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Credit derivatives pre- and in crisis: The importance of properly accounting for extreme scenarios in valuation

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This work expresses the author's opinion

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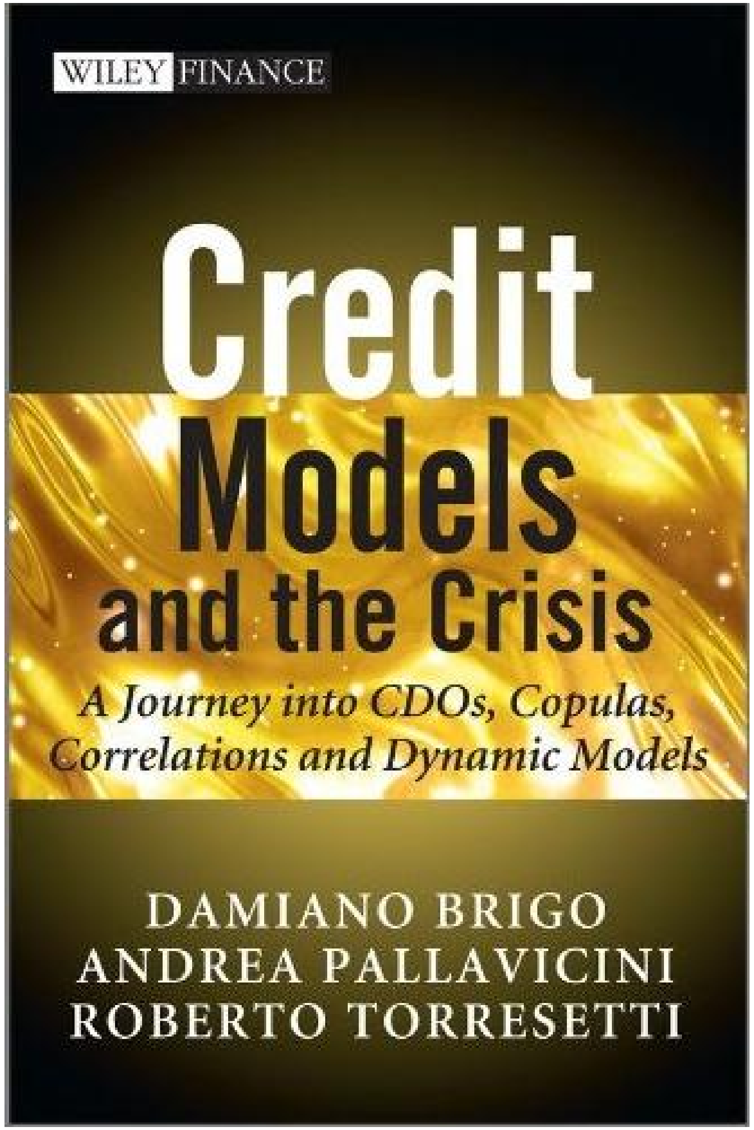

1: Forthcoming
April 2010

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2: See also
"Credit Models and the crisis
or: How I learned to stop
worrying and love the CDOs".
Available at
arXiv.org, ssrn.com,
defaultrisk.com

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3: Related papers in:
Mathematical Finance,
Risk Magazine, IJTAF
(see references below)

Outline

- Derivatives and Models to be blamed for the crisis? Two case studies
- Credit Index Options: flaws and "singularities" in the market formula
- Credit Crisis and Mis-pricing between earlier and No-Arbitrage Formulas
- CDO's: consistent calibration across capital structure and maturity
- Results and armageddon-warnings pre-crisis
- Enlarging the picture: Is the crisis really due to poor modeling?
- Conclusions

First case: The Credit Index options

The credit crisis started in 2007 has reduced dramatically the liquidity for the majority of multiname credit derivatives.

One exception has been the *Credit Index Option* market, supported by the sharp increase in historical and implied volatility.

During the credit crisis, *Payer* options have protected investors from the rise of the credit indices, and trading has restarted on these products just two weeks after the beginning of the crisis in July 2007.

The Credit Index

Portfolio of n names with initial notional $N(0) = 1$. Each name has notional $\frac{1}{n}$. τ_i default time of name i, with associated loss $\left(1 - Rec^i\right)\frac{1}{n}$. Total portfolio loss by t is denoted by $L(t)$.

To simplify notation we assume a flat recovery R across names, but the model works also under the more general assumption.

At time t, the Outstanding Notional is $N_t = 1 - L(t) / (1 - R)$.

An index position has two legs.

Protection leg, offered by the protection seller to the protection buyer, pays protection payments in case of defaults.

Premium leg, offered by the protection buyer to the protection seller, pays fee payments on the outstanding notional left at each (quarterly) payment date.

The Index payments

The discounted payoff of the Protection Leg is

$$Prot_t^{T_A,T_M} = \int_{T_A}^{T_M} D(t,u)\, dL(u) \approx \sum_{j=A+1}^{M} D(t,T_j) \left[L(T_j) - L(T_{j-1})\right]$$

By $D(t,T)$ we indicate the discount factor from T to t. Its expectation is the corresponding bond price, $P(t,T) = \mathbb{E}^Q \left[D(t,T)\mathbf{1} \middle| \mathcal{F}_t \right]$.

The discounted payoff of the Premium Leg is

$$Prem_t^{T_A,T_M}(K) = \left\{ \sum_{j=A+1}^{M} D(t,T_j)\, \alpha_j \left(1 - \frac{L(T_j)}{(1-R)}\right) \right\} K$$

The quantity in curly brackets is called $DV_t^{T_A,T_M}$.

Evaluating Premium and Protection Legs

The value of the two legs is computed by expectation under the risk neutral probability measure $\mathbb{Q}$. We use the notation $\Pi\left(X_t\right) = \mathbb{E}\left[X_t \middle| \mathcal{F}_t\right]$.

The Payer Forward Index starting at T_A and lasting until T_M has a price

$$\Pi\left(I_t^{T_A,T_M}\left(K\right)\right) := \mathbb{E}\left[I_t^{T_A,T_M}\left(K\right)\middle| \mathcal{F}_t\right] := \mathbb{E}\left[Prot_t^{T_A,T_M} - Prem_t^{T_A,T_M}\left(K\right)\middle| \mathcal{F}_t\right]$$

Substituting the payoffs, one sees that the Index value does not depend on the Loss distribution but only on the Expected Loss $\boxed{\mathbb{E}_t\left[L\left(T\right)\right]}$ at different maturities.

Index Options

A **payer** Index Option with inception 0, strike K and exercise T_A, written on an index with maturity T_M, gives the right but no obligation to *enter at* T_A *into the running Index with final payment at* T_M **as protection buyer** *paying a fixed rate* K.

