Ranking the Predictive Performances of Value At Risk Methods

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Abstract

We propose a novel ranking model and a complementary predictive ability test statistic to investigate the forecasting performances of different Value at Risk (VaR) methods. The ranking model develops a unified framework which penalizes excessive capital allocation, autocorrelation of violations and violation magnitudes through an anisometric structure. In addition, as an alternative to existing predictive ability tests which compare the forecasting methods two at a time, our complementary test statistic considers all methods at the same time to confirm whether the chosen method outperforms the competing methods. The results show that asymmetric methods such as CAViaR Asymmetric and EGARCH generate the best performing forecasts. This suggests that the performance of VaR methods does not depend entirely on whether they are parametric, non-parametric, semi-parametric or hybrid; but rather if they can effectively model the asymmetry of the underlying data or not.

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1 Introduction

Forecasting is of fundamental importance to all the sciences and accurate forecasting is essential for making sound decisions. Evaluating forecasting accuracy is of great importance for those who are primarily interested in analyzing and distinguishing between competing prediction methods. Also, predictive performance and the adequacy of methods are closely linked as method inadequacy causes predictive failure.

Given the ever-increasing importance of risk management, risk prediction plays a significant role in finance. The concept of Value-at-Risk (VaR) has emerged as the standard measure of market risk. VaR can be defined as the determined quantile of the future return, conditional on the available information set. The main task is to forecast a value; VaR_t at each period, which will exceed the future return with $\Theta \in (0, 1)$ probability. For a brief explanation, let $\{x_t\}_{t=1}^{T}$ be the time series of returns where T is the end of the sample. Then, following the approach of Artzner, Delbaen, Eber and Heath (1999) and by letting $\Omega_{t-\tau}$ be the information set up to $t - \tau$, the risk measure can be defined as follows: $VaR_t = -\inf\{q \in \mathbb{R} : Pr(x_t \leq q|\Omega_{t-\tau}) > \Theta\}$.

In simpler terms, regarding VaR as the maximum potential loss with a given Θ probability, a quantile measure relatively far out in the left tail of the return distribution is searched for some specified future time. Existing methods for calculating VaR can be classified into four main groups as follows: Nonparametric methods, Semiparametric methods, Parametric methods and Hybrid methods.

Evaluating Value at Risk methods' forecasting accuracy in an objective and effective framework is crucial for both efficient capital allocation and loss prediction. Hence, finding an adequate method is of obvious importance for both the regulators and the risk managers' needs for financial stability and profitability.

Many forecasting accuracy assesments for VaR methods exist through the use of different criteria such as empirical coverage probabilities, error estimating loss functions and autocorrelation tests. Several other types of descriptive information such as the mean of violations and the maximum of violations can also be considered (Berkowitz and O'Brien, 2001). It is important to note that evaluating forecasts based solely on one criterion yields limited information regarding method accuracy. Thus, in literature, it is a common approach to present the results of each evaluation criterion seperately and then determine the *best* performing method. However, it is likely to observe different *best* methods for different evaluation criteria. Thus, it becomes difficult to make conclusive remarks.

The Dynamic Quantile (DQ) test (Engle and Manganelli, 2004) proposes an easy test statistic which incorporates important evaluation criteria and hence can be used for method selection. It stands out as a powerful tool in testing whether VaR estimates from different methods satisfy the requirements of independent unexpected losses, the correct fraction of unexpected losses and unbiasedness. Here, the aim is not to rank different methods from best to worse, but to statistically test whether a method at hand satisfies these requirements. Thus, if more than one forecasting method passes this test, the DQ test does not distinguish which one is relatively more accurate.

As an alternative approach, predictive ability tests are widely used when comparing forecasting methods and to confirm whether the chosen method outperforms the competing forecasting methods. Equal predictive ability (EPA) tests were proposed by Diebold and Mariano (1995) and West (1996), in which the latter considers forecasting methods including estimated parameters. Harvey, Leybourne and Newbold (1997) suggest a framework which considers small sample properties. Harvey and Newbold (2000) propose a test for comparing multiple nested methods and McCracken (2000) consideres the cases with estimated parameters and non-differentiable loss functions. As an extension to equal predictive ability tests, in the framework of White (2000) and Hansen (2005), a test for superior predictive ability (SPA) is examined. The distinction is that the latter leads to a composite null hypothesis and former leads to a simple null hypothesis. Other important works include Clark and McCracken (2001), Corradi, Swanson and Olivetti (2001), Clark and West (2006). [1] Both EPA and SPA require the choice of a fixed benchmark method and compare methods two at a time, instead of all at the same time.

Any reasonable methodology in evaluating the forecasting accuracy of VaR methods should take into consideration the concerns of both the regulators and the risk managers while being able to rank the VaR methods. Accordingly, our methodology takes the following steps: 1) Determining the important evaluation criteria 2) Developing a model which incorporates all these evaluation criteria 3) Providing meaningful results to establish the quality of the forecasting estimates 4) Providing complementary test statistics for statistical validation.

With these in mind, we propose a new ranking model and a complementary predictive ability test statistic to investigate the performances of different VaR methods. In addition, we implement the DQ test and White's SPA test as supporting information. Twelve different VaR methods (from four main categories of VaR methods) are implemented using equity index data from eleven emerging markets and seven developed markets. Our results suggest that asymmetric methods such as CAViaR Asymmetric (Engle and Manganelli, 2004) [2]

[1] Refer to the original papers for detailed discussions.

[2] The unknown parameters are found by the regression quantile framework introduced by Koenker and Basset (1978).

and EGARCH (Nelson, 1991) tend to generate more accurate forecasts.

This paper is organized as follows. Section 2 introduces our ranking model. Section 3 provides information on predictive ability testing and presents our test statistic. Section 4 explains the empirical results. Section 5 is the conclusion.

2 Loss Functions and Ranking Model

A reasonable loss function should provide a numerical value which reflects both the regulatory supervisors' and risk managers' concerns on potential forecast errors. To sustain financial stability, government regulators would need firms to generate few unexpected losses or violations; meaning few returns below VaR measures. They would also need the unexpected losses to be small in magnitude and be unautocorrelated. Disastrous events (i.e bankruptcy) are more probable as a result of huge unexpected losses occuring in rapid successions (Christoffersen and Pelletier, 2004). On the other side, risk managers would also consider the profitability of their firms and would need smaller scaled VaR measures for efficient capital allocation. The delicate balance can be reached when a VaR method is accurate at forecasting the volatility evolution.

Appropriate loss functions for the evaluation of forecast methods are discussed in Lopez and Diebold (1996), Hansen (2005), Awartani and Corradi (2005), Chiu, Lee and Hung (2005), Giacomini and White (2006) and Fuertes, Izzeldin, Kalotychou (2009). Loss functions commonly used in literature include:

Examples of Forecast Evaluating Loss Functions

Let $VaR_{j,t}$ be the VaR forecast at time t with model j and x_t be the realized return at time t. Also, let $l_{j,t}$ be any loss function.

1. *Mean Squared Error:* $l_{j,t}$=$(x_t - VaR_{j,t})^2$
2. *Absolute Error:* $l_{j,t}$=$|x_t - VaR_{j,t}|$
3. *Asymmetric Linear:* $l_{j,t}$=$(\Theta - 1(x_t < VaR_{j,t}))(x_t - VaR_{j,t})$
4. *Linex:* $l_{j,t}$=$exp(\alpha(x_t - VaR_{j,t})) - \alpha(x_t - VaR_{j,t}) - 1$ with $\alpha \in \Re$
5. *Logarithmic:* $l_{j,t}$=$(ln(x_t) - ln(VaR_{j,t}))^2$
6. *Direction of Change:* $l_{j,t}$=$1\{(x_{t+1} - x_t) \neq (VaR_{j,t+1} - x_t)\}$
7. *Kolmogorov-Smirnov:* $l_{j,t}$=$sup_{y\in\Re}| \int_{-\infty}^{y} [x_t(z) - VaR_{j,t}(z)]dz|$
8. *Kullback-Leibler:* $l_{j,t}$=$\int_{-\infty}^{\infty} log[VaR_{j,t}(z)/x_t(z)]x_t(z)dz$
9. *Hansen's VaR Specific:* $l_{j,t}$=$| \int_{-\infty}^{VaR_{j,t}} x_t(z)dz - \Theta|$

A general framework to represent a reasonable loss function, regarding the VaR measure, can be presented by considering the returns both above and below the VaR measure as proposed by Lopez (1998). More specifically, let $VaR_{j,t}$ be the VaR forecast at time t with model j. Also, let $l_{j,t}$ be any loss function.

Then, the following can be written:

$$l_{j,t} = \begin{cases} f(x_t, VaR_{j,t}) & \text{if } x_t \geq VaR_{j,t} \\ h(x_t, VaR_{j,t}) & \text{if } x_t < VaR_{j,t} \end{cases} \tag{1}$$

In this representation, x_t is the realized return at time t. Thus, considering multiple VaR methods, the numerical values generated by a reasonable loss function can be used to rank the predictive accuracy of these methods.

Note that, common regulatory forecasting assesments, which only regard unconditional violation frequencies are flawed (Kuester, Mittnik and Paolella, 2006). A reasonable loss function should incorporate the concerns of both the regulators and the risk managers. A complete model should not only consider the number of unexpected losses, but also the magnitudes of unexpected losses and the clusterings of unexpected losses. In addition, the model should also favour efficient capital allocation. To the best of our knowledge, there is no loss function which considers all these concerns and ranks the VaR methods accordingly. In the literature, this is the reason why it is common to use a number of loss functions providing complementary information instead of a single loss function which incorporates all the appropriate information to evaluate VaR methods.

2.1 Ranking Model

Following the general framework of the loss function given in (1), we propose a model which divides the return space into two: The *safe space* and the *violation space*. The safe space considers the realized returns above the VaR measure; whereas the violation space considers the unexpected losses.

In the violation space, a cluster is defined as the sequence of unexpected losses with a finite length of z; that is $z \leq Z$. Furthermore, there may be more than one cluster in this space; say $h \leq H$ number of clusters. A cluster by itself and the interaction of clusters with each other all contribute to autocorrelation. A cluster can be modeled as compounded unexpected losses and the interaction between the clusters can be modeled by an inversely proportional function of the distance between the clusters. Then, such an approach penalizes the autocorrelation of unexpected losses.

