept of development in economics:
  - the global management of resources;
  - adopting very long-term horizons.
- "Free-rider" phenomena constitute obstacles to implementation of sustainable development models. These phenomena arise between different actors in the same generation, but also an intergenerational level, between present and future generations.



Most scientific experts predict a complete melting of Arctic ice by summer 2040-2060.

This disappearance would have negative effects on wildlife, but could, in the meantime, have positive effects on shipping and resource exploitation.

Find the Ivar Ekeland's article on [www.louisbachelier.org](http://www.louisbachelier.org)



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- Ivar Ekeland, Yiming Long and Qinlong Zhou, "If not us, who? And if not now, when? - Growth theory and sustainable development" <https://www.ceremade.dauphine.fr/~ekeland/lectures/TIGER-31.pdf>



# Should the installation of non-emitting power generation equipment be subsidized?

What are the effects on private investment decisions of introducing a subsidy encouraging investment in non-emitting electricity production technologies? This is the question René Aïd, Nizar Touzi and Ren Zhen-Jie set out to answer by means of modelling. They seek to establish what would be the best balance of such a subsidy over time to achieve a given GHG (greenhouse gas) emissions reduction target in the power sector at the end date.

Governments today have two main tools for reducing GHG emissions: subsidizing non-emitting generation technologies or developing allocated allowances systems, through emissions permits markets. The effects of these policies are often complex and difficult to assess, given the diversity of electricity generation technologies and the specific characteristics of the sector. Whether they be carbon-emitting (coal, oil, gas) or renewable and non-emitting (solar, wind, geothermal, hydro, etc.), the various technologies have specific properties in terms of fixed investment and operational costs and in terms of equipment life-times. In addition, the energy mix should be adapted to the specificity of the electricity sector, i.e. a non-storable good, the demand for which has a strong seasonal component with annual peaks that are difficult to anticipate. In practice, one also needs to take into account the interactions

between the different sectors directly affected by emissions markets (electricity, construction, etc.) as well as people's expectations. The use of detailed models, covering a maximum amount of data, for analysing these phenomena makes it difficult to isolate a specific effect.

### A stylized model

In this study, the authors instead decided to use a very simple model, in order to isolate certain basic mechanisms that govern the dynamics of investment in a new electricity generation technology. It is assumed that there are only two technologies. The first, corresponding to one hundred per cent of the equipment at the beginning of the period, is carbon-emitting and has a low investment cost, but a relatively high operating cost. The new, non-emitting, technology calls for a relatively high-cost investment but has a lower opera-

ting cost. The perspective is then taken of a benevolent planner who aims to achieve a specific emissions reduction target at a given end-date, by subsidizing the new non-emitting production technology. The carbon market is also represented in the model, with actors being able to buy or sell allowances according to their needs. The price is endogenous and results from the average expectation of the state of the constraint at the end date. The planner's decisions depend on the demand for and changes in electricity prices; these changes are random but are (downwardly) affected by replacing carbon-emitting production equipment by non-carbon-emitting equipment. The planner also constantly takes into account the relative level of non-emitting capacities compared to emitting capacities, as well as electricity prices and carbon prices. He seeks to achieve his emissions reduction target at least cost, i.e. by minimizing the total generating cost of electricity and the amount of subsidies provided.

### A complex play of expectations

The planner's decisions derive from the combination of two distinct logics, corresponding to the two types of equations – forward and backward – that make up the model. On the one hand, the planner calculates the optimal proportion of the subsidy over time, based on all the information available at present and (probabilistically) in the future. On the other, there is an adaptive dimension: at any time, the planner may revise the calculation according to the penetration level of the new technology.

The combination of these two types of expectation generates a relatively complex dynamic, given the simplicity of the original assumptions. The initial numerical simulations have made this dynamic apparent, and have already produced a striking result.

**“ it is important to take into account the feedback effects in the emissions allowances market ”**

In the numerical simulations, investment in non-carbon-emitting energy equipment leads in all cases to a rapid depreciation in the price of emissions allowances: the prospect of a cleaner energy mix, and thus of a relative glut of allowances, leads to a fall in their price. This trend continues until the final penetration target of the new technology is reached.