However, the option buyer would give away protection from inception 0 to maturity T_A. In order to attract investors, standard Credit Index Option payoff includes the payment of the losses from the option inception to T_A as well (**front end protection**)

$$F_t^{T_A} = D(t, T_A) L(T_A) \quad \Pi\left(F_t^{T_A}\right) = \mathbb{E}\left[D(t, T_A) L(T_A) | \mathcal{F}_t\right].$$

The market approach

$$\tilde{I}_t^{T_A,T_M}(K) = Prot_t^{T_A,T_M} - Prem_t^{T_A,T_M}(K) + F_t^{T_A}.$$

It is natural to give a new spread definition, setting to zero $\Pi\left(\tilde{I}_t^{T_A,T_M}(K)\right)$.
This leads to the *Loss-Adjusted Market Index Spread*

$$\tilde{S}_t^{T_A,T_M} = \left[\Pi\left(Prot_t^{T_A,T_M}\right) + \Pi\left(F_t^{T_A}\right)\right] / \Pi\left(DV_t^{T_A,T_M}\right)$$

that allows to write the option price as

$$\mathbb{E}\left[D(t,T_A)\left(\Pi\left(DV_{T_A}^{T_A,T_M}\right)\left(\tilde{S}_{T_A}^{T_A,T_M} - K\right)\right)^+ \middle| \mathcal{F}_t\right].$$

"Numeraire" $\Pi\left(DV_{T_A}^{T_A,T_M}\right)$ and lognormal $\tilde{S}_t^{T_A,T_M}$: Mkt Index Option Form.:

$$\Pi\left(DV_t^{T_A,T_M}\right) Black\left(\tilde{S}_t^{T_A,T_M}, K, \tilde{\sigma}^{T_A,T_M}\sqrt{T_A}\right). \qquad (1)$$

The three flaws of the Standard Formula

1. The definition of $\tilde{S}_t^{T_A,T_M}$ is valid only when the denominator

$$\Pi\left(DV_t^{T_A,T_M}\right) = \sum_{j=A+1}^{M} \mathbb{E}\left[D(t,T_j)\,\alpha_j\left(1 - \frac{L(T_j)}{(1-R)}\right)\middle|\mathcal{F}_t\right]$$

   is different from zero. Since $\Pi\left(DV_t^{T_A,T_M}\right)$ is the price of a portfolio of defaultable assets, it can go to zero.

2. When $\Pi\left(DV_t^{T_A,T_M}\right) = 0$ the pricing formula (1) is undefined.

3. Since it is not strictly positive, $\Pi\left(DV_t^{T_A,T_M}\right)$ would lead to the definition of a pricing measure not equivalent to the standard risk-neutral measure.

Subfiltration Pricing without armageddon

We adapt a technique used earlier by Jamshidian (2004) and Brigo (2005) for single name CDS options, making use of a *subfiltration structure*, separating default free information from information on the default event. This is based on the filtration-switching (FS) formula.

To effectively use FS in index options context, define the last to default time

$$\hat{\tau} = \max(\tau_1, \tau_2, \ldots, \tau_n)$$

and define a new filtration $\hat{\mathcal{H}}_t$ comprising all information except the last default occurrence.

Using a number of advanced probabilistic techniques described in the full paper, one can arrive at the following rigorous formula for index options.

The Arbitrage-free Formula at time 0

$$\Pi\left(Option_0^{T_A,T_M}(K)\right) =$$

$$= \Pi\left(DV_0^{T_A,T_M}\right)\left[Bl\left(\tilde{S}_0^{T_A,T_M} - \frac{\boxed{(1-R)\,P(0,T_A)\,\mathbb{Q}(\hat{\tau}\le T_A)}}{\Pi\left(DV_0^{T_A,T_M}\right)}, K, \hat{\sigma}^{T_A,T_M}\sqrt{T_A}\right)\right.$$

$$\left. + \frac{\boxed{(1-R)\,P(0,T_A)\,\mathbb{Q}(\hat{\tau}\le T_A)}}{\Pi\left(DV_0^{T_A,T_M}\right)}\right]$$

This formula has a general version at time t with a spread $\hat{S}_t^{T_A,T_M}$ that always exists, being defined under the filtration that does not see the armageddon time $\hat{\tau}$.

Now we apply the formula in practice.

The Arbitrage-free Formula at time 0

Notice that the use of a Black formula is not necessary. We could use any martingale dynamics with smile and jumps, leading to a richer formula.

Also, we will resort to base correlation for the armageddon probabilities but this is also not necessary, we could resort to a full fledged dynamic loss model (later).

However, we aim at showing that properly dealing with the armageddon event with two of the models that are closest to the traders daily practice already makes a huge difference post-crisis.

Credit Index Options before and after 2007 subprime crisis

For computing $\hat{\tau}$ probabilities, one needs the correlations associated to the most senior tranche $\left[\frac{n-1}{n}(1-R),(1-R)\right]$ (very thin), at a short maturity. Market agrees correlation increases with seniority and decreases with maturity. We expect a correlation higher than the highest level quoted by the CDX market, $\rho_{30\%}$. However we consider a range of equally spaced correlations in-between i-Traxx and CDX most seniors, because

1) Crossover is a less senior Index, usually more subject to idiosyncratic risk. However, analysis of the most senior index in the North America market, the CDX.NA.IG, and the most junior, CDX.NA.HY, shows that this difference is very low for very senior tranches, since their risk is related mainly to systemic credit events.

2) Secondly, this choice tends to *underestimate* the probability of $\hat{\tau}$, compared to the standard market approach of extrapolating correlations. Thus the relevance of the new formula will be underestimated.

Options on i-Traxx Europe Main - 2007 vs 2008

In the next table we report the market inputs. The bid-offer spread for options in March 08 was in the range 5-8 bps.

	March-09-07	March-11-08
Spot Spread 5y: $S_0^{9m,5y}$	22.50 bp	154.50 bp
Forward Spread Adjusted 9m-5y: $\tilde{S}_0^{9m,5y}$	23.67 bp	163.60 bp
Implied Volatility, $K = \tilde{S}_0^{9m,5y} \times 0.9$	52%	108%
Implied Volatility, $K = \tilde{S}_0^{9m,5y} \times 1.1$	54%	113%
Correlation 22% I-Traxx Main: $\rho_{0.22}^I$	0.545	0.912
Correlation 30% CDX IG: $\rho_{0.3}^C$	0.701	0.999
Annuity 9m-5y: $\Pi\left(\gamma_0^{9m,5y}\right)$	3.993	3.912

Market Inputs: : March-09-07 (left), March-11-08 (right)