In addition, the magnitudes of the unexpected losses can simply be modelled as the difference between the unexpected loss and the VaR measure at each time point. Defining the magnitude of unexpected losses as $\epsilon_t = (VaR_t - x_t)$ for $x_t < VaR_t$, the total loss from the violation space is written as follows:

$$\Phi = \sum_{i=1}^{h-1} \sum_{m=1}^{h-i} \left\{ \frac{1}{k_{i,i+m}} \left\{ \prod_{b=1}^{z_i} (1 + \epsilon_{b,i}) \prod_{b=1}^{z_{i+m}} (1 + \epsilon_{b,i+m}) - 1 \right\} \right\} \tag{2}$$

In this representation, z_i is the length of the i^{th} cluster and $k_{i,i+m}$ is the length between i^{th} and $(i+m)^{th}$ clusters. It can be seen that as the length between clusters increase, the penalization factor decreases. Note that, any combination of cluster interactions are taken into account.

In the space remaining above the VaR measure, that is the safe space; over-capital allocation should be penalized as it generates less profitability for the firm, which is undesirable for the risk managers. Note that, over-capital allocation does *not* generate an actual unexpected loss, but only generates less profitability for the firm. Thus, penalization of autocorrelation of returns remaining above VaR measure does not apply in this space as this would otherwise overestimate the penalization power of the safe space.

As VaR measure only deals with the left tail of a distribution, only the negative returns should be considered with respect to the calculated VaR measure. By definition, a positive VaR measure is not meaningful and therefore, constructing the penalization by regarding the positive returns is not entirely correct. Thus, only the distance between the negative returns remaining above VaR_t and the VaR_t itself are considered. This constitutes the magnitude of the over-capital allocation. The safe space penalization factor is structured as follows:

$$\Psi = \sum_{t=1}^{T} \left[\mathbf{1}(x_t > VaR_t | x_t < 0) \right] (x_t - VaR_t) \tag{3}$$

The weighting of the two spaces is a fundamental issue. It is important to understand that as the quantile; Θ decreases, the ratio of the safe space to the violation space increases. Hence, the weighting scheme should be a function of the chosen quantile. It is also important to note that in the context of VaR, underestimation of the VaR measure should be less desirable than overestimating it, as underestimation generates actual unexpected losses and overestimation only generates less profitability and no actual unexpected losses. This suggests an anisometric weighting of the spaces. Following the approach of the Asymmetric Linear Loss Function, the safe space can be weighted with Θ and the violation space can be weighted with (1-Θ).

Finally, as a scaling parameter of the sum of the two weighted spaces, we use the total number of observations. Then our resulting penalization function that ranks the methods is structured as follows:

$$RM(\theta, \Phi, \Psi) = \frac{1}{T} \left[(1-\theta)\Phi + \theta\Psi \right] \tag{4}$$

The ratio of the value generated by a particular method to the sum of all the values generated by all methods can be calculated. More specifically, let RM_j be the value generated by the ranking model for the j^{th} VaR method. The ratio for the j^{th} method amongst n number of methods can be calculated as follows: $Ratio_j = \frac{RM_j}{\sum_{i=1}^{n} RM_i}$. The higher the ratio is, the worse the corresponding forecasting method is. If every method is equally accurate in forecasting the VaR measure, then this ratio would be $\frac{1}{n}$ as $RM_j = RM_i, \forall i \in [1, n] \subset \mathbb{N}$. This ratio approach is closely linked to our predictive ability test statistic.

3 Predictive Ability Tests

In comparing the forecasting methods in a statistically meaningful way, predictive ability testing can be used as a complementary framework to a ranking framework. In order to demonstrate where our alternative predictive ability test statistic stands, the frameworks of Diebold's equal predictive ability test (1995) and White's superior predictive ability test (2000) are explained.

Let two forecasts be defined as $\{f_{i,t}\}_{t=1}^{T}$ and $\{f_{j,t}\}_{t=1}^{T}$ for the time series $\{x_t\}_{t=1}^{T}$. Also, let the corresponding forecast errors be defined as $\{e_{i,t}\}_{t=1}^{T}$ and

$\{e_{j,t}\}_{t=1}^{T}$. An arbitrary loss function of the actual data and forecasts can be defined as $l(x_t, f_{i,t})$. In most applications, the loss function will be a direct function of the forecast erros; $l(x_t, f_{i,t})$=$l(e_{i,t}) \equiv l_{i,t}$. Then, the null hypothesis of equal forecast accuracy for two forecasts can be written as follows: $E[l_{i,t}] = E[l_{j,t}]$. If the loss differential series is defined as; $\{\kappa_t\}_{t=1}^{T} \equiv \{l_{i,t} - l_{j,t}\}_{t=1}^{T}$, then the null hypothesis can be written as $H_0 \equiv$ E$[\kappa_t]$=0. Diebold (1995) proposes various ways of testing this hypothesis.

As an extension, White (2000) developed a framework for comparing n number of forecasting methods and proposed a test for SPA also known as the reality check (RC) for data snooping. In this framework, a benchmark method is chosen and a *vector* of differential loss series, corresponding to all forecasting methods is created. Let the differential loss series be defined as $\{\kappa_{j,t}\}_{t=1}^{T} \equiv \{l_{1,t} - l_{j,t}\}_{t=1}^{T}$ for $l_{1,t}$ being the loss function associated with the forecasting errors of the *benchmark* method. Then, the vector is generated: $\kappa_t^* = (\kappa_{2,t}, ..., \kappa_{n,t})$ and the null hypothesis of interest is $H_0 \equiv E[\kappa_t^*] \leq 0$. Then, the question of interest is whether any alternative forecast method is better than the benchmark method, or equivalently, whether the best alternative forecasting method is better than the benchmark. White (2000) and Hansen (2005) propose different test statistics for testing the null hypothesis.

Note that, in these frameworks, the differential loss series are calculated by considering two forecasting methods at a time, instead of all at the same time. Hence, the differential loss series is a function of two methods: $\kappa_{j,t} \equiv g(l_{1,t}, l_{j,t})$.

3.1 Alternative Predictive Ability Test

In our framework, the null hypothesis is the same: the loss series generated by a benchmark forecasting method is statistically no worse than the others. In order to consider all methods at the same time, one alternative way of defining the loss series can be through ratios instead of the spreads of two methods. Let the following be defined: $\kappa_{j,t} \equiv \zeta(l_{1,t}, ..., l_{j,t}, ..., l_{n,t})$.

This function becomes the ratio of the loss series generated by a particular forecasting method over the sum of all loss series from the forecasting methods, instead of loss differentials of two methods. Then, the ratio-loss series is defined as follows: $\{\kappa_{j,t}\}_{t=1}^{T} = \{\frac{l_{j,t}}{\sum_{i=1}^{n} l_{i,t}}\}_{t=1}^{T}$. Here, a total of n methods are considered. If all the forecasting methods are equally accurate, then this ratio would be $\frac{1}{n}$. Then, our null hypothesis becomes the following: $H_0 \equiv E[\kappa_{j,t}] = \frac{1}{T}\sum_{t=1}^{T} \kappa_{j,t} \leq \frac{1}{n}$.

A similar binomial testing criteria proposed by Diebold (1995) can be used to create the test statistic: $W_j = \sum_{t=1}^{T} \mathbf{1}(\kappa_{j,t} > \frac{1}{n})$. According to the Central Limit Theorem and assuming *iid* of the generated ratio-loss series, the test statistic can be written as follows: $\hat{W}_j = \frac{W_j - pT}{\sqrt{p(1-p)T}} \sim N(0,1)$.

In this framework, $p = 0.5$ and the proof can be found in the Appendix B. The main advantages of such a structure are that all the methods can be considered at the same time and it is still easy to implement. Also, for both the EPA and the SPA testing, a total of $\frac{n(n-1)}{2}$ differential loss series must be generated to carry out the tests. On the other hand, for our predictive ability test, n number of ratio-loss series is enough to test n number of methods. This allows a computational complexity of order $\frac{n-1}{2}$ for $n > 2$.

Our predictive ability testing framework is sensitive to exceedingly poor forecasting methods. If there exists m number of exceedingly inaccurate forecasting methods amongst n number of methods for $m << n$, other unsuccessful methods may still pass the test. More specifically, at each time point, the magnitude of the loss generated by the exceedingly inaccurate methods will be so large that the magnitude of the loss generated by another unsuccessful method will be relatively too small. Then, the ratios for an unsuccessful method may remain under $\frac{1}{n}$ and such a method may pass the test. In such cases, our predictive ability testing should be carried out once again without including them.

4 Empirical Results

In this analysis, equity index data from eleven emerging markets and seven developed markets are analyzed. The emerging markets are as follows: Brazil (IBOV Index), Chile (IPSA Index), Colombia (IGBC Index), the Czech Republic (PX Index), Hungary (BUX Index), Mexico (MEXBOL Index), Poland (WIG Index), Russia (INDEXCF Index), Turkey (XU100 Index), South Africa (JALSH Index), and Argentina (MERVAL Index). The developed markets are as follows: England (UKX Index), the US (INDU Index), France (CAC Index), Spain (IBEX Index), Germany (DAX Index), Japan (NKY Index), and Holland (AEX Index).

The data is taken from Datastream as daily observations from the beginning of 1995 until mid July of 2009, considering only business days. Therefore, amongst the other major crises, the data at hand also includes the recent subprime mortgage crisis, which is an important stress testing period for the VaR methods. The first 500 days is determined to be the out of sample data and the size of the rolling window is also 500. The twelve VaR methods implemented on the return series and their brief explanations can be found in Appendix A. The quantile levels which we tested for each VaR method are $\Theta = 0.05$ and $\Theta = 0.01$. [3]

[3]For the sake of brevity, only the results for $\Theta = 0.05$ are presented in this paper. The results of $\Theta = 0.01$, which are very similar to that of $\Theta = 0.05$, are available from the authors

4.1 Descriptive Statistics of Returns Series

Through the pioneering works of Mandelbrot (1963) and Fama (1965), the empirical characteristics of financial return data are well known. They can be listed as follows: 1) The distributions have a level of skewness; they are asymmetric 2) The distributions have high kurtosis; they are leptokurtic 3) Squared returns have severe autocorrelation.

Table 1 demonstrates the skewness, kurtosis and Jarque-Bera test results for the return series of emerging and developed markets. It can clearly be observed that none of the underlying return series is normally distributed. There is severe kurtosis and the distributions are asymmetric with considerable skewness. These observations are key factors to understand why certain VaR methods might be better performing. In figures 1-2, the VaR values calculated by each method against the returns of Brazil and Spain are plotted, respectively.