### The conditions for an effective emissions reduction policy

What lessons can be drawn from the dynamics of such a highly stylized model? First of all, it suggests that if we want to subsidize investment in a particular non-emitting technology, it is important to take into account the feedback effects in the emissions allowances market. In practice, the effectiveness of this policy would be strengthened by a joint intervention by the public authorities in the emissions markets, with a view to reducing the quantity of emission allowances on an ongoing basis, as the new electricity generating technology develops, so as to support the price of allowances. In the absence of such a compensatory measure, the indirect effect of the subsidy in clean technology would run counter to the original purpose, since the effectiveness of the tradable allowances system would, at least in the short term, be significantly reduced.

## Key points

- Subsidizing the installation of non-emitting power plants can be counter-productive, if actors expect a further decline in the price of tradable emissions allowances.
- The combination of two instruments – subsidizing a new technology and a tradable allowances system – can generate complex dynamics in the short term.
- In this model, investment in non-carbon-emitting energy equipment leads in all cases to a rapid depreciation in the price of emission allowances.

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### Nizar Touzi

Nizar Touzi has been Professor of Applied Mathematics at the Ecole Polytechnique since 2006, and is vice-president of his department. He is co-director of the Changing Markets Chair and vice-president of the Finance and Sustainable Development Chair. He has published more than 80 papers in international journals and a book (Fields Institute Monographs, Springer). His work has earned him numerous distinctions and awards, including an Advanced Grant from the European Research Council in 2012. He is currently President of the Bachelier Finance Society.

## METHODOLOGY

Technically, the model deploys a system of forward-backward stochastic differential equations. The standpoint is adopted of a benevolent planner who is looking for the optimal non-emitting technology strategy to achieve a specified emissions level by a given date. The planner's only action lever is the level of subsidy, which depends on the demand for electricity, the relative non-emitting capacity compared to emitting capacities, and the price of electricity. The current price of emission allowances is endogenous and corresponds to the average expectations as to the price at the end date.

## Recommendations

- Take account of the retroactive effects of GHG reduction measures on existing regulatory mechanisms, particularly allowances markets.
- The effectiveness of a subsidy for a new non-emitting technology would be enhanced by a joint intervention by governments in emissions allowances market, so as to support the price of these allowances.



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Find the Nizar Touzi's article on [www.louisbachelier.org](http://www.louisbachelier.org)

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## BIOGRAPHY



### Delphine Lautier

Delphine Lautier is Professor of Finance at University Paris-Dauphine. A member of the Scientific and Steering Committees of the Chair "Finance and Sustainable Development", she is a researcher at DRM-Finance (Université Paris-Dauphine) and at the Fime Lab (University Paris-Dauphine, CREST, EDF). Since 2000, she has also been an associate research fellow at Mines Paris-Tech. Her main areas of research are commodity derivative markets and term structures. She has published a number of books and articles on this topic.

### Bertrand Villeneuve

Bertrand Villeneuve is Professor of Economics at Paris-Dauphine University, where he heads the financial economics team. He is a specialist in market structures. A graduate from Centrale Paris, formerly of the Toulouse School of Economics and a holder of the higher aggregation qualification, he is scientific director of the Risk Foundation and associate editor of the Journal of Risk and Insurance.

## Understanding the interactions between commodity derivatives markets and physical markets

What is the impact on physical commodity markets of operations implemented in futures markets, where financial contracts based on the price movements of these commodities are traded? Does speculation have a stabilising or destabilising effect on prices? It is these questions, among others, that Ivar Ekeland, Delphine Lautier and Bertrand Villeneuve seek to illuminate in their model.

Based on the paper "A simple equilibrium model for a commodity market with spot trades and futures contracts" by Ivar Ekeland, CEREMADE, Paris-Dauphine University, Delphine Lautier, DRM Finance Paris-Dauphine University, and Bertrand Villeneuve, Leda, Paris-Dauphine University.

The link between physical commodity markets and derivatives (futures) markets, where individuals try to cover themselves against price movements of commodities, is a longstanding and unresolved question, that regularly recurs in public debate, and all the more sharply when commodity prices are high. Such was the conclusion, for example, of the recent FAO report (2011), cited by the authors. The report also underlined the need for research to improve our understanding of the workings of these markets.

While the various links between prices in the physical market and prices in the paper market have long been clearly understood (through theories of normal backwardation and of storage), there is no unified theoretical framework to gather them together. The aim of Ivar Ekeland, Delphine Lautier and Bertrand Villeneuve is to make good this absence by developing a model,

which, though admittedly very stylized, provides an overall picture of the main relationships governing price changes in commodity physical and futures markets. To this end, they have taken into account the differentiated behaviour of the actors operating in these markets – industry, stockholders, speculators – and their interactions.