Options on i-Traxx Europe Main - March 2007

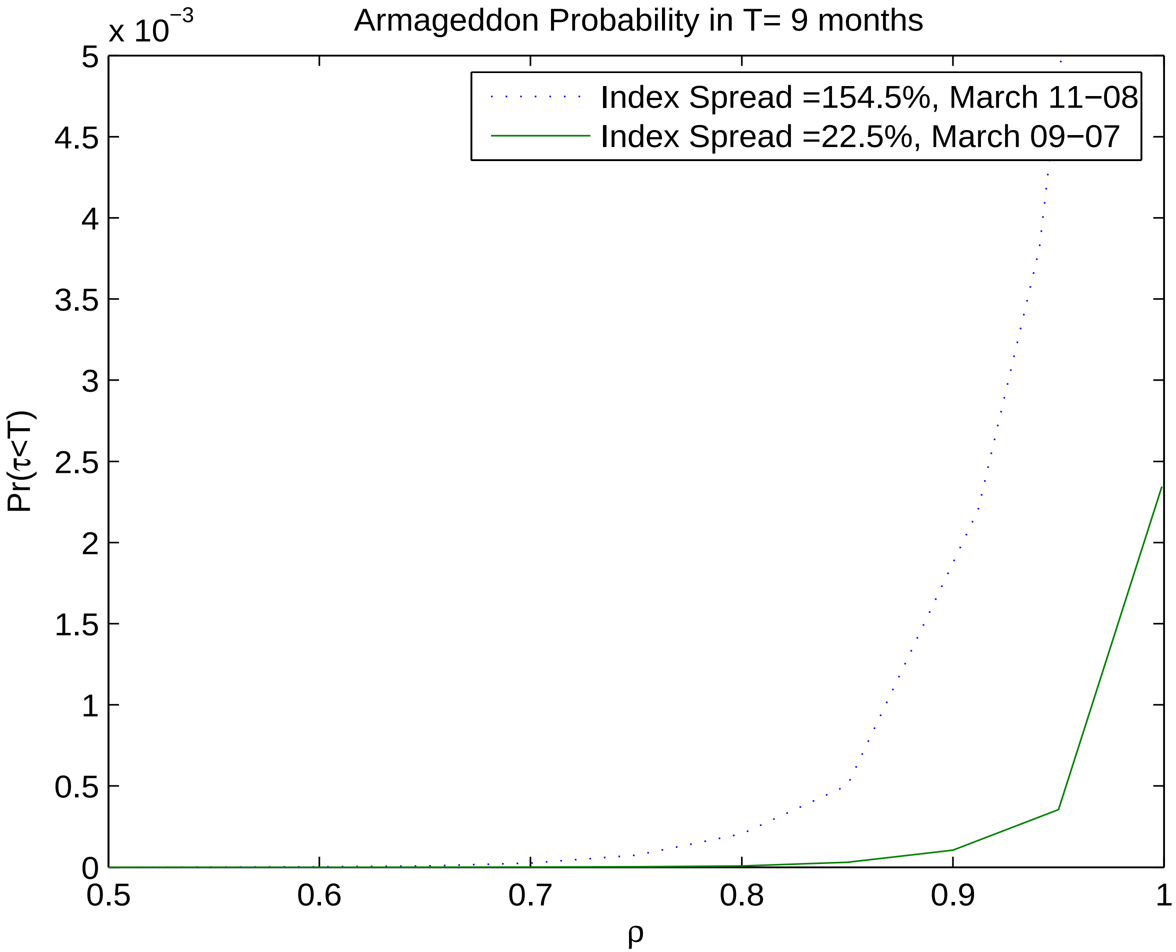
Strike (Call)	26	21
Market Formula	**23.289**	**11.619**
No-Arb. Form. $\rho = 0.545$	23.289	11.619
No-Arb. Form. $\rho = 0.597$	23.289	11.619
No-Arb. Form. $\rho = 0.649$	23.289	11.618
No-Arb. Form. $\rho = 0.701$	23.286	11.614
Strike (Put)	26	21
Market Formula	**13.840**	**21.069**
No-Arb. Form. $\rho = 0.545$	13.840	21.069
No-Arb. Form. $\rho = 0.597$	13.840	21.069
No-Arb. Form. $\rho = 0.649$	13.840	21.069
No-Arb. Form. $\rho = 0.701$	13.843	21.071

March-09-07 Options on i-Traxx 5y, Maturity 9m

Options on i-Traxx Europe Main - March 2008

Strike (Call)	180	147
Market Formula	**286.241**	**189.076**
No-Arb. Form. $\rho = 0.912$	277.668	179.624
Difference	**8.573**	**9.453**
No-Arb. Form. $\rho = 0.941$	271.460	172.769
Difference	**14.781**	**16.307**
No-Arb. Form. $\rho = 0.970$	258.887	158.862
Difference	**27.354**	**30.215**
No-Arb. Form. $\rho = 0.999$	212.867	107.630
Difference	**73.374**	**81.447**

March-11-08 Options on i-Traxx 5y, Maturity 9m


x 10^−3
Armageddon Probability in T= 9 months
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Pr(τ<T)
Index Spread =154.5%, March 11−08
Index Spread =22.5%, March 09−07
0.5
0.6
0.7
0.8
0.9
1
ρ

Figure

Second Case: CDO's (iTraxx, CDX...)

CDO's have been among the most mentioned derivatives in relation with the crisis.

Their liquidity has dramatically diminished since the crisis begun.

TRANCHE: default leg pays to protection buyer the *tranched-loss* increments each time one or more defaults impact the tranche or until maturity. Tranched loss is

$$L_t^{A,B} = [0 \times 1_{\{L_t < A\}} + (L_t - A)1_{\{A \leq L_t < B\}} + (B - A)1_{\{L_t \geq B\}}]/(B - A)$$

Default legs are balanced by periodic payments on outstanding notional of an "equilibrium spread", $S^{A,B}$ (the premium leg).

Market quoted fair spreads for tranches or sometimes upfronts with fixed spreads.

The GPL and GPCL Models

We model the total number of defaults in the pool by t as

$$Z_t := \sum_{j=1}^{n} \delta_j Z_j(t)$$

(for integers δ_j) where Z_j are independent Poissons. This is consistent with the Common Poisson Shock framework, where defaults are linked by a Marshall Olkin copula.

$$\text{Example :} \quad n = 125, \quad Z_t = 1\ Z_1(t) + 2\ Z_2(t) + \ldots + 125\ Z_{125}(t).$$

If Z_1 jumps there is just one default (idiosyncratic), if Z_{125} jumps there are 125 ones and the whole pool defaults one shot (total systemic risk), otherwise for other Z_i's we have intermediate situations (sectors).

The GPL and GPCL Models

Default clusters? Thrifts in the early 90s at the height of the loan and deposit crisis.

Airliners after 2001.

Autos and financials more recently. In particular, from the September, 7 2008 to the October, 8 2008, a time window of one month, we witnessed seven credit events occurring to major financial entities: Fannie Mae, Freddie Mac, Lehman Brothers, Washington Mutual, Landsbanki, Glitnir, Kaupthing.

Moreover, S&P issued a request for comments related to changes in the rating criteria of corporate CDO. Tranches rated 'AAA' should be able to withstand the default of the largest single industry in the pool with zero recoveries. The proposed changes to S&P's rating criteria imply admitting as a stressed but plausible scenario that a cluster of defaults in the objective measure exists.

The GPL and GPCL Models

Problem: infinite defaults. Solution 1: **GPL:** Modify the aggregated pool default counting process so that this does not exceed the number of names, by simply capping Z_t to n, regardless of cluster structures:

$$C_t := \min(Z_t, n)$$

Solution 2: **GPCL**. Force clusters to jump only once and deduce single names defaults consistently.

The first choice is ok at top level but it does not really go down towards single names. The second choice is a real top down model, but combinatorially more complex.

Calibration

The GPL model is calibrated to the market quotes observed on March 1 and 6, 2006. Deterministic discount rates are listed in Brigo, Pallavicini and Torresetti (2006). Tranche data and DJi-TRAXX fixings, along with bid-ask spreads, are

	Att-Det	March, 1 2006		March, 6 2006		
		5y	7y	3y	5y	7y
Index		35(1)	48(1)	20(1)	35(1)	48(1)
Tranche	0-3	2600(50)	4788(50)	500(20)	2655(25)	4825(25)
	3-6	71.00(2.00)	210.00(5.00)	7.50(2.50)	67.50(1.00)	225.50(2.50)
	6-9	22.00(2.00)	49.00(2.00)	1.25(0.75)	22.00(1.00)	51.00(1.00)
	9-12	10.00(2.00)	29.00(2.00)	0.50(0.25)	10.50(1.00)	28.50(1.00)
	12-22	4.25(1.00)	11.00(1.00)	0.15(0.05)	4.50(0.50)	10.25(0.50)
Tranchlet	0-1	6100(200)	7400(300)			
	1-2	1085(70)	5025(300)			
	2-3	393(45)	850(60)			