4.2 Results of Ranking Model

The values generated by our ranking model can be seen in tables 2 and 4. In tables 3 and 5, the methods corresponding to the values are sorted in ascending order. It can be observed that for 10 out of 11 emerging markets and 7 out of 7 developed markets, EGARCH and CAViaR Asymmetric methods are ranked the highest in their performance power. Our ranking model results confirm the works of Nelson (1991), Glosten, Jagannathan and Runkle (1993) and Hentschel (1995) among others, which state the importance of asymmetries in market wide equity index returns as also mentioned in Andersen, Bollerslev, Diebold and Ebens (2001). The volatility tends to increase with bad news and decrease with good news; hence, an asymmetric modelling structure becomes an essential issue. There are two major explanations for this behaviour. Black (1976) and Christie (1982) explain the phenomenon with leverage effect, where a large negative return raises financial and operating leverage which increases equity return volatility. Campbell and Hentschel (1992) discuss that if the market risk premium is an increasing function of volatility, then due to the volatility feedback effect, negative returns would raise future volatility more than that of positive returns.

Both in emerging and developed markets, Historical Simulation and Monte Carlo Simulation methods do not perform particularly well. These two methods are most commonly used in financial institutions (Christoffersen, Berkowitz and Pelletier, 2007) and do not seem to be the best methods to use. [4]

upon request.

[4]Regarding $\Theta = 0.01$, Historical Simulation and Monte Carlo Simulation methods perform relatively better when compared to their performance regarding $\Theta = 0.05$. This is also the case for RiskMetrics (RiskMetrics, 1996) and GARCH (Bollerslev, 1986) methods.

It is clear that Extreme Value Theory and Variance/Covariance methods are the worst methods in their performance power as they are ranked within the last three places for 10 out of 11 emerging markets and 7 out of 7 developed markets. They overestimate the VaR and still manage to get severe unexpected losses. Specifically for developed markets, the Hybrid method seems to be the worst performing method as it is ranked in the very last place for 5 out of 7 developed markets.

From our ranking model, we have found out that asymmetric methods such as EGARCH and CAViaR Asymmetric perform the best in forecasting the VaR measure. Therefore, regarding the asymmetric distribution of the returns for each country, the performance of VaR methods does not depend entirely on whether they are parametric, non-parametric, semi-parametric or hybrid; but rather if they can model the asymmetric structure or not.

4.3 Results of Dynamic Quantile Test

The results of the Dynamic Quantile test proposed by Engle and Manganelli (2004) can be seen in tables 6 and table 7 for emerging and developed markets, respectively. The results of our ranking model and the DQ test (2004) are highly consistent. It can be observed that CAViaR Asymmetric method seems to be the only method which succesfully passes the test for every country. In addition, Extreme Value Theory and Variance/Covariance methods do not pass the test for any country for the same confidence level.

It can also be observed that Historical Simulation and Monte Carlo Simulation methods do not pass the test for any country either. Our ranking model does not favor these two methods, but it still does not rank them as the worst methods. This suggests that these two methods allow for efficient capital allocation which our ranking model favors to some extent, but there is an autocorrelation of unexpected losses which the DQ test does not tolerate.

It is also interesting to observe that the Hybrid method passes the DQ test for 6 out of 7 developed markets, although it is ranked as the worst method in our ranking model for 5 out of 7 developed markets. This suggests that the Hybrid method is not successful for efficient capital allocation which our ranking model punishes.

4.4 Results of Predictive Ability Tests

Tables 8 and 10 demonstrate the results of our predictive ability test statistic for emerging and developed markets, respectively. For each market, we reject that the Extreme Value Theory method is statistically no worse than others.

Apart from the Extreme Value Theory and Variance/Covariance methods, all other methods seem to have passed the test for each emerging market. This suggests that these two methods are exceedingly poor compared to others and thus, the test should be carried out once again without including them. From table 9, it can be observed that CAViaR Asymmetric is the only method which passes our test statistic for every country.

For developed markets, the test should be carried out once again without including Extreme Value Theory method. From table 11, it can be observed that EGARCH and CAViaR Asymmetric methods are the only methods which pass our test statistic for every country. These results are consistent both with our ranking model and the DQ test.

White's SPA test (2000) results regarding emerging and developed markets can be seen in tables 12 and 13, respectively. It can be seen that for every country, we fail to reject that CAViaR Asymmetric method is no worse than other methods. Hence, White's SPA test (2000) is confirmed by our alternative predictive ability test statistic and our ranking model.

5 Conclusion

Regulators and risk managers have an abundance of Value at Risk methods to choose from. Thereby, from the perspective that succesful VaR methods should satisfy both the concerns of regulators and the risk managers, we have developed a model which ranks the methods accordingly. As a complementary statistical validation of our model, we propose our own predictive ability test statistic and also implemented White's SPA test (2000). Additionally, the Dynamic Quantile test (2004) is used for further analysis.

We implemente twelve different VaR methods on eleven emerging markets and seven developed markets. The results suggest that asymmetric methods such as CAViaR Asymmetric and EGARCH perform the best. The distributions of the underlying data are asymmetric for every country considered in this analysis. Therefore, the performance of VaR methods does not depend entirely on whether they are parametric, non-parametric, semi-parametric or hybrid; but rather if they can model the asymmetry of the underlying data or not.

6 Appendix A

6.1 VaR Methods Used in Analysis

Nonparametric Methods:

1. Variance-Covariance Method: $VaR_{t+\tau} = H_t^{-1}(\Theta)\sqrt{w'\Sigma_t w}\sqrt{\tau}$ where w is the weight of each asset, Σ_t is the covariance matrix, H_t is the cumulative standard normal distribution function.

2. Historical Simulation Method: $VaR_{t+\tau}$ is given by the empirical Θ-quantile up to time t; which can be defined as $Q_\Theta(x_t, x_{t-\tau}, ..., x_0)$.

3. Monte Carlo Simulation Method: $VaR_{t+\tau}$ is given by the empirical Θ-quantile up to time t; which can be defined as $Q_\Theta(x_t, x_{t-\tau}, ..., x_0)$. The returns are generated by simulation: $x_t = \mu dt + \sigma\epsilon\sqrt{dt}$ where $\epsilon \sim N(0,1)$.

Semiparametric Methods:

1. Adaptive CAViaR Method: $VaR_t = \beta_0 + \sum_{i=1}^{p}\beta_i VaR_{t-i} + \sum_{j=1}^{q}\beta_j l(x_{t-j})$ where p and q are the determined lags.

2. Symmetric CAViaR Method: $VaR_t = \beta_1 + \beta_2 VaR_{t-1} + \beta_3|(x_{t-1})|$

3. Asymmetric CAViaR Method: $VaR_t = \beta_1 + \beta_2 VaR_{t-1} + \beta_3(x_{t-1})^+ + \beta_4(x_{t-1})^-$ where the functions are $(y)^+ = max(y,0)$ and $(y)^- = -min(y,0)$.

4. Indirect GARCH CAViaR Method: $VaR_t = (\beta_1 + \beta_2 VaR_{t-1}^2 + \beta_3(x_{t-1})^2)^{\frac{1}{2}}$

5. Extreme Value Theory Method: Through the Generalized Pareto Distribution $VaR_{t+\tau} = \nu - \frac{\hat{\delta}}{\hat{\psi}}\left(1 - \left[\Theta(\frac{T}{K})\right]^{-\hat{\psi}}\right)$ where K is the number of exceedences, ν is the threshold, $\hat{\psi}$ is the estimated shape parameter, $\hat{\delta}$ is the estimated scale parameter.

Parametric Methods:

1. GARCH Method: $VaR_{t+\tau} = H_t^{-1}(\Theta)\sigma_t\sqrt{\tau}$ where H_t is the cumulative standard normal distribution function and $\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p}\alpha_i\epsilon_{t-i}^2 + \sum_{i=1}^{q}\beta_i\sigma_{t-i}^2$. In here, $\epsilon_{t-i} = x_{t-i} - \mu_{t-i}$ where $\mu_{t-i} = \frac{1}{t-i}\sum_{m=1}^{t-i} x_m$.

2. EGARCH Method: $VaR_{t+\tau} = H_t^{-1}(\Theta)\sigma_t\sqrt{\tau}$ where H_t is the cumulative standard normal distribution function and $\ln\sigma_t^2 = \alpha_0 + \alpha_{1a}\frac{\epsilon_{t-1}}{\sigma_{t-1}} + \alpha_{1b}\left(\frac{|\epsilon_{t-1}|}{\sigma_{t-1}} - E[\frac{|\epsilon_{t-1}|}{\sigma_{t-1}}]\right) + \beta_1\ln\sigma_{t-1}^2$. In here, $\epsilon_{t-1} = x_{t-1} - \mu_{t-1}$ where $\mu_{t-1} = \frac{1}{t-1}\sum_{m=1}^{t-1} x_m$.

3. RiskMetrics Method: $VaR_{t+\tau} = H_t^{-1}(\Theta)\sigma_t\sqrt{\tau}$ where H_t is the cumulative standard normal distribution function and $\sigma_t^2 = (1-\lambda)\epsilon_{t-1}^2 + \lambda\sigma_{t-1}^2$. In here, λ=0.94, $\epsilon_{t-1} = x_{t-1} - \mu_{t-1}$ where $\mu_{t-1} = \frac{1}{t-1}\sum_{m=1}^{t-1} x_m$.

Semiparametric Methods:

1. Hybrid Method: The Historical Simulation and RiskMetrics methods are combined to produce the Hybrid method. $VaR_{t+\tau} = \sum_{i=t-M+\tau}^{t} x_i\mathbf{1}\left(\sum_{j=1}^{M} S(i)\mathbf{1}\left\{x_{t+\tau-j\leq x_j}\right\} = \Theta\right)$. In here, assign weights to most recent M number of returns; $x_t, x_{t-\tau}, ...x_{t-M+\tau}$ with the following vector: $S = \frac{1-\lambda}{1-\lambda^M}, \frac{1-\lambda}{1-\lambda^M}\lambda, , ..., \frac{1-\lambda}{1-\lambda^M}\lambda^{M-1}$ where $\lambda = 0.97$ or $\lambda = 0.99$. The *S(i)* is the i^{th} element of the vector *S*, which is the weight associated with the return x_i.

7 Appendix B

7.1 Proof of p=0.5:

For the sake of short notations, let the loss function given above be defined without time dependency as follows: $l_{j,t} \equiv \upsilon_j$.