A simple model that nonetheless give a complete picture of the functioning of the markets

The characteristics of the model can be summarized as follows. It is concerned with the change between "today" (period 1) and "tomorrow" (period 2) in the price of a commodity in the physical market and in the futures market. In the physical market (the "spot" market), commodities are traded for immediate delivery: it is the place where producers meet users. Producers stock up or

destock, while users have industrial needs pertaining to the transformation of the raw material. Stockers buy today and deliver tomorrow, industrial players plan their future production today, hence their needs for tomorrow. These two functions are distinct and in a sense have opposite signs: the one plans a sale in advance when destocking, the other plans a purchase to meet their commitments.

In the futures market financial contracts are traded: agents seeking to hedge against rising prices in the future meet others fearing a decline in prices. Together they thus provide mutual insurance enabling them to buy or sell, according to their needs, a certain amount of the good at a price set today. These trades concern industrial users and holders of stocks, who are naturally complementary, but the market also attracts a another type of agent, namely speculators, who are interested not in the commodity itself, but in the risk they may take on in return for remuneration. A fourth agent category, spot traders, operate only in the spot market, in accordance with their immediate needs and with prices, which they help set.

### A model of great explanatory richness

The model describes a situation of simultaneous equilibria: in period 1 in the physical and paper markets, and in period 2 on the spot market. These equilibria depend on the decisions taken in period 1 by the three categories of agents, depending on their anticipation of the state of the spot market in period 2. The value of the model derives from to the possibility of comparing different equilibrium situations, when certain parameters are varied (what economists call an exercise in "comparative statics"). In this respect, the model reveals great explanatory richness, in view of the wide variety of situations it can generate. Thus, at equilibrium,

the spot price may be lower or higher than the futures price; stock holders may or may not choose to build up stocks; industrial users may or may not decide to hedge themselves in the futures markets. It all depends on how market fundamentals combine – the actors involved or technologies, for example – and on one-off shocks in the physical market. Intuitively, if this very simplified model can generate so many different configurations corresponding to real situations observed in commodity markets, is because it focuses on the right mechanisms and the relations that actually govern the interaction between the physical market and the futures market. The model also allows an analysis in terms of well-being, in the sense that it can compare the utilities of different types of agent in different situations. In particular it allows the utility of speculation and the utility of hedging to be clearly separated. Over and beyond the bullish effect – in general – on the spot price of speculators' activity, such activity must also be viewed as enhancing hedging possibilities for other agents (which underlies the approach in terms of utility).

More specifically, the model allows the impact of an increase in the number of speculators on other categories of agents to be measured. One important finding of this work is that industrial users and stock holders have conflicting interests in terms of hedging, regarding the entry of new speculators. In short, the arrival of speculators has no effect when the positions of industrial users and stock holders coincide perfectly, which is rare. In other cases, if one of the categories has greater needs than the other category can offer it, the arrival of new speculators is in its interest, since this will give it access to cheaper hedging. The other category loses out, since it benefited from its relative scarcity.

## Key points

- A stylized model that accounts for a wide variety of situations and deals in a unified framework with the main interactions between the different actors involved in these markets.
- In this model, while the financialisation of the market increases the possibility of hedging for industry and stock holders, it has opposite effects on these two categories of agent.



## Further reading...

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Find the Delphine Lautier's and Bertrand Villeneuve's article on [www.louisbachelier.org](http://www.louisbachelier.org)

## METHODOLOGY

Technically, the model is a rational expectations equilibrium model. It is assumed that agents share the same information about possible future states and the likelihood of these different states occurring.

It is a partial equilibrium model: it is concerned with changes in the price of a commodity, and ignores interactions with other markets.

The model allows the influence of certain parameters to be addressed by comparing different equilibrium situations ("comparative statics").

## Recommandations

- To assess the impact of future markets on physical markets, it is necessary to take into account not only the effects of speculation on commodity prices, but also the (often ignored) physical effects allowed by the hedging opportunities that these markets offer to key players.



## Introducing the scarcity function to anticipate the market price of electricity

How to model spot and futures contract prices in the electricity market is a long-standing issue. Electricity is distinctive in that it is not a storable commodity. So it is difficult to apply models that are specific to other markets. In the classical literature, there are various price calculation models that involve calculating spot and futures contract prices. In their research, René Aïd, Luciano Campi and Nicolas Langrené have modelled changes in electricity prices by taking into account events that are specific to this market.

Based on the paper by René Aïd, Luciano Campi and Nicolas Langrené “A structural risk-neutral model for pricing and hedging power derivatives” and on an interview with Luciano Campi.