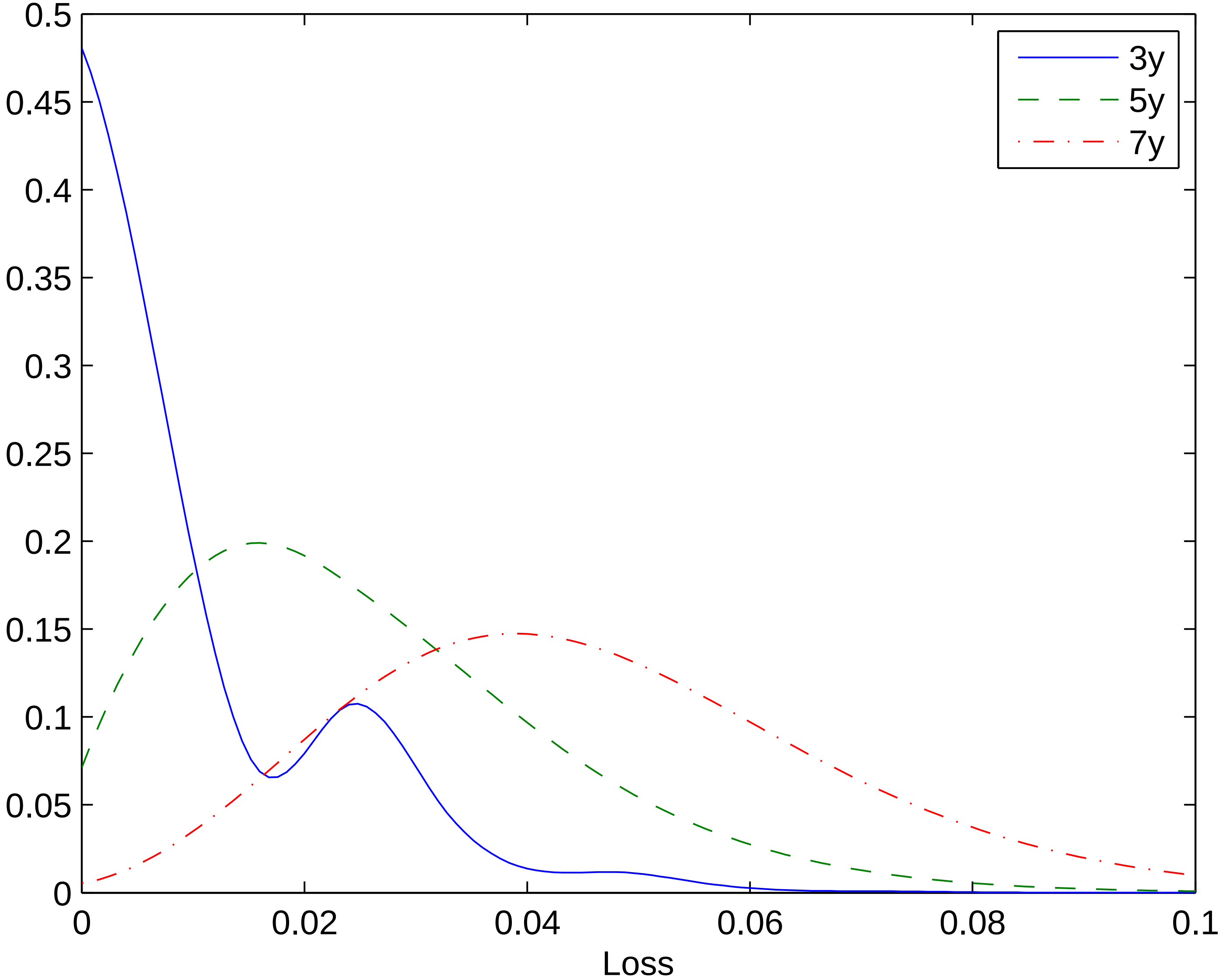
Calibration: All standard tranches up to seven years

As a first calibration example we consider standard DJi-TRAXX tranches up to a maturity of 7y with constant recovery rate of 40%.

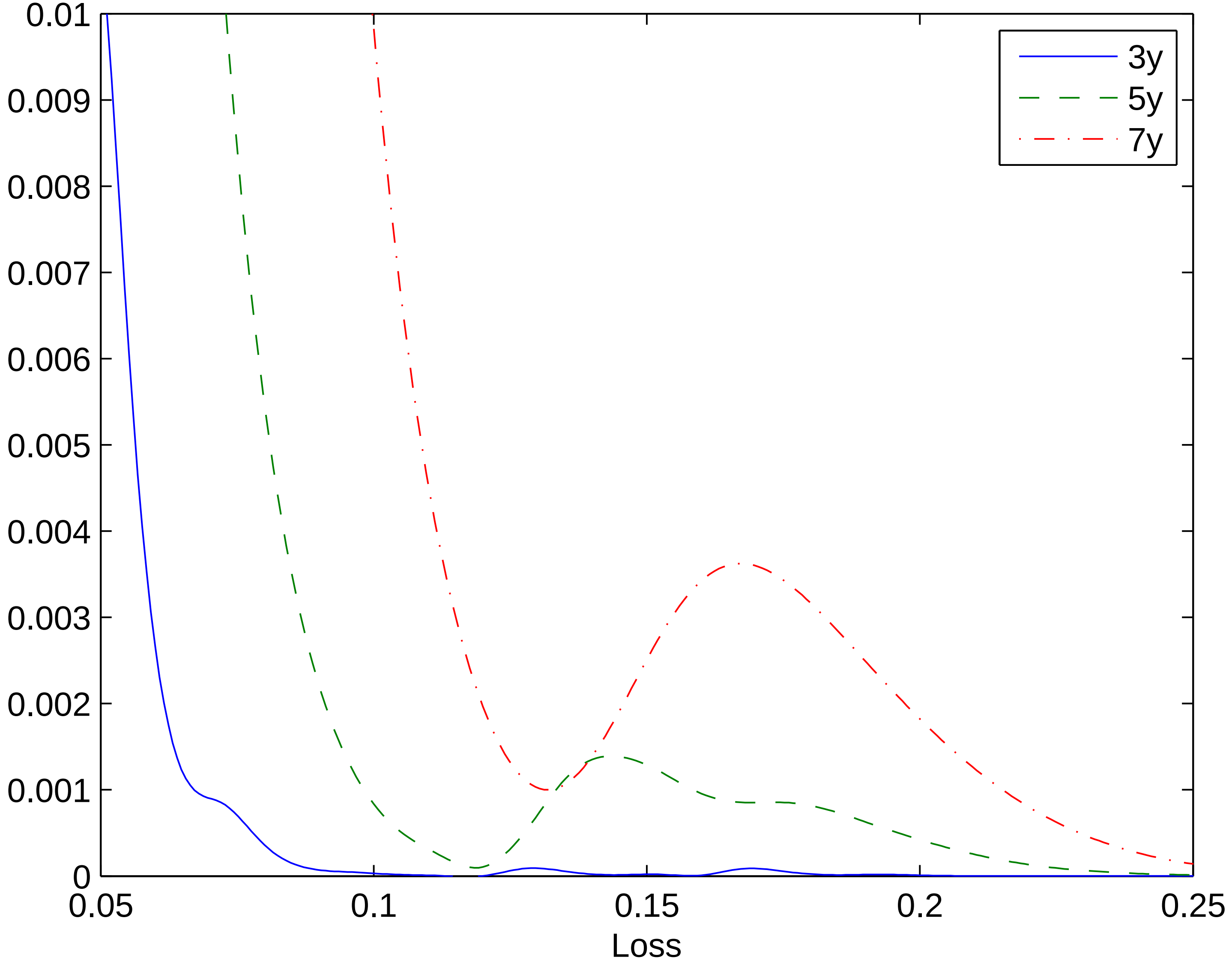
The calibration procedure selects five Poisson processes. The 18 market quotes used by the calibration procedure are almost perfectly recovered. In particular all instruments are calibrated within the bid-ask spread (we show the ratio calibration error / bid ask spread).