In a two method model proposed by Diebold(1995), the following assumption is made:

$$Pr(\upsilon_1 > \upsilon_2) = Pr(\upsilon_2 > \upsilon_1) = \frac{1}{2} \tag{5}$$

The above argument implies the following statement:

$$Pr(\frac{\upsilon_1}{\upsilon_2} > 1) = Pr(\frac{\upsilon_2}{\upsilon_1} > 1) = Pr(\frac{\upsilon_1}{\upsilon_2} < 1) = \frac{1}{2} \tag{6}$$

Then, extend this idea further by letting a summation in the denominator and show that the following statement holds:

$$Pr(\frac{\upsilon_1}{\upsilon_1+\upsilon_2} > \frac{1}{2}) = Pr(\frac{\upsilon_2}{\upsilon_1+\upsilon_2} > \frac{1}{2}) = Pr(\frac{\upsilon_1}{\upsilon_2} < 1) = \frac{1}{2} \tag{7}$$

This can simply be shown as:

$$Pr(\frac{\upsilon_2}{\upsilon_1+\upsilon_2} > \frac{1}{2}) = Pr(\frac{\upsilon_1}{2}+\frac{\upsilon_2}{2} < \upsilon_2) = Pr(\frac{\upsilon_1}{2} < \frac{\upsilon_2}{2}) = Pr(\frac{\upsilon_1}{\upsilon_2} < 1) = \frac{1}{2} \tag{8}$$

This framework can be extended to n number of methods considered at the same time. Let $n = 3$. Then, the following can be written:

$$Pr(\frac{\upsilon_1}{\upsilon_1+\upsilon_2+\upsilon_3} > \frac{1}{3}) = Pr(\frac{\upsilon_2}{\upsilon_1+\upsilon_2+\upsilon_3} > \frac{1}{3}) = Pr(\frac{\upsilon_3}{\upsilon_1+\upsilon_2+\upsilon_3} > \frac{1}{3}) \tag{9}$$

Also note that, the following must hold:

$$Pr(\frac{\upsilon_1}{\upsilon_1+\upsilon_2} > \frac{1}{2}) = Pr(\frac{\upsilon_2}{\upsilon_2+\upsilon_3} > \frac{1}{2}) = Pr(\frac{\upsilon_3}{\upsilon_1+\upsilon_3} > \frac{1}{2}) = ... \tag{10}$$

$$= Pr(\frac{\upsilon_2}{\upsilon_3} < 1) = Pr(\frac{\upsilon_1}{\upsilon_2} < 1) = \frac{1}{2} \tag{11}$$

Not all of the inequalities are demonstrated here, but the idea should be understood. Now, reagrding $n = 3$, from (9), the following equalities can be written:

$$Pr(\frac{\upsilon_1}{3} + \frac{\upsilon_3}{3} < \frac{2\upsilon_2}{3}) = Pr(\upsilon_1 + \upsilon_3 < 2\upsilon_2) = Pr(\frac{\upsilon_1}{\upsilon_2} + \frac{\upsilon_3}{\upsilon_2} < 2) = Pr\Phi_1 \tag{12}$$

$$Pr(\frac{\upsilon_1}{3} + \frac{\upsilon_2}{3} < \frac{2\upsilon_3}{3}) = Pr(\upsilon_1 + \upsilon_2 < 2\upsilon_3) = Pr(\frac{\upsilon_1}{\upsilon_3} + \frac{\upsilon_2}{\upsilon_3} < 2) = Pr\Phi_2 \tag{13}$$

We know that $Pr\Phi_1 = Pr\Phi_2$ and we also know that the following can be written:

$$Pr\Phi_1 = Pr\Phi_2 = Pr(\frac{\upsilon_1}{\upsilon_2} < 1) = Pr(\frac{\upsilon_3}{\upsilon_2} < 1) = \frac{1}{2} \tag{14}$$

Using forward induction, the proof can be extended to n greater than 3. Then, the following can be written:

$$Pr(\frac{\nu_i}{\sum_{j=1}^{n} \nu_j} < \frac{1}{n}) = Pr(\frac{\nu_i}{\sum_{j=1}^{n} \nu_j} > \frac{1}{n}) = \frac{1}{2} \tag{15}$$

Briefly, the ratio considering n methods can either be greater or smaller than $\frac{1}{n}$ with equal probability; which is $p = 0.5$.

References

[1] Andersen T., Bollerslev T., Diebold F.X., Ebens H., "The Distribution of Realized Stock Return Volatility", *Journal of Financial Economics*, 2001, *Vol. 61 pp. 43-76.*

[2] Artzner P., Delbaen F., Eber J., Heath D., "Coherent Measures of Risk", *Mathematical Finance*, 1999, *Vol. 9 pp. 203-228.*

[3] Awartani B.M.A., Corradi V., "Predicting the Volatility of the S&P-500 Stock Index via GARCH Models: The Role of Asymmetries", *International Journal of Forecasting*, 2005, *Vol. 21 pp. 167-183.*

[4] Berkowitz J., O'Brien J., "How Accurate are the Value-at-Risk Models at Commercial Banks?", *Journal of Finance*, 2002, *Vol. 57 pp. 1093-1111.*

[5] Black F., Studies of Stock Price Volatility Changes, *Proceedings of the 1976 Meeting of Business and Economic Statistics Section, American Statistical Association*, 1976, *pp. 177-181.*

[6] Bollerslev T., "Generalized Autoregressive Conditional Heteroskedasticity", *Journal of Econometrics*, 1986, *Vol. 31 pp. 307-327.*

[7] Campbell J. Y., Hentschel L., "No News is Good News: An Asymmetric M odel of Changing Volatility in Stock Returns", *Journal of Financial Economics*, 1992, *Vol. 31 pp. 281-318.*

[8] Christie A. A., "The Stochastic Behavior of Common Stock Variances: Value, Leverage and Interest Rate Effects", *Journal of Financial Economics*, 1982, *Vol. 10 pp. 407-432.*

[9] Christoffersen P.F, Pelletier D., "Backtesting Value-at-Risk: A Duration Based Approach", *Journal of Financial Econometrics*, 2004, *Vol. 2 pp. 84-108.*

[10] Christoffersen P.F, Berkowitz J., Pelletier D., "Evaluating Value-at-Risk Models with Desk-Level Data", *North Carolina State University, Working Paper Series No. 010*, 2006.

[11] Chiu C.L., Lee M.C., Hung J. C., "Estimation of Value-at-Risk Under Jump Dynamics and Asymmetric Information", *Applied Financial Economics*, 2005, *Vol. 15 pp. 1095-1106.*

[12] Clark T. E., McCracken M.W., "Tests of Equal Forecast Accuracy and Encompassing for Nested Models", *Journal of Econometrics*, 2001, *Vol. 105 pp. 85-110.*

[13] Clark T. E., West K.D., "Using Out-of-Sample Mean Squared Prediction Errors to Test the Martingale Difference Hypothesis", *Journal of Econometrics*, 2006, *Vol. 135 pp. 155-186.*

[14] Corradi V., Swanson R. N, Olivetti C., "Predictive Ability with Cointegrated Variables", *Journal of Econometrics*, 2001, *Vol. 104 pp. 315-358.*

[15] Diebold F. X. and Mariano R. S., "Comparing Predictive Accuracy", *Journal of Business & Economic Statistics*, 1995, *Vol. 13, pp. 253-265.*

[16] Engle R. F., and Manganelli S., "CAViaR: Conditional Autoregressive Value at Risk by Regression Quantiles", *Journal of Business & Economic Statistics*, 2004, *Vol. 22, pp. 367-381.*

[17] Fama E., "The Behavior of Stock Market Prices", *Journal of Business*, 1965, *Vol. 38 pp. 34-105.*

[18] Fuertes A.M, Izzeldin M., Kalotychou E., "On Forecasting Daily Stock Volatility: The Role of Intraday Information and Market Conditions", *International Journal of Forecasting*, 2009, *Vol. 25 pp. 259-281.*

[19] Giacomini R., White H., "Tests of Conditional Predictive Ability", *Econometrica*, 2006, *Vol. 74 pp. 1545-1578.*

[20] Glosten L. R., Jagannathan R., Runkle D.E., "On the Relation Between the Expected Value and the Volatility of the Nominal Excess Return on Stocks", *Journal of Finance*, 1993, *Vol. 48 pp. 1779-1801.*

[21] Hansen P. R., "A Test for Superior Predictive Ability", *Journal of Business & Economic Statistics*, 2005, *Vol. 23 pp. 365-380.*

[22] Harvey D. I., S. J. Leybourne, and P. Newbold, "Testing the Equality of Prediction Mean Squared Errors", *International Journal of Forecasting*, 1997, *Vol. 13, pp. 281-291.*

[23] Harvey D., P. Newbold, "Tests for Multiple Forecast Encompassing", *Journal of Applied Econometrics*, 2000, *Vol. 15 pp. 471-482.*

[24] Hentschel L., "All in the Family: Nesting Symmetric and Asymmetric GARCH Models", *Journal of Financial Economics*, 1995, *Vol. 39 pp. 71-104.*

[25] Koenker R., Basset G., "Regression Quantiles", *Econometrica*, 1978, *Vol. 46 pp. 33-50.*

[26] Kuester K., Mittnik S., Paolella M. S., "Value-at-Risk Prediction: A Comparison of Alternative Strategies", *Journal of Financial Econometrics*, 2006, *Vol. 4 pp. 53-89.*

[27] Lopez J., "Methods for Evaluating Value-at-Risk Estimates", *Federal Reserve Bank of New York*, *Research Paper No. 9802*, 1998.

[28] Lopez J. and Diebold F. X., "Forecast Evaluation and Combination", *Federal Reserve Bank of New York*, *Research Paper No. 9525*, 1995.

[29] Mandelbrot B., "The Variations of Certain Speculative Prices", *Journal of Business*, 1963, *Vol. 36 pp. 394-419.*

[30] McCracken M. W., "Robust Out-of-Sample Inference", *Journal of Econometrics*, 2000, *Vol. 99 pp. 195-223.*

[31] Nelson D., "Conditional Heteroscedasticity in Asset Returns: A New Approach", *Econometrica*, 1991, *Vol. 59 pp. 347-370.*

[32] RiskMetrics, *Technical Document*, Morgan Guarantee Trust Company of New York, 1996.