## BIOGRAPHY



### Luciano Campi

Luciano Campi is Reader in Statistics at London School of Economics since September 2013. He previously was Professor of Applied Mathematics at LAGA, University Paris 13, and Associate Professor at University Paris-Dauphine. His main research interests are stochastic calculus, market frictions, information asymmetry, credit risk and energy markets. He has two PhDs. One from University of Paris 6 and one from the University of Padua. He is the author of many research articles in peer-reviewed international journals. He is a regularly invited as speaker to international conferences on financial mathematics and stochastic processes.

The liberalization of the electricity market in Europe and the United States starting in the 1990s has led many researchers to focus on modelling price changes in this market. The fact that electricity cannot be stored has given rise to many problems. In the classical literature, two approaches have been generally used. The first entails directly modelling the forward curve and from that deducing the spot price. Although this pragmatic approach models the prices of the available hedging instruments, it fails to make the link between fuels and energy prices. The second approach seeks to derive forward prices, under risk-neutral probability, from spot prices. But fact remains that in this approach most authors use an exogenous dynamic for spot prices. In their research, René Aïd, Luciano

Campi and Nicolas Langrené go further and propose a new model of price changes in electricity futures contracts. They then seek to answer the question of how to construct a complete model corresponding to the pricing constraints of derivatives. To do this, they take into account not only the price of fuels, but also the demand for electricity, the production capacity of power plants, and the notion of scarcity.

### A specific market that requires special modelling

The modelling procedure used for equity markets does not work on the electricity market. To build a model of electricity market prices, the researchers went through the fuel market, where the model traditionally applied to equities can be

used. The introduction of the production function then enabled them to link fuels and electricity forward contracts.

### Linking the fuel market to the electricity market through a production function

In the model, René Aïd, Luciano Campi and Nicolas Langrené considered five factors, including two fuels (coal and gas) and demand for electricity. For each parameter, they applied two capacity processes. The formula for electricity forward prices is thus written as a linear combination of the forward fuel prices. The researchers also introduced the no-arbitrage argument: to determine the price of derivatives contracts, they considered expectation in relation to risk-neutral probability. By taking into account the notion of scarcity, they were able to reproduce price spikes. When demand is close to capacity, then the function obtained is very high. This function drives prices up.

### Introducing the scarcity function to explain price spikes

The marginal cost of production is the cost of the last, and highest priced, fossil fuel used. On the spot market, i.e. paid for in cash, the price of electricity does not always correspond to the marginal cost of production. In the case of peak demand, for example, if power plant production is close to the maximum overall capacity, prices will then be driven up. In order to include price spikes in their model,

the researchers introduced a multiplier coefficient, which allows them to calculate the difference between the electricity spot price and the marginal cost of production when demand approaches the limit of capacity. This factor represents the scarcity of production capacity. This scarcity function is also applicable to futures markets. Through their model, the researchers were able to establish the following fact: the price of a forward contract at time “t” corresponds to the mathematical expectation of the spot price at the maturity of the contract.

### In the event of tension in the market, the electricity price deviates from the price of a fuel portfolio

With this model, the researchers were able to establish the following relationship: any electricity forward contract behaves like a portfolio of forward contracts on fuel. Provided, however, that all of these fuels have a maturity date that is not close: the equivalence between electricity future and a futures portfolio on fuels is confirmed if the maturity of the contract is distant. In a tighter market, for example when production capacity is approaching its maximum, the scarcity function introduced by the researchers makes the electricity price deviate from the price of the fuel portfolio.

“any electricity forward contract behaves like a portfolio of forward contracts on fuel.”

Find the Luciano Campi's article on [www.louisbachelier.org](http://www.louisbachelier.org)

## Key points

- The modelling methodology appropriate for equities markets is applicable to the electricity market, if a production function is introduced that enables a link to be made between the fuel market and the electricity market.
- The price of a forward contract at the time “t” corresponds to the mathematical expectation of the spot price that would be found at the maturity of that contract.
- In the event of market stress, the electricity price deviates from the price of a fuel portfolio.



## Further reading...

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- R. Aïd., L. Campi, N. Langrené and H. Pham (2012). A probabilistic numerical method for optimal multiple switching problem and application to investments in electricity generation. arXiv preprint arXiv:1210.8175.