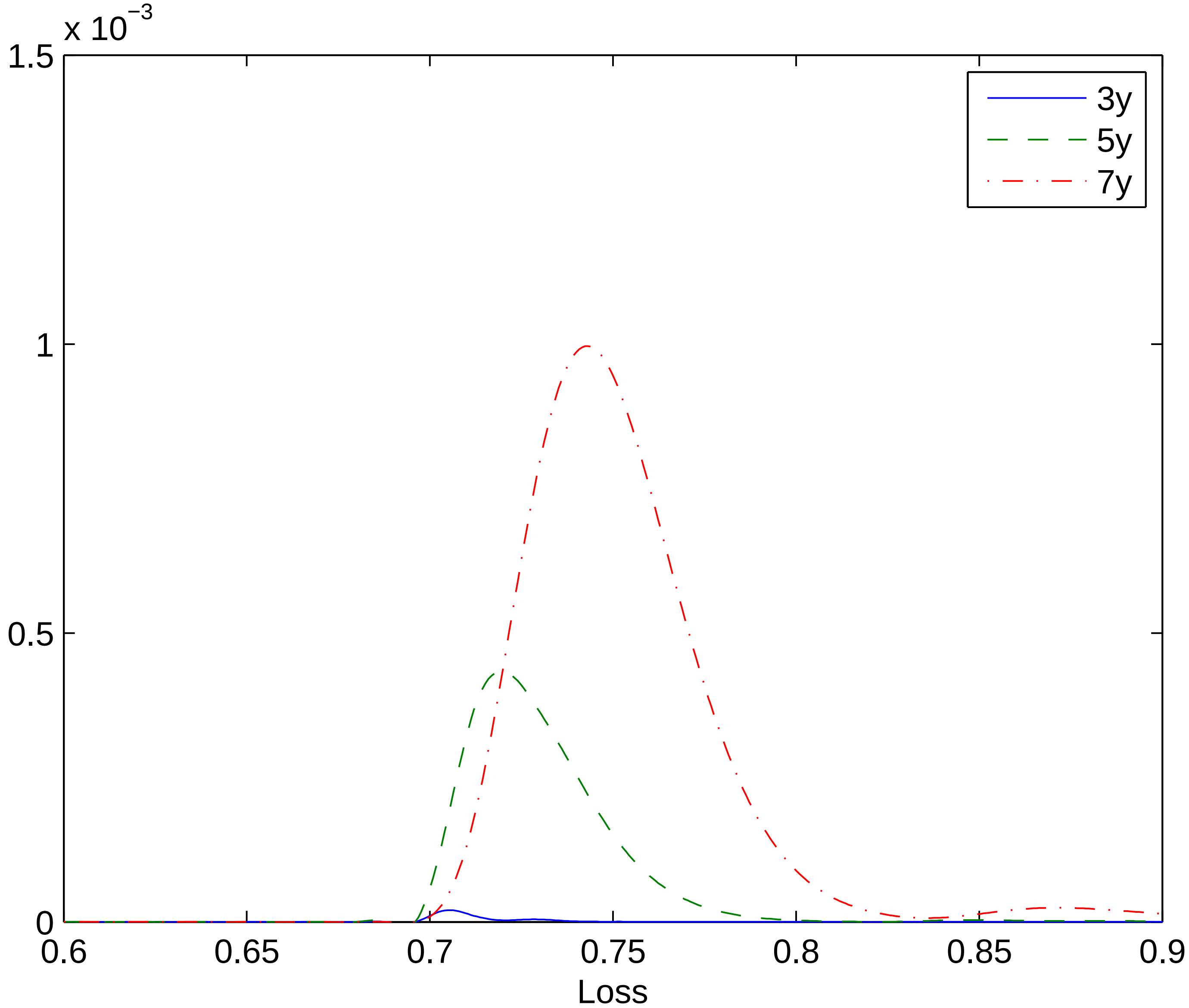
	Att-Det	**Maturities**		
		3y	5y	7y
Index		-0.4	-0.2	-0.9
Tranche	0-3	0.1	0.0	-0.7
	3-6	0.0	0.0	0.7
	6-9	0.0	0.0	-0.2
	9-12	0.0	0.0	0.0
	12-22	0.0	0.0	0.2

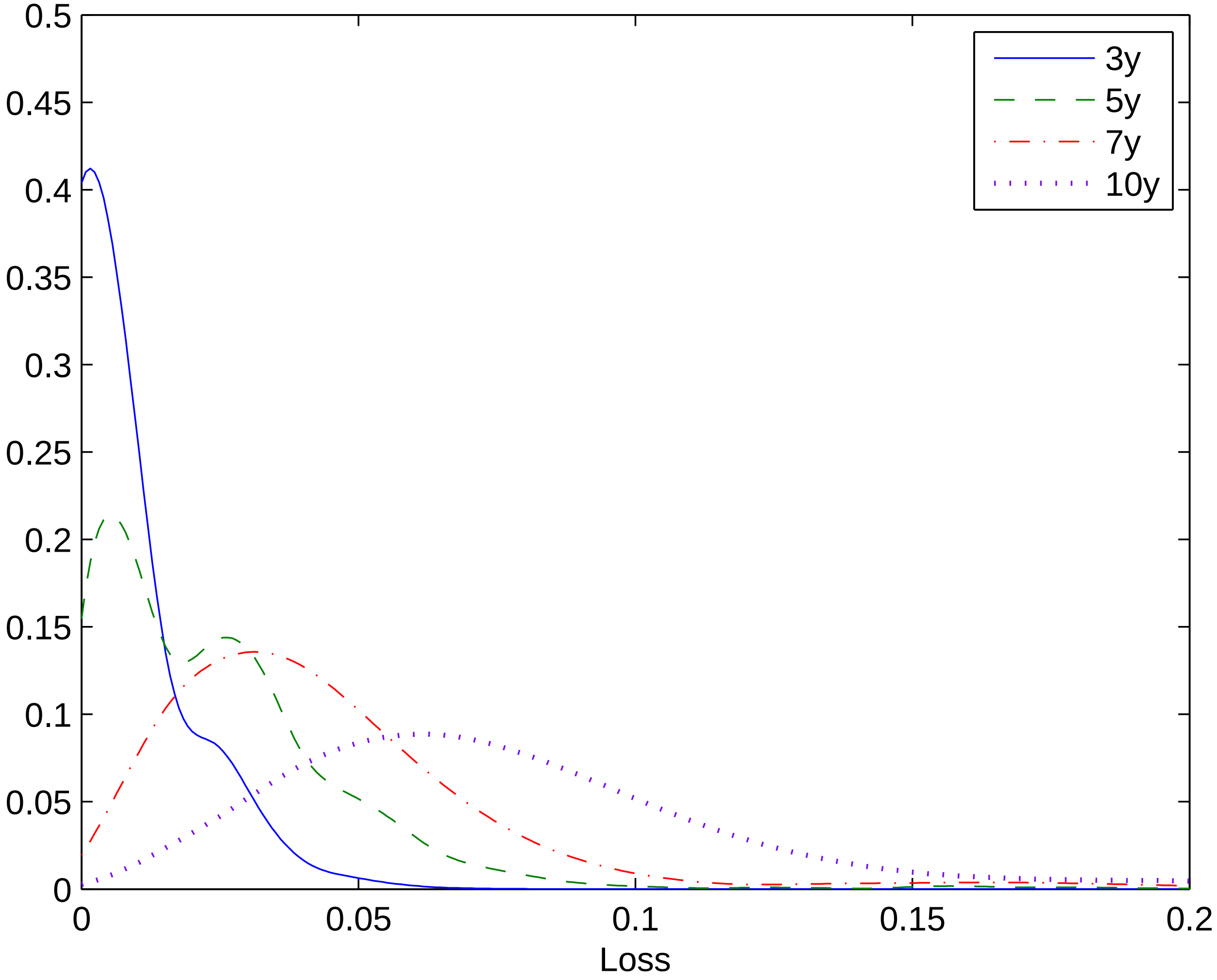
$\boldsymbol{\delta}$	$\boldsymbol{\Lambda(T)}$		
	3y	5y	7y
1	0.535	2.366	4.930
3	0.197	0.266	0.267
16	0.000	0.007	0.024
21	0.000	0.003	0.003
88	0.000	0.002	0.007