[33] West K. D., "Asymptotic Inference About Predictive Ability" , *Econometrica*, 1996, *Vol. 64, pp. 1067-1084.*

[34] White H., "A Reality Check for Data Snooping", *Econometrica*, 2000, *Vol. 68 pp. 1097-1126.*

8 Empirical Result Tables and Figures

Table 1: Descriptive Statistics of Return Series

	IBOV Index	IPSA Index	IGBC Index	PX Index	BUX Index	MEXBOL Index
Skewness	0,22	0,40	-0,03	-0,18	0,06	0,35
Kurtosis	5,43	12,34	14,01	15,50	9,56	7,90
J-B Test	R	R	R	R	R	R
Maximum	0,1466	0,1253	0,1582	0,1316	0,1409	0,1101
Minimum	-0,1139	-0,0603	-0,1046	-0,1494	-0,1188	-0,0701

	WIG Index	INDEXCF Index	XU100 Index	JALSH Index	MERVAL Index
Skewness	-0,36	0,62	0,01	-0,07	-0,39
Kurtosis	5,69	19,37	5,89	5,70	8,18
J-B Test	R	R	R	R	R
Maximum	0,0627	0,2869	0,1289	0,0707	0,1101
Minimum	-0,0795	-0,1866	-0,0862	-0,073	-0,1215

	UKX Index	INDU Index	CAC Index	IBEX Index	DAX Index	NKY Index	AEX Index
Skewness	0,13	0,34	0,36	0,15	0,48	-0,16	0,08
Kurtosis	11,85	13,67	12,47	11,68	13,18	11,88	12,89
J-B Test	R	R	R	R	R	R	R
Maximum	0,0984	0,1108	0,1118	0,1065	0,114	0,1415	0,1055
Minimum	-0,0885	-0,0787	-0,0904	-0,0914	-0,0716	-0,1141	-0,0914

Figure 1: VaR vs. Brazil Return Series

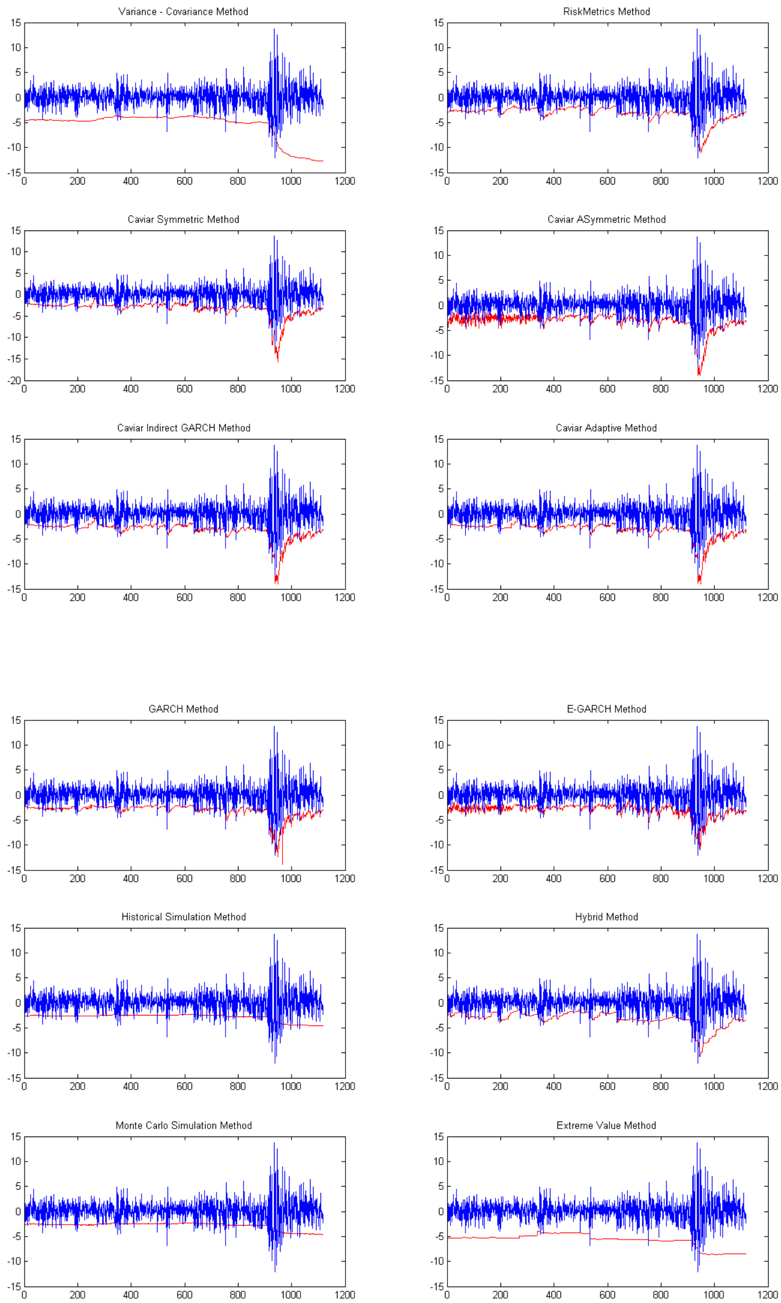

Figure 2: VaR vs. Spain Return Series

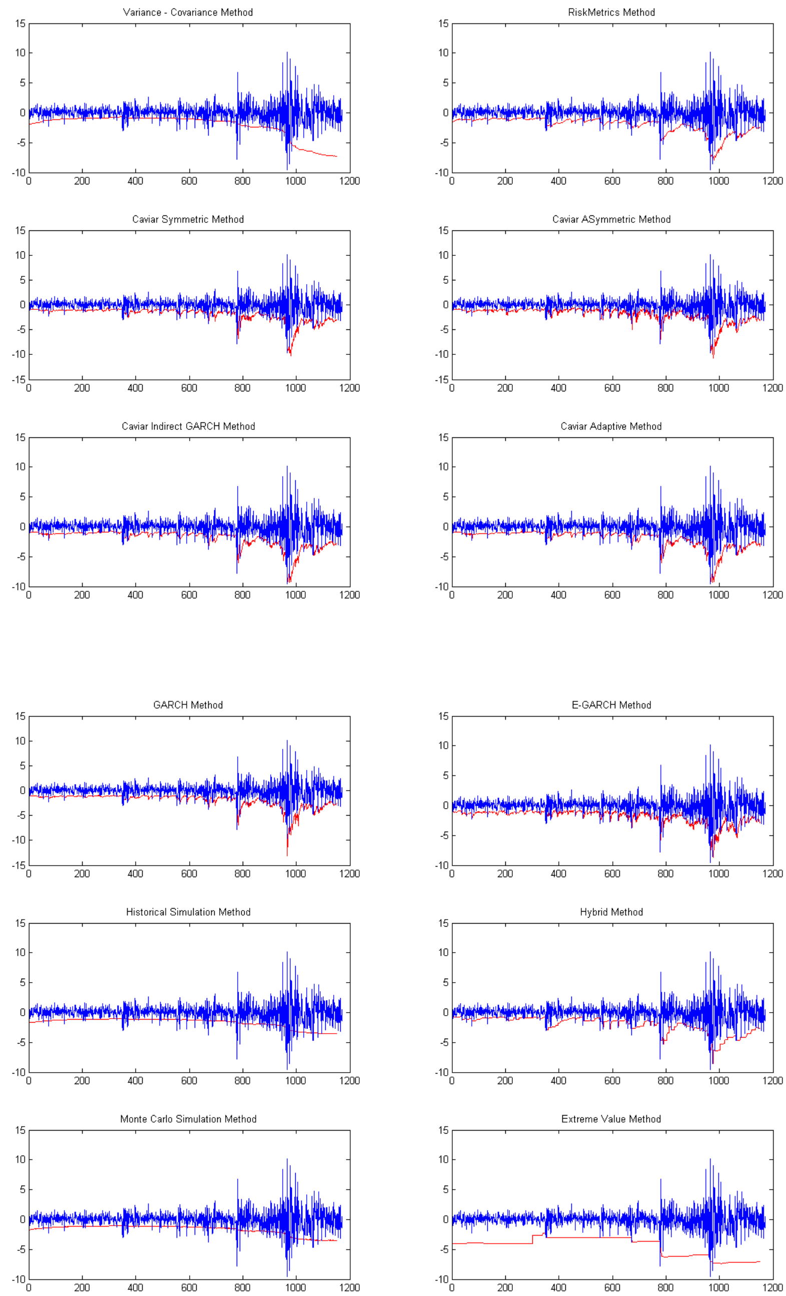

Table 2: Ranking Model Results - Emerging Markets

	IBOV Index		IPSA Index		IGBC Index		PX Index		BUX Index		MEXBOL Index	
Variance Covariance	0,28204	10,72%	0,13266	7,50%	0,25683	10,88%	0,19351	9,00%	0,20605	9,56%	0,32637	10,38%
RiskMetrics	0,20630	7,84%	0,11907	6,73%	0,16663	7,06%	0,15761	7,33%	0,16080	7,46%	0,30816	9,80%
CAViaR Symmetric	0,20573	7,82%	0,18737	10,59%	0,16522	7,00%	0,15661	7,28%	0,16491	7,65%	0,31591	10,04%
CAViaR Asymmetric	0,18373	6,98%	0,12758	7,21%	0,14578	6,18%	0,17883	8,32%	0,16102	7,47%	0,16619	5,28%
CAViaR Indirect GARCH	0,20431	7,77%	0,18239	10,31%	0,16399	6,95%	0,15563	7,24%	0,16116	7,48%	0,18402	5,85%
CAViaR Adaptive	0,20433	7,77%	0,18240	10,31%	0,16400	6,95%	0,15565	7,24%	0,16117	7,48%	0,18403	5,85%
GARCH	0,20926	7,95%	0,11227	6,34%	0,16223	6,88%	0,15409	7,17%	0,16181	7,51%	0,18952	6,02%
E GARCH	0,19014	7,23%	0,10662	6,02%	0,16370	6,94%	0,14792	6,88%	0,16001	7,42%	0,17888	5,69%
Historical Simulation	0,21910	8,33%	0,11995	6,78%	0,19292	8,18%	0,17140	7,97%	0,17933	8,32%	0,44188	14,05%
Hybrid Model	0,21491	8,17%	0,16762	9,47%	0,20120	8,53%	0,21214	9,86%	0,20032	9,29%	0,18161	5,77%
Monte Carlo Simulation	0,21846	8,30%	0,11975	6,77%	0,19296	8,18%	0,17142	7,97%	0,17892	8,30%	0,43711	13,90%
Extreme Value Theory	0,29251	11,12%	0,21204	11,98%	0,38405	16,28%	0,29573	13,75%	0,25993	12,06%	0,23184	7,37%