## METHODOLOGY

The model developed for the spot price of electricity model is based on fundamental factors such as the demand for electricity, capacity, and the fuels used. The production function linking these factors and the price of electricity is chosen so as to reproduce price spikes. The theory of no-arbitrage produces a closed formula for the price of electricity futures as a weighted average of fuel futures prices, where the weights are based on non-tradable factors such as demand and capacity. The “local risk minimization” approach generates quasi-explicit formulas for option prices, like the mixture of Black-Scholes formulas.

## Recommendations

- Taking into account the scarcity function is essential for the correct modelling of electricity prices in futures markets.



## How should the optimal trading dates in illiquid markets be determined?

Physical markets are less liquid than equity markets. So people operating in commodities markets tend to trade less frequently than their colleagues specializing in stocks or bonds. When is it best to make investment decisions in these illiquid markets? What is the optimal intervention frequency for ensuring the best risk hedging? The model developed by Emmanuel Gobet and Nicolas Landon offers a mathematical answer to these questions.

Based on an interview with Emmanuel Gobet and on the paper “Almost sure optimal hedging strategy”, co-authored by Emmanuel Gobet of the Ecole Polytechnique and Nicolas Landon (doctoral student at the Ecole Polytechnique), with the support of GDF Suez.

Empirically, financiers operating in commodities markets decide their trading frequency themselves. Such intervention is often based on their experience of the market and may be daily, weekly or monthly, depending on the case. Emmanuel Gobet and Nicolas Landon have developed a mathematical model for determining an optimal trading frequency. Their work initially involved defining a study framework that would allow them to define the optimal moment for intervention in illiquid markets. They implemented their study with data from the GDF Suez group, which uses futures market techniques with a view to hedging against fluctuations in the price of gas and oil. The

two researchers set out to develop a strategy to minimize the error due to continuous market monitoring. To do this, they established an optimality criterion, which they have tested. In a second stage, they elaborated a strategy allowing this criterion to be minimized.

**Gamma, a good indicator of changes in portfolios in response to the market**

The optimality criterion is related to gamma. In an investment portfolio, different variables change over time. Gamma represents the rate of change of delta and expresses the acceleration with which a security loses or gains value in response to an

underlying movement. Empirically, market operators know that the higher the gammas, the more they must be sensitive to market fluctuations. Thus gamma is a good indicator of changes in portfolios in accordance with market movements, and professionals know that when a position has a high gamma, they need to be in front of their screens and be vigilant and responsive. For there will be more changes with a high gamma than with a low gamma. The strategy developed by Emmanuel Gobet and Nicolas Landon proposes intervening in accordance with the gammas. While traders in practice have felt that gammas influence when

“ In fact, this approach can be applied to all financial markets ”

it is appropriate to intervene and have used them accordingly, these researchers have, by means of their model, been able to quantify this phenomenon.

**BOX: Determining an intervention threshold using the gamma function**

By incorporating the position's gamma function into their model, Emmanuel Gobet and Nicolas Landon were able to define an indicator. Expressed as an intervention signal, this tells the market operator when to change his position in response to price changes (up or down) of more or less than X percent. Thus, so long as the market has not shifted above or below this threshold, operators do not change their position. The percentage is defined according to the asset's gamma and allows triggering thresholds to be calculated. When the percentage is exceeded, operators can then adjust their positions. Intervention times are then optimized, since traders only operate when these thresholds are crossed. For the first time, the model developed by Emmanuel Gobet and

Nicolas Landon opens up the possibility of automated intervention strategies.

**A model applicable to a combination of several commodities**

Going further, Emmanuel Gobet and Nicolas Landon have applied their model to an asset portfolio. The formula thus developed can specify triggering thresholds applicable to a mixed basket of assets. The function will depend on a matrix constructed from each asset's gamma function.

For example, a manager with both gas and oil in the same portfolio would be able, through the model, to

determine the intervention threshold, which would be the resultant of a calculation made on the basis of each asset's gamma function and of cross gammas. This single intervention threshold is applicable to the entire portfolio. Thus, even with multiple risks, the matrices constructed from gamma functions can determine an appropriate intervention threshold for a portfolio combining a number of assets.

**An intervention threshold also applicable to liquid markets**

For Emmanuel Gobet, “the commodities application was only a pretext for this work. In fact, this approach can be applied to all financial markets, including the most liquid”. Based on the different gamma functions, the matrix developed can define an intervention threshold applicable to all asset portfolios, whether they be illiquid or, conversely, highly liquid. The model developed by the two researchers offers traders the ability to set automated intervention thresholds, irrespective of the market in which they operate.