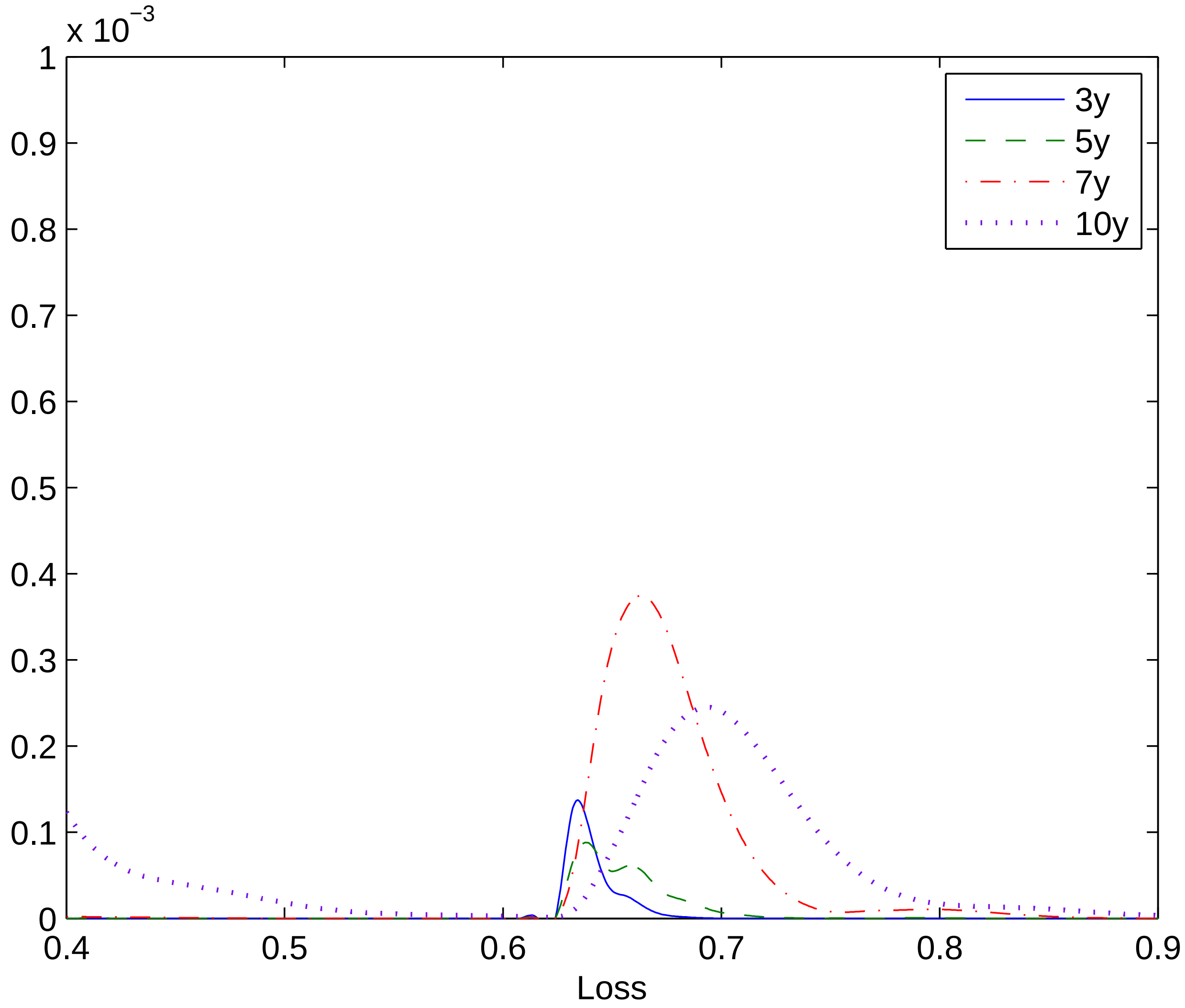
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.02
0.04
0.06
0.08
0.1
Loss
3y
5y
7y


0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
0.05
0.1
0.15
0.2
0.25
Loss
3y
5y
7y