	WIG Index		INDEXCF Index		XU100 Index		JALSH Index		MERVAL Index	
Variance Covariance	0,17509	8,27%	0,44167	12,96%	0,29761	11,08%	0,15710	8,57%	0,26027	7,12%
RiskMetrics	0,16396	7,74%	0,30050	8,82%	0,19573	7,28%	0,15692	8,56%	0,38737	10,59%
CAViaR Symmetric	0,17441	8,23%	0,24078	7,07%	0,20202	7,52%	0,13485	7,36%	0,35905	9,82%
CAViaR Asymmetric	0,16417	7,75%	0,23240	6,82%	0,19510	7,26%	0,14209	7,75%	0,17210	4,71%
CAViaR Indirect GARCH	0,17212	8,13%	0,29547	8,67%	0,19711	7,34%	0,13794	7,53%	0,18428	5,04%
CAViaR Adaptive	0,17215	8,13%	0,29548	8,67%	0,19712	7,34%	0,13796	7,53%	0,18430	5,04%
GARCH	0,16470	7,77%	0,23653	6,94%	0,19534	7,27%	0,15496	8,46%	0,33621	9,19%
E GARCH	0,17262	8,15%	0,23277	6,83%	0,19405	7,22%	0,13315	7,26%	0,17360	4,75%
Historical Simulation	0,17658	8,34%	0,23498	6,90%	0,24571	9,14%	0,13704	7,48%	0,54273	14,84%
Hybrid Model	0,17365	8,20%	0,25693	7,54%	0,20568	7,66%	0,16155	8,81%	0,19463	5,32%
Monte Carlo Simulation	0,17599	8,31%	0,23562	6,92%	0,24748	9,21%	0,13622	7,43%	0,54533	14,91%
Extreme Value Theory	0,23290	10,99%	0,40387	11,85%	0,31392	11,68%	0,24301	13,26%	0,31781	8,69%

Table 3: Rankings - Emerging Markets

	IBOV Index	**IPSA Index**	**IGBC Index**	**PX Index**	**BUX Index**	**MEXBOL Index**
1st	CAViaR Asymmetric	E GARCH	CAViaR Asymmetric	E GARCH	E GARCH	CAViaR Asymmetric
2nd	E GARCH	GARCH	GARCH	GARCH	RiskMetrics	E GARCH
3rd	CAViaR Indirect GARCH	RiskMetrics	E GARCH	CAViaR Indirect GARCH	CAViaR Asymmetric	Hybrid Model
4th	CAViaR Adaptive	Monte Carlo Simulation	CAViaR Indirect GARCH	CAViaR Adaptive	CAViaR Indirect GARCH	CAViaR Indirect GARCH
5th	CAViaR Symmetric	Historical Simulation	CAViaR Adaptive	CAViaR Symmetric	CAViaR Adaptive	CAViaR Adaptive
6th	RiskMetrics	CAViaR Asymmetric	CAViaR Symmetric	RiskMetrics	GARCH	GARCH
7th	GARCH	Variance Covariance	RiskMetrics	Historical Simulation	CAViaR Symmetric	Extreme Value Theory
8th	Hybrid Model	Hybrid Model	Historical Simulation	Monte Carlo Simulation	Monte Carlo Simulation	RiskMetrics
9th	Monte Carlo Simulation	CAViaR Indirect GARCH	Monte Carlo Simulation	CAViaR Asymmetric	Historical Simulation	CAViaR Symmetric
10th	Historical Simulation	CAViaR Adaptive	Hybrid Model	Variance Covariance	Hybrid Model	Variance Covariance
11th	Variance Covariance	CAViaR Symmetric	Variance Covariance	Hybrid Model	Variance Covariance	Monte Carlo Simulation
12th	Extreme Value Theory	Extreme Value Theory	Extreme Value Theory	Extreme Value Theory	Extreme Value Theory	Historical Simulation

	WIG Index	**INDEXCF Index**	**XU100 Index**	**JALSH Index**	**MERVAL Index**
1st	RiskMetrics	CAViaR Asymmetric	E GARCH	E GARCH	CAViaR Asymmetric
2nd	CAViaR Asymmetric	E GARCH	CAViaR Asymmetric	CAViaR Symmetric	E GARCH
3rd	GARCH	Historical Simulation	GARCH	Monte Carlo Simulation	CAViaR Indirect GARCH
4th	CAViaR Indirect GARCH	Monte Carlo Simulation	RiskMetrics	Historical Simulation	CAViaR Adaptive
5th	CAViaR Adaptive	GARCH	CAViaR Indirect GARCH	CAViaR Indirect GARCH	Hybrid Model
6th	E GARCH	CAViaR Symmetric	CAViaR Adaptive	CAViaR Adaptive	Variance Covariance
7th	Hybrid Model	Hybrid Model	CAViaR Symmetric	CAViaR Asymmetric	Extreme Value Theory
8th	CAViaR Symmetric	CAViaR Indirect GARCH	Hybrid Model	GARCH	GARCH
9th	Variance Covariance	CAViaR Adaptive	Historical Simulation	RiskMetrics	CAViaR Symmetric
10th	Monte Carlo Simulation	RiskMetrics	Monte Carlo Simulation	Variance Covariance	RiskMetrics
11th	Historical Simulation	Extreme Value Theory	Variance Covariance	Hybrid Model	Historical Simulation
12th	Extreme Value Theory	Variance Covariance	Extreme Value Theory	Extreme Value Theory	Monte Carlo Simulation

Table 4: Ranking Model Results - Developed Markets

	UKX Index		INDU Index		CAC Index		IBEX Index		DAX Index		NKY Index		AEX Index	
Variance Covariance	0,03992	9,99%	0,04315	11,42%	0,03844	8,14%	0,03805	8,32%	0,03669	7,93%	0,04945	9,57%	0,03883	9,31%
RiskMetrics	0,03008	7,53%	0,02845	7,53%	0,03135	6,64%	0,03262	7,13%	0,03328	7,19%	0,03943	7,63%	0,03235	7,75%
CAViaR Symmetric	0,03034	7,59%	0,02746	7,26%	0,03087	6,54%	0,03229	7,06%	0,03204	6,92%	0,03991	7,72%	0,03317	7,95%
CAViaR Asymmetric	0,02893	7,24%	0,02743	7,25%	0,02995	6,35%	0,02993	6,54%	0,03113	6,73%	0,03750	7,26%	0,03098	7,43%
CAViaR Indirect GARCH	0,03109	7,78%	0,02772	7,33%	0,03000	6,36%	0,03195	6,99%	0,03101	6,70%	0,03944	7,63%	0,03358	8,05%
CAViaR Adaptive	0,03109	7,78%	0,02773	7,34%	0,03000	6,36%	0,03196	6,99%	0,03102	6,70%	0,03945	7,63%	0,03359	8,05%
GARCH	0,02936	7,35%	0,02845	7,53%	0,04585	9,71%	0,03162	6,91%	0,03243	7,01%	0,03798	7,35%	0,03169	7,60%
E GARCH	0,02705	6,77%	0,02703	7,15%	0,04525	9,59%	0,03067	6,71%	0,02940	6,35%	0,03843	7,44%	0,02949	7,07%
Historical Simulation	0,02977	7,45%	0,02867	7,59%	0,03050	6,46%	0,03198	6,99%	0,03225	6,97%	0,04089	7,91%	0,03230	7,74%
Hybrid Model	0,04891	12,24%	0,04795	12,68%	0,08240	17,46%	0,08398	18,36%	0,09389	20,29%	0,05560	10,76%	0,04013	9,62%
Monte Carlo Simulation	0,02979	7,46%	0,02880	7,62%	0,03053	6,47%	0,03196	6,99%	0,03205	6,93%	0,04068	7,87%	0,03230	7,74%
Extreme Value Theory	0,04322	10,82%	0,03518	9,31%	0,04688	9,93%	0,05029	11,00%	0,04766	10,30%	0,05805	11,23%	0,04881	11,70%

Table 5: Rankings - Developed Markets

	UKX Index	INDU Index	CAC Index	IBEX Index	DAX Index	NKY Index	AEX Index
1st	E GARCH	E GARCH	CAViaR Asymmetric	CAViaR Asymmetric	E GARCH	CAViaR Asymmetric	E GARCH
2nd	CAViaR Asymmetric	CAViaR Asymmetric	CAViaR Indirect GARCH	E GARCH	CAViaR Indirect GARCH	GARCH	CAViaR Asymmetric
3rd	GARCH	CAViaR Symmetric	CAViaR Adaptive	GARCH	CAViaR Adaptive	E GARCH	GARCH
4th	Historical Simulation	CAViaR Indirect GARCH	Historical Simulation	CAViaR Indirect GARCH	CAViaR Asymmetric	RiskMetrics	Monte Carlo Simulation
5th	Monte Carlo Simulation	CAViaR Adaptive	Monte Carlo Simulation	Monte Carlo Simulation	CAViaR Symmetric	CAViaR Indirect GARCH	Historical Simulation
6th	RiskMetrics	GARCH	CAViaR Symmetric	CAViaR Adaptive	Monte Carlo Simulation	CAViaR Adaptive	RiskMetrics
7th	CAViaR Symmetric	RiskMetrics	RiskMetrics	Historical Simulation	Historical Simulation	CAViaR Symmetric	CAViaR Symmetric
8th	CAViaR Indirect GARCH	Historical Simulation	Variance Covariance	CAViaR Symmetric	GARCH	Monte Carlo Simulation	CAViaR Indirect GARCH
9th	CAViaR Adaptive	Monte Carlo Simulation	E GARCH	RiskMetrics	RiskMetrics	Historical Simulation	CAViaR Adaptive
10th	Variance Covariance	Extreme Value Theory	GARCH	Variance Covariance	Variance Covariance	Variance Covariance	Variance Covariance
11th	Extreme Value Theory	Variance Covariance	Extreme Value Theory	Extreme Value Theory	Extreme Value Theory	Hybrid Model	Hybrid Model
12th	Hybrid Model	Hybrid Model	Hybrid Model	Hybrid Model	Hybrid Model	Extreme Value Theory	Extreme Value Theory