### Key points

- Gamma is a good indicator of changes in the portfolio in response to markets.
- Using the gamma function, it is possible to determine intervention thresholds that automate the market operators' strategies.
- This model can be used for a portfolio of assets.
- The model can also be applied to assets that are more liquid than commodities.



### Further reading...

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- E. Benhamou, E. Gobet and M. Miri. Analytical formulas for local volatility model with stochastic rates. Quantitative Finance, Vol.12(2), pp.185-198, 2012.

## BIOGRAPHY



### Emmanuel Gobet

Emmanuel Gobet is Professor of Applied Mathematics at the Ecole Polytechnique, with some fifty international scientific publications to his name. His work focuses on probabilistic numerical methods, stochastic analysis, the statistics of processes and financial mathematics. He regularly collaborates with financial institutions and insurance and energy companies.

## METHODOLOGY

It is a question here of optimizing the moment of intervention for hedging a financial contract. Continuous intervention leads to perfect risk hedging, whereas if done intermittently a residual risk remains. The paper provides two main optimality results.

1. Given a range of intervention frequency, whatever the strategy, it is not possible to reduce residual risk below a certain threshold, explicit in the contract's gamma function.
2. The model provides an optimal intervention strategy that attains this minimum. The optimal strategy is based on triggering thresholds that depend non-trivially on gamma.
3. For this purpose, new probabilistic convergence results are shown, valid on most models and contracts to be hedged.

## Recommendations

- While gamma is a good indicator of changes in response to the market, the gamma function is an even more accurate indicator that allows triggering thresholds for intervention to be determined.
- These thresholds are effective not only on a single illiquid asset, but on a portfolio of assets, whether they be relatively illiquid or highly liquid.

Find the Emmanuel Gobet's article on [www.louisbachelier.org](http://www.louisbachelier.org)



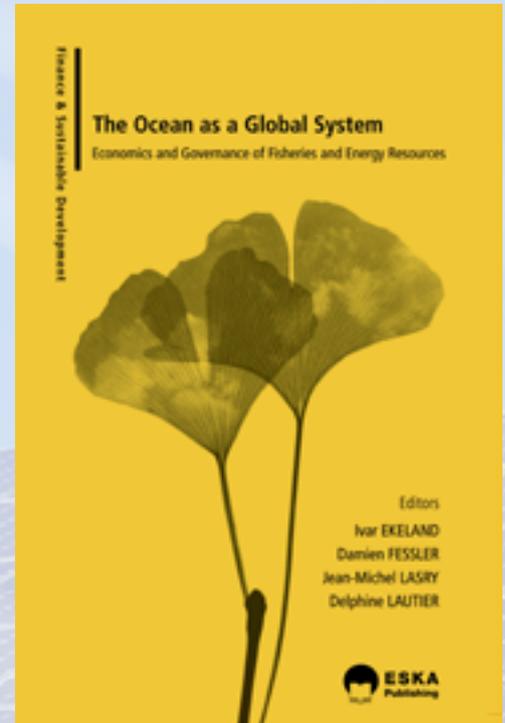
**CHAIR**

Finance & Sustainable Development

## **The Ocean as a Global System** Economics and Governance of Fisheries and Energy Resources

Ivar Ekeland, Damien Fessler, Jean-Michel Lasry et Delphine Lautier, Eska, 2012

This book gathers together contributions from the Symposium “The Ocean: Green Shipping and Sustainable Energy”, held in Paris on 28-29 April 2011. The aim of the conference was to address critical issues regarding the ocean, considered successively as a global ecosystem, as a global energy system and as a global regulation system. The first part of the book is concerned with the current state and the future of fisheries. The second part deals with energy-related maritime activities, while the third offers a global perspective on these issues.



## **The Economics of Sustainable Development**

Jean-Michel Lasry, Delphine Lautier & Damien Fessler (editors), The Economics of Sustainable Development, Economica, 2010

This book brings together the lecture notes from the seminars held at the Finance and Sustainable Development Chair from 2007 to 2009. The first part is concerned with climate change policies, with particular emphasis on the relevance of traditional economic theories. The second part deals with economic concepts and methodologies associated with sustainable development. It presents the precautionary principle, discusses the concept of externalities and introduces the methodology of mean field games. The third part is devoted to models and empirical applications in various fields, such as water resources, gas emissions, and a sustainable development path. The fourth part deals with carbon markets, along with their theoretical justification and historical development. The last part focuses on socially responsible investment and gives insights in regard to its definition, its links with sustainable development, and the way it could be used for investment strategies.