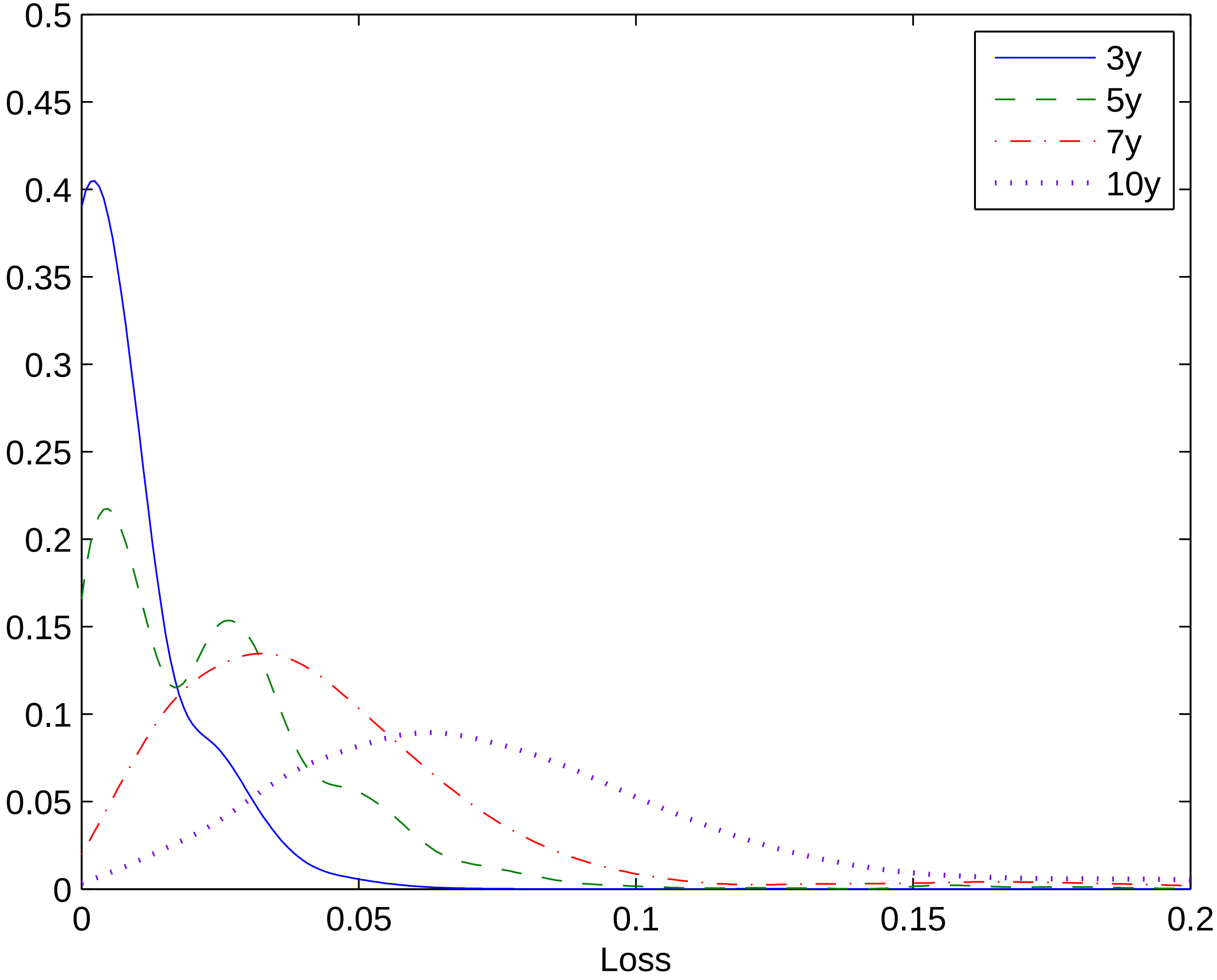

x 10^−3
1.5
1
0.5
0
0.6
0.65
0.7
0.75
0.8
0.85
0.9
Loss
3y
5y
7y



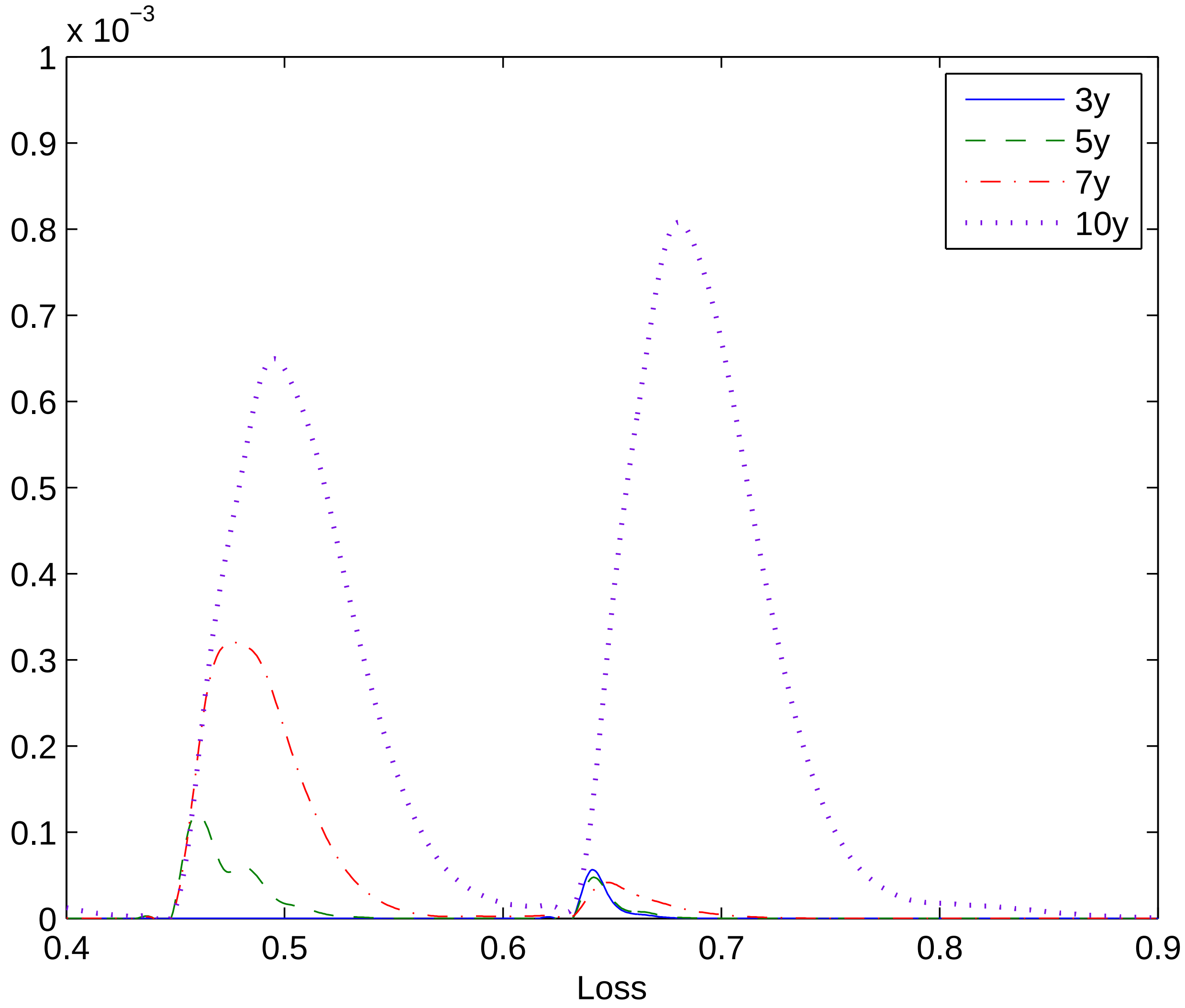
2: October 2 2006, GPL, Calibration up to 10y