Table 6: DQ Test - Emerging Markets

	IBOV Index		IPSA Index		IGBC Index		PX Index		BUX Index		MEXBOL Index	
Variance Covariance	0,00000	R	0,00000	R	0,00000	R	0,00000	R	0,00003	R	0,00000	R
RiskMetrics	0,02854	R	0,00789	R	0,00000	R	0,16721	R	0,25141	NR	0,01675	R
CAViaR Symmetric	0,37478	NR	0,03424	R	0,00352	R	0,17403	R	0,18555	NR	0,04075	R
CAViaR Asymmetric	0,06993	NR	0,13041	NR	0,78217	NR	0,97378	NR	0,07320	NR	0,29234	NR
CAViaR Indirect GARCH	0,14648	NR	0,01290	R	0,04820	R	0,28161	R	0,43637	NR	0,04320	R
CAViaR Adaptive	0,14648	NR	0,01290	R	0,04820	R	0,28161	R	0,43637	NR	0,04320	R
GARCH	0,01394	R	0,71061	NR	0,12265	NR	0,94603	R	0,42884	NR	0,03426	R
E GARCH	0,00819	R	0,68648	NR	0,00017	R	0,58518	NR	0,34636	NR	0,11117	NR
Historical Simulation	0,00000	R	0,00000	R	0,00000	R	0,00000	R	0,00000	R	0,00000	R
Hybrid Model	0,11738	NR	0,00063	R	0,00000	R	0,00796	R	0,18975	NR	0,00849	R
Monte Carlo Simulation	0,00000	R	0,00000	R	0,00000	R	0,00000	R	0,00000	R	0,00000	R
Extreme Value Theory	0,00000	R	0,00000	R	0,00000	R	0,00000	R	0,00000	R	0,00000	R

	WIG Index		INDEXCF Index		XU100 Index		JALSH Index		MERVAL Index	
Variance Covariance	0,00141	R	0,00000	R	0,00000	R	0,00000	R	0,00000	R
RiskMetrics	0,00849	R	0,00555	R	0,10992	NR	0,02825	R	0,01625	R
CAViaR Symmetric	0,06851	NR	0,24772	NR	0,00183	R	0,20644	NR	0,24970	NR
CAViaR Asymmetric	0,37648	NR	0,68646	NR	0,13506	NR	0,15560	NR	0,75137	NR
CAViaR Indirect GARCH	0,02403	R	0,43165	NR	0,01059	R	0,15340	NR	0,39509	NR
CAViaR Adaptive	0,02403	R	0,43165	NR	0,01059	R	0,15340	NR	0,39509	NR
GARCH	0,11424	NR	0,39368	NR	0,50542	NR	0,25116	NR	0,07366	NR
E GARCH	0,03119	R	0,46134	NR	0,59903	NR	0,70180	NR	0,00564	R
Historical Simulation	0,00000	R	0,00000	R	0,00001	R	0,00000	R	0,00000	R
Hybrid Model	0,08092	NR	0,00775	R	0,00012	R	0,00148	R	0,00002	R
Monte Carlo Simulation	0,00000	R	0,00000	R	0,00001	R	0,00000	R	0,00000	R
Extreme Value Theory	0,00000	R	0,00000	R	0,00000	R	0,00000	R	0,00000	R

Table 7: DQ Test - Developed Markets

	UKX Index		INDU Index		CAC Index		IBEX Index		DAX Index		NKY Index		AEX Index	
Variance Covariance	0	R	0	R	7,54E-12	R	9,1E-12	R	1,24E-10	R	7,22E-12	R	0	R
RiskMetrics	0,626853	NR	0,131516	NR	0,03938	NR	0,40095	NR	0,237469	NR	0,029592	NR	0,007764	R
CAViaR Symmetric	0,183826	NR	0,433675	NR	0,013783	NR	0,156922	NR	0,141244	NR	7,55E-05	R	0,402186	NR
CAViaR Asymmetric	0,10292	NR	0,022286	NR	0,965885	NR	0,133344	NR	0,40687	NR	0,617651	NR	0,711276	NR
CAViaR Indirect GARCH	0,083612	NR	0,066084	NR	0,136877	NR	0,250787	NR	0,240118	NR	0,011609	NR	0,538702	NR
CAViaR Adaptive	0,083612	NR	0,066084	NR	0,136877	NR	0,250787	NR	0,240118	NR	0,011609	NR	0,538702	NR
GARCH	0,320813	NR	0,001457	R	0,024383	NR	0,370207	NR	0,306511	NR	0,017988	NR	0,081464	NR
E GARCH	0,005876	R	0,063105	NR	0,022732	NR	0,025745	NR	7,45E-05	R	0,001554	R	0,20295	NR
Historical Simulation	2,54E-07	R	0	R	6,66E-16	R	1,32E-09	R	8,22E-13	R	2,95E-13	R	2,71E-14	R
Hybrid Model	0,026981	NR	0,189439	NR	0,01167	NR	0,043485	NR	0,1673	NR	0,147783	NR	0,003422	R
Monte Carlo Simulation	1,06E-07	R	0	R	1,22E-15	R	1,36E-09	R	8,11E-13	R	1,65E-13	R	2,75E-14	R
Extreme Value Theory	1,64E-10	R	8,83E-09	R	4,53E-12	R	5,63E-09	R	1,38E-11	R	1,82E-08	R	6,71E-11	R

In this table "R" stands for Reject and "NR" stands for Not Reject the Hypothesis

Table 8: Alternative Predictive Ability Test Statistic - All Methods Included - Emerging Markets

	IBOV Index		IPSA Index		IGBC Index		PX Index		BUX Index		MEXBOL Index	
Variance Covariance	27,31770	R	-13,76733	NR	24,77651	R	8,58153	R	23,06261	R	5,16721	R
RiskMetrics	-19,05956	NR	-13,35104	NR	-19,38504	NR	-18,16918	NR	-15,17745	NR	-13,08043	NR
CAViaR Symmetric	-23,36816	NR	-23,69884	NR	-22,59569	NR	-23,19973	NR	-23,95191	NR	-17,03704	NR
CAViaR Asymmetric	-17,80288	NR	-20,60639	NR	-24,04957	NR	-20,18140	NR	-21,28400	NR	-14,49772	NR
CAViaR Indirect GARCH	-24,20594	NR	-21,32003	NR	-23,92841	NR	-24,62013	NR	-23,00332	NR	-19,51730	NR
CAViaR Adaptive	-24,20594	NR	-21,32003	NR	-23,92841	NR	-24,62013	NR	-23,00332	NR	-19,51730	NR
GARCH	-26,12087	NR	-19,17911	NR	-25,13998	NR	-22,90382	NR	-21,40258	NR	-19,93068	NR
E GARCH	-20,55560	NR	-20,84427	NR	-26,65443	NR	-23,73238	NR	-21,34329	NR	-19,10392	NR
Historical Simulation	-26,59960	NR	-18,04918	NR	-11,38871	NR	-22,01607	NR	-19,56468	NR	-26,83998	NR
Hybrid Model	-13,91318	NR	-14,42150	NR	-16,90133	NR	-14,79575	NR	-12,56883	NR	-8,35612	NR
Monte Carlo Simulation	-26,47992	NR	-17,81130	NR	-11,20698	NR	-21,77934	NR	-20,03898	NR	-26,72187	NR
Extreme Value Theory	27,37755	R	28,75379	R	28,71409	R	28,99967	R	28,93202	R	28,55254	R

	WIG Index		INDEXCF Index		XU100 Index		JALSH Index		MERVAL Index	
Variance Covariance	10,54381	R	28,11483	R	29,37285	R	3,09108	R	26,70750	R
RiskMetrics	-13,38708	NR	-24,06009	NR	-19,75991	NR	-10,69989	NR	-20,91838	NR
CAViaR Symmetric	-19,07363	NR	-24,47749	NR	-25,21911	NR	-25,85807	NR	-24,02183	NR
CAViaR Asymmetric	-20,37680	NR	-22,21161	NR	-22,66753	NR	-21,87533	NR	-20,91838	NR
CAViaR Indirect GARCH	-20,02139	NR	-22,62901	NR	-25,63448	NR	-20,56757	NR	-23,78310	NR
CAViaR Adaptive	-20,02139	NR	-22,62901	NR	-25,63448	NR	-20,56757	NR	-23,78310	NR
GARCH	-21,44303	NR	-26,44523	NR	-25,93118	NR	-16,46594	NR	-27,66240	NR
E GARCH	-17,94817	NR	-28,05520	NR	-25,99052	NR	-16,88205	NR	-27,36399	NR
Historical Simulation	-20,67298	NR	-19,58796	NR	-26,22787	NR	-23,18309	NR	-22,47010	NR
Hybrid Model	-11,07692	NR	-15,05619	NR	-14,95345	NR	-14,97985	NR	-15,24864	NR
Monte Carlo Simulation	-20,37680	NR	-19,34944	NR	-25,93118	NR	-22,94532	NR	-22,52979	NR
Extreme Value Theory	29,67668	R	26,92226	R	29,31351	R	29,00859	R	28,97539	R

Table 9: Alternative Predictive Ability Test Statistic - EVT and Var/Covar Methods Excluded - Emerging Markets

	IBOV Index		IPSA Index		IGBC Index		PX Index		BUX Index		MEXBOL Index	
RiskMetrics	-0,44881	NR	1,15967	NR	-4,60395	NR	4,85301	R	9,24876	R	5,16721	R
CAViaR Symmetric	2,00469	R	-3,12218	NR	-7,51170	NR	-13,31617	NR	-8,71518	NR	4,04519	R
CAViaR Asymmetric	-1,64564	NR	-12,57793	NR	-12,84259	NR	-7,04278	NR	-9,96020	NR	-3,51370	NR
CAViaR Indirect GARCH	2,72279	R	-8,41502	NR	-9,51079	NR	-13,73045	NR	-10,73093	NR	0,08858	NR
CAViaR Adaptive	2,78264	R	-8,35555	NR	-9,51079	NR	-13,73045	NR	-10,73093	NR	0,14763	NR
GARCH	-1,64564	NR	0,50550	NR	-14,11473	NR	4,26118	R	11,79810	R	-1,68303	NR
E GARCH	-5,29598	NR	-5,91728	NR	-8,90500	NR	2,72242	R	8,24088	R	-4,69478	NR
Historical Simulation	-2,42359	NR	5,67939	R	13,32722	R	7,33869	R	0,83002	NR	-5,40343	NR
Hybrid Model	4,51804	R	-1,57596	NR	-4,78568	NR	1,95304	NR	-5,27654	NR	4,28140	R
Monte Carlo Simulation	-2,00469	NR	5,85781	R	13,14548	R	7,87134	R	1,00788	NR	-5,46248	NR