3: October 2 2006, GPL tail



4: October 2 2006, GPCL, Calibration up to 10y



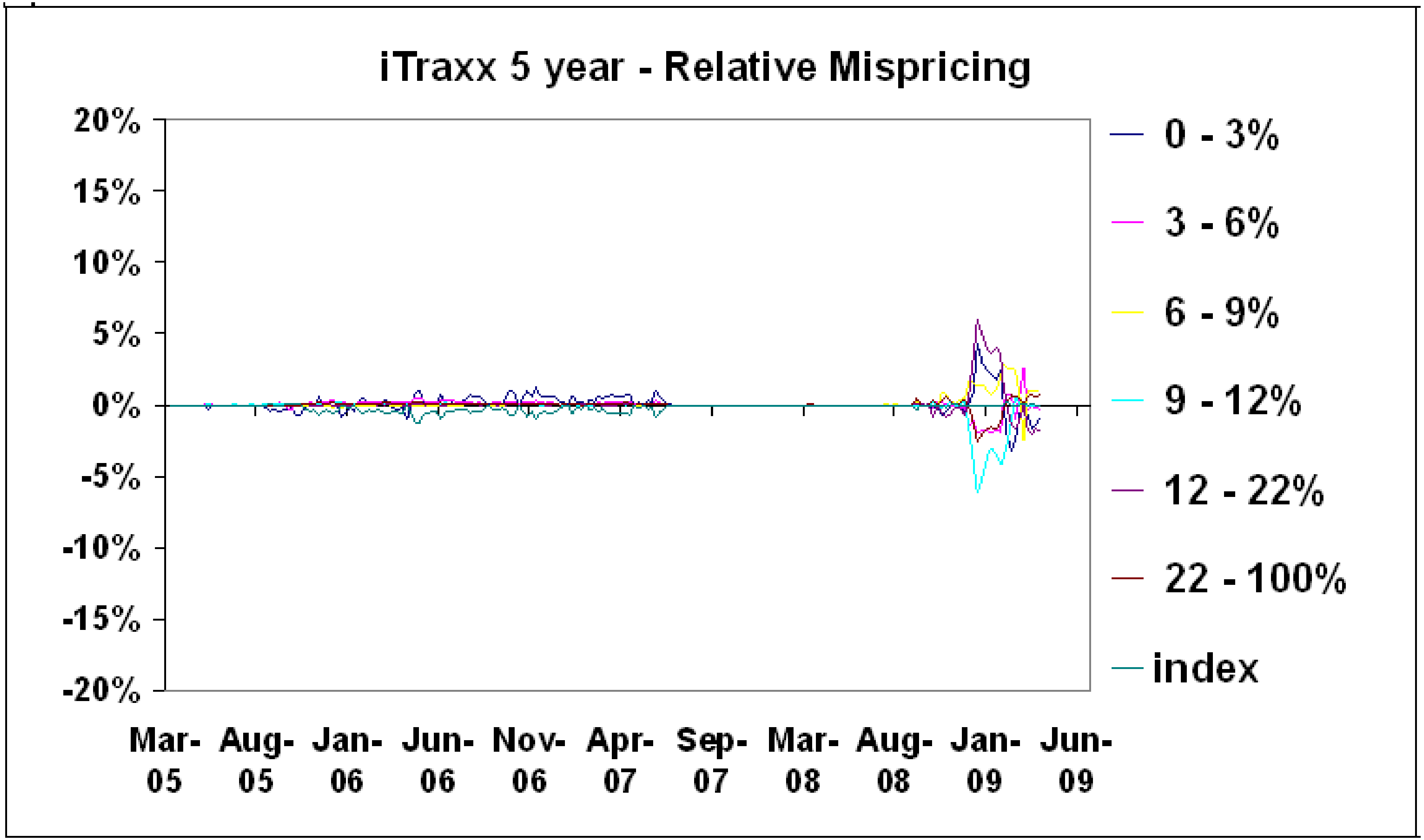
5: October 2 2006, GPCL tail

Calibration comments

Notice the large portion of mass concentrated near the origin, the subsequent modes (default clusters) when moving along the loss distribution for increasing values, and the bumps in the far tail.

Modes in the tail represent risk of default for large sectors. This is systemic risk as perceived by the dynamical model from the CDO quotes. With the crisis these probabilities have become larger, but they were already observable pre-crisis. Difficult to get this with parametric copula models.

History of calibration in-crisis with a different parametrization (α's fixed a priori):


iTraxx 5 year - Relative Mispricing
20%
15%
10%
5%
0%
-5%
-10%
-15%
-20%
Mar-05
Aug-05
Jan-06
Jun-06
Nov-06
Apr-07
Sep-07
Mar-08
Aug-08
Jan-09
Jun-09
0 - 3%
3 - 6%
6 - 9%
9 - 12%
12 - 22%
22 - 100%
index

The broad picture

The market has been using simplistic approaches for credit derivatives.

Also, CDO's are divided in Cash and Synthetics.

CDO's on morgages (RMBS and CDO of RMBS) involve thousands of assets and have complex path-dependent payouts ("waterfall") while being valued with single homogeneous scenarios or very primitive assumptions.

Synthetic corporate CDO's have simple standardized payouts and a lot of research has been done to come up with complex models to deal with them beyond the basic and average notion of "implied (base) correlation" related to the Gaussian copula. (A formula killed wall street?)

However, almost no research on mortgage CDO's.

The broad picture

The quant community was aware of the modeling limitations and much on the research to try and enhance the models in the synthetic context had appeared before the crisis

We gave two examples of extreme event risks as embedded in credit index option valuation and in CDO quotes, the latter pre-crisis.

For many other CDO's, for example cash, or on bespoke corporate pools (mapping??) or on residential mortgages (RMBS), i.e. some of the products that triggered the crisis, the problem is in the data more than in the models.

The broad picture

Bespoke corporate pools have no data from which to infer default "correlation" and dubious mapping methods are used.

Often data for valuation in mortgages CDO's (RMBS and CDO of RMBS) are dubious and at times distorted by fraud (see FBI Mortgage fraud report, 2007).

http://www.fbi.gov/publications/fraud/mortgage_fraud07.htm

6: The above photos are from condos that were involved in a mortgage fraud. The appraisal described recently renovated condominiums to include Brazilian hardwood, granite countertops, and a value of 275,000 USD

At times it is not even clear what is in the portfolio: From the offering circular of a huge RMBS (more than 300.000 mortgages)

Type of property	% of Total
Detached Bungalow	2.65%
Detached House	16.16%
Flat	13.25%
Maisonette	1.53%
Not Known	$\boxed{2.49}$%
New Property	0.02%
Other	0.21%
Semi Detached Bungalow	1.45%
Semi Detached House	27.46%
Terraced House	34.78%
Total	100.00%

Conclusions

So is the crisis due to models inadequacy?

Is the crisis due to quantitative analysts pride and unawareness of models limitations?

We have shown in this talk that quants have been aware of the limitations and of extreme risks before the crisis.

Lack of/corrupted Data and blindly strong business drive are often factors not to be underestimated

Methodology certainly needs to be improved but blaming the models for the crisis appears to be the result of a very limited point of view.

References

- D. Brigo, A. Pallavicini, R. Torresetti (2007). Cluster-based extension of the generalized poisson loss dynamics and consistency with single names. International Journal of Theoretical and Applied Finance, Vol 10, n. 4. Also in: A. Lipton and Rennie (Editors), Credit Correlation - Life After Copulas, World Scientific, 2007.

- D. Brigo, A. Pallavicini, R. Torresetti (2006). CDO calibration with the dynamical Generalized Poisson Loss model. ssrn.com. Published later in Risk Magazine, June 2007 issue.

- Morini, M. and Brigo, D. (2007). No-Armageddon Arbitrage-free Equivalent Measure for Index options in a credit crisis. Forthcoming in *Mathematical Finance*.

- Morini, M. and Brigo, D. (2009). Last option before the Armageddon, *Risk Magazine*, September issue.

- Brigo, Pallavicini, Torresetti (2009). Credit Models and the Crisis or: How I learned to stop worrying and love the CDO's. Available at ssrn.com, arXiv.org, defaultrisk.com

- Torresetti, R., Brigo, D., and Pallavicini, A. (2006a). Implied Expected Tranched Loss Surface from CDO Data. Available at ssrn.com.

- Torresetti, R., Brigo, D., and Pallavicini, A. (2006b). Implied Correlation in CDO Tranches: A Paradigm to be Handled with Care. Available at ssrn.com.

- Torresetti, R., and Pallavicini, A. (2007). Stressing Rating Criteria Allowing for Default Clustering: the CPDO case. Available at ssrn.com.

- Torresetti, R., Brigo, D., and Pallavicini, A. (2006). Risk Neutral Versus Objective Loss Distribution and CDO Tranches Valuation. Available at ssrn.com, updated version appeared in the *Journal of Risk Management in Financial Institutions*, January-March 2009 issue.

- Brigo, D., Pallavicini, A. and Torresetti, R. (2010). Credit Models and the Crisis: A journey into CDOs, Copulas, Correlations and Dynamic Models. Wiley, Chichester.