	WIG Index		INDEXCF Index		XU100 Index		JALSH Index		MERVAL Index	
RiskMetrics	12,73550	R	-8,85483	NR	-1,72083	NR	4,39884	R	-8,80303	NR
CAViaR Symmetric	-3,31715	NR	-6,29080	NR	-4,56911	NR	-6,36049	NR	-13,93564	NR
CAViaR Asymmetric	-11,78774	NR	-5,69452	NR	-10,44368	NR	-9,74879	NR	-0,08952	NR
CAViaR Indirect GARCH	-5,98272	NR	-4,74046	NR	-7,53606	NR	-4,04218	NR	-14,23405	NR
CAViaR Adaptive	-5,92349	NR	-4,68084	NR	-7,47672	NR	-3,98274	NR	-14,17437	NR
GARCH	11,84698	R	-5,39638	NR	-0,35603	NR	-0,53499	NR	-3,07360	NR
E GARCH	9,18141	R	-2,29570	NR	-3,08563	NR	1,72387	NR	4,56564	R
Historical Simulation	-1,36240	NR	7,30449	R	6,82399	R	-0,95110	NR	7,43035	R
Hybrid Model	0,17770	NR	-0,68573	NR	0,35603	NR	3,56663	R	-0,38793	NR
Monte Carlo Simulation	-1,18470	NR	7,48337	R	6,64598	R	-1,42665	NR	7,43035	R

In this table "R" stands for Reject and "NR" stands for Not Reject the Hypothesis

Table 10: Alternative Predictive Ability Test Results - All Methods Included - Developed Markets

	UKX Index		INDU Index		CAC Index		IBEX Index		DAX Index		NKY Index		AEX Index	
Variance Covariance	-14,4725	NR	-18,1317	NR	-14,173	NR	-16,9706	NR	-9,44611	NR	9,484683	R	-11,7768	NR
RiskMetrics	-9,87432	NR	-10,2638	NR	-13,5301	NR	-9,95842	NR	-13,7291	NR	-16,952	NR	-15,1082	NR
CAViaR Symmetric	-20,1908	NR	-7,48337	NR	-16,8615	NR	-23,4524	NR	-20,711	NR	-16,8315	NR	-17,5629	NR
CAViaR Asymmetric	-14,3546	NR	-12,6892	NR	-14,5822	NR	-19,7401	NR	-15,3719	NR	-16,3498	NR	-15,9849	NR
CAViaR Indirect GARCH	-20,3676	NR	-13,1033	NR	-17,0369	NR	-22,2739	NR	-21,4738	NR	-18,5177	NR	-21,5372	NR
CAViaR Adaptive	-20,3676	NR	-13,1033	NR	-17,0369	NR	-22,2739	NR	-21,4738	NR	-18,5177	NR	-21,5372	NR
GARCH	-14,8852	NR	-19,5514	NR	-22,823	NR	-21,3311	NR	-21,6498	NR	-18,3371	NR	-23,7581	NR
E GARCH	-15,7105	NR	-12,0385	NR	-19,1409	NR	-19,4454	NR	-20,7697	NR	-20,8663	NR	-21,5956	NR
Historical Simulation	-17,7148	NR	-15,0555	NR	-18,4396	NR	-25,8683	NR	-16,7214	NR	-13,8808	NR	-16,8031	NR
Hybrid Model	-10,1691	NR	-7,12843	NR	-11,1923	NR	-12,5511	NR	-10,8542	NR	-11,1106	NR	-11,4846	NR
Monte Carlo Simulation	-17,7738	NR	-14,5822	NR	-18,1473	NR	-25,9862	NR	-16,78	NR	-14,0614	NR	-16,5693	NR
Extreme Value Theory	28,38498	R	27,36016	R	28,78444	R	29,52171	R	28,74902	R	27,91207	R	28,95978	R

Table 11: Alternative Predictive Ability Test Results - EVT Method Excluded - Developed Markets

	UKX Index		INDU Index		CAC Index		IBEX Index		DAX Index		NKY Index		AEX Index	
Variance Covariance	-10,28698	NR	-16,06115	NR	-6,39979	NR	-11,07801	NR	3,16826	R	16,22935	R	-4,99710	NR
RiskMetrics	5,33508	R	0,85778	NR	-1,19813	NR	4,06586	R	-0,70406	NR	-5,44993	NR	-3,18528	NR
CAViaR Symmetric	-5,86564	NR	8,37073	R	-1,54881	NR	-6,59966	NR	-5,28043	NR	-6,17257	NR	3,00995	R
CAViaR Asymmetric	-5,21718	NR	-2,21839	NR	-4,23730	NR	-6,77644	NR	-2,34686	NR	-7,49742	NR	-7,10114	NR
CAViaR Indirect GARCH	-6,69096	NR	10,20460	R	-3,30217	NR	-8,42636	NR	-4,51770	NR	-6,35323	NR	-4,41264	NR
CAViaR Adaptive	-6,69096	NR	10,26376	R	-3,30217	NR	-8,42636	NR	-4,51770	NR	-6,35323	NR	-4,41264	NR
GARCH	2,26962	R	8,90314	R	0,84746	NR	7,01214	R	-1,64280	NR	-8,22006	NR	-5,11399	NR
E GARCH	-2,09277	NR	-2,80996	NR	-8,85450	NR	-3,06413	NR	-10,44352	NR	-11,11063	NR	-10,08186	NR
Historical Simulation	0,44213	NR	0,73946	NR	0,26301	NR	-0,23570	NR	-2,11217	NR	-4,72729	NR	4,17886	R
Hybrid Model	2,97703	R	0,02958	NR	0,84746	NR	0,53033	NR	1,29077	NR	-0,27099	NR	-0,61368	NR
Monte Carlo Simulation	0,61899	NR	0,56199	NR	0,08767	NR	-0,05893	NR	-1,93616	NR	-5,20905	NR	4,12041	R

In this table ”R” stands for Reject and ”NR” stands for Not Reject the Hypothesis

Table 12: White's Superior Predictive Ability Test Results - Emerging Markets

	IBOV Index		IPSA Index		IGBC Index		PX Index		BUX Index		MEXBOL Index		WIG Index		INDEXCF Index		XU100 Index		JALSH Index		MERVAL Index	
Variance Covariance	3,05289	R	1,31789	R	4,44744	R	2,17269	R	2,10045	R	1,36494	R	0,99446	R	7,10223	R	3,51756	R	1,22961	R	3,34474	R
RiskMetrics	0,40338	R	0,48483	R	1,51424	R	0,58353	R	0,32055	R	0,50065	R	0,36981	R	0,65786	R	0,41351	R	0,40302	R	0,55764	R
CAViaR Symmetric	0,09008	R	0,23039	R	0,66846	R	0,33638	R	0,21444	R	0,39187	R	0,21855	R	0,37727	R	0,27756	R	0,09865	R	0,18981	R
CAViaR Asymmetric	0,00000	NR	0,00000	NR	0,00000	NR	0,00000	NR	0,00000	NR	0,00000	NR	0,00000	NR	0,00000	NR	0,00000	NR	0,00000	NR	0,00000	NR
CAViaR Indirect GARCH	0,09384	R	0,27846	R	0,75380	R	0,22180	R	0,14805	R	0,31572	R	0,24150	R	0,40752	R	0,22239	R	0,12723	R	0,12463	R
CAViaR Adaptive	0,09470	R	0,27892	R	0,75398	R	0,22251	R	0,14848	R	0,31619	R	0,24246	R	0,40803	R	0,22284	R	0,12783	R	0,12515	R
GARCH	0,49065	R	0,34887	R	0,92035	R	0,39062	R	0,40124	R	0,53125	R	0,37290	R	0,85044	R	0,40350	R	0,28433	R	0,56544	R
E GARCH	0,57585	R	0,25318	R	0,94324	R	0,33057	R	0,42498	R	0,51276	R	0,35818	R	0,95476	R	0,35205	R	0,27577	R	0,83809	R
Historical Simulation	1,57647	R	1,09317	R	2,68346	R	1,76140	R	1,45965	R	1,24542	R	0,94484	R	2,52765	R	1,10614	R	1,08517	R	1,74733	R
Hybrid Model	0,55281	R	0,63060	R	1,64072	R	0,90365	R	0,57404	R	0,58897	R	0,47916	R	1,41691	R	0,68516	R	0,48448	R	0,79341	R
Monte Carlo Simulation	1,56483	R	1,08931	R	2,68306	R	1,76593	R	1,46663	R	1,24872	R	0,94691	R	2,53475	R	1,10445	R	1,08498	R	1,74902	R
Extreme Value Theory	3,26667	R	3,44458	R	7,01298	R	4,76659	R	3,31375	R	2,69614	R	2,82752	R	5,76420	R	3,93987	R	3,37321	R	4,44928	R

Table 13: White's Superior Predictive Ability Test Results - Developed Markets

	UKX Index		INDU Index		CAC Index		IBEX Index		DAX Index		NKY Index		AEX Index	
Variance Covariance	1,719042	R	2,024724	R	1,598724	R	1,299364	R	1,356528	R	2,099302	R	2,056339	R
RiskMetrics	0,525919	R	0,396567	R	0,444196	R	0,347004	R	0,405592	R	0,577594	R	0,586389	R
CAViaR Symmetric	0,204585	R	0,183695	R	0,224292	R	0,15821	R	0,162483	R	0,364466	R	0,237889	R
CAViaR Asymmetric	0	NR	0	NR	0	NR	0	NR	0	NR	0	NR	0	NR
CAViaR Indirect GARCH	0,231235	R	0,172785	R	0,233231	R	0,114493	R	0,165788	R	0,217143	R	0,231138	R
CAViaR Adaptive	0,231856	R	0,173573	R	0,233904	R	0,115264	R	0,166472	R	0,217723	R	0,231835	R
GARCH	0,499139	R	0,408657	R	0,465265	R	0,345905	R	0,347715	R	0,568023	R	0,415556	R
E GARCH	0,512005	R	0,486542	R	0,54385	R	0,433804	R	0,456227	R	0,590832	R	0,404215	R
Historical Simulation	1,350044	R	1,58379	R	1,378183	R	1,14744	R	1,235454	R	1,823354	R	1,782938	R
Hybrid Model	0,56915	R	0,455024	R	0,543592	R	0,443063	R	0,441763	R	0,646393	R	0,50811	R
Monte Carlo Simulation	1,353237	R	1,581832	R	1,386454	R	1,152203	R	1,232682	R	1,820497	R	1,784669	R
Extreme Value Theory	2,71249	R	1,802337	R	2,808771	R	3,227168	R	2,833599	R	3,590131	R	3,249566	R

In this table "R" stands for Reject and "NR" stands for Not Reject the Hypothesis