Contagion in Financial Networks: A Random Graph Model

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December 2, 2009

EXTENDED ABSTRACT

Abstract

We study the phenomenon of financial contagion in the banking system in relation to its role in liquidity creation. Contagion and the corresponding fragility of financial markets stems from the interconnections banks establish to protect themselves from liquidity shocks. We investigate how the structure of interbank connections is related to the contagion risk of defaults, given the exogenous default of a bank of set of banks. We study both the optimal design of networks that minimize contagion risk as well as how to optimally inject capital into such a network so as to reduce contagion risk.

Keywords: network, random graph, systemic risk, default contagion, financial stability.

1 Preliminaries

Financial institutions use the interbank market to develop relationships that protect them against liquidity risk, see [Kahn and Santos, 2005],[Cocco et al., 2009]. Moreover, an intricate web of claims and obligations links the balance sheets of a wide variety of intermediaries, such as banks and hedge funds, into a network structure. Contagion and the corresponding fragility of financial markets stems from these interconnections. Indeed, [Cifuentes et al., 2005] stress the knock-on effect of an initial default of a financial

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institution on asset prices can trigger further rounds of default as other financial entities are forced to write down the value of their assets. In this paper we study the phenomenon of financial contagion in the banking system with a particular focus on how the structure of liabilities between banks, and the size and cross-correlation across shocks in the system, relate to systemic risk.

As part of our investigation, we propose an alternative approach to gauging systemic risk based on explicit modeling of counterparty relationships between banks as a weighted graph. We show that our approach leads to regulatory leverage and capital requirements that are substantively different from those based on distribution-based risk measures. Based on recent empirical data on bank leverage ratios and the structure of interbank lending networks, we propose a model for such a graph and examine the stability of the financial market under a number of regulatory scenarios.

This paper applies statistical methods from network theory to develop a general model of contagion in complex financial systems. Our framework explicitly accounts for the nature and scale of aggregate and idiosyncratic shocks, the complexity of network structure, and allows asset prices to interact with balance sheets. We also characterize conditions on idiosyncratic shocks, the value-at-risk of assets under management, and the underlying interbank network structures, under which contagion is submodular (as well as supermodular).

2 Related Work

Prior work on contagion examines mostly small network structures. [Allen and Gale, 2000] obtain results using a four-bank network, while [Freixas et al., 2000] model a system with a simple hub-and-spoke structure. The generality of insights based on small networks with symmetric structures to real-world default contagion is debatable. Furthermore, the literature largely fails to distinguish the probability of contagious default from its potential impact. This paper examines contagion in the context of large-scale and heterogenous graph structures.

Links between banks have two opposing effects on contagion. Increasing the number of links increases the opportunities for spreading liquidity shocks among counterparties, but also facilitates the ways through which contagion may spread. In a recent paper, [Gai and S.Kapadia, 2008] analyze how the network structure affects this trade-off. Using a random graph model in which links are formed randomly according to some distribution, they find that financial systems exhibit a robust-yet-fragile tendency: while greater connectivity reduces the likelihood of widespread default, the impact on the financial system, should problems occur, could be on a significantly larger scale than hitherto. We aim to extend this approach in several directions.

In the model proposed by [Gai and S.Kapadia, 2008], the total amount of assets and liabilities of a bank is kept fixed, irrespective of the network structure. Our model has no such requirement. In practice the amount of interbank assets and liabilities are related positively with the number of links. Intuitively, this correlation should hamper the risk-sharing benefit from forming links. Even if the relationship between asset and liability and the number of links is sufficiently strong, one may suspect that dense networks are vulnerable to contagion.

A second extension is to allow for heterogeneity in the size of interconnnected financial institutions. Once heterogeneity is introduced, a particularly important question is to assess the contagion risk initiated by a *given* bank. The dynamics of contagion is likely to be highly influenced by the starting point, that is by which banks fail initially. This means that a measure of systematic risk based on the expected contagion size given an expected bank picked at random is crude and may be misleading. A better measure is to condition a systemic risk indicator on the initial failures. We do this and find that analysis is similar to the derivation of *influence sets* from 'viral marketing' [Kempe et al., 2003].

In what follows, we very briefly describe our model.

3 The Model

We describe a simple model in which financial institutions draw some risky revenues from their activities, are endowed with capital, and are linked through claims on each other.

Consider n financial institutions, called banks for simplicity. A bank is endowed with some capital e_i for all $i \in V$. Let $\tilde{z}_i$ represent the (risky) revenue that i expects from its activities excluding the inter-banking relationships. From a balance sheet perspective, $\tilde{z}_i$ is equal to the asset values (stocks + loans to consumers) minus the consumers' deposits.

These banks are interconnected by their bilateral liabilities to each other, the union of which defines a liability network $\mathbf{W} = (V, E)$ in which $V = \{1, ..., n\}$ is the set of banks, and E the set of links.

Given $\mathbf{W}$, w_{ij} represents the magnitude of i's debt obligation toward j. Two banks that are not linked have no obligations toward each other: For (ij) not in E, $w_{ij} = w_{ji} = 0$. The liability network, in its membership and makeup, are exogenous.

Given the bank's endowment and the extent of its incoming and outgoing liabilities, the solvency condition for bank i when no bank defaults is:

$$z_i + e_i + \sum_{j \in V} w_{ij} - \sum_{j \in V} w_{ji} > 0. \tag{1}$$

In the event of i's default, all nodes for whom i is a creditor pay their debts. However, all nodes that are owed money by the defaulting node i receive nothing. That a defaulting node's creditors lose money can engender a default among these banks (which are neighbors). Defaults can then spread sequentially through the system: i.e. an exogenous failure at $t = 0$ at some bank travels through the network and, perhaps, to a significant number of banks. Indeed, we can have a contagion effect on defaults.

The risk of contagion stems from idiosyncratic shocks (that knock out a bank in the first time step) as well as imperfect insurance (leading to defaults in subsequent time steps), i.e. depending on the cross-correlation or size of shocks across banks.

In what follows, we consider the baseline scenario, in which there is no macro-economic factor. Dealing strictly with idiosyncratic shocks, the $\tilde{z}_i$ are independent, shocks spread sequentially through the system: i.e. an exogenous failure at $t = 0$ at some bank travels through the network and, perhaps, to all other banks.

Contagion We describe the process by which the default of some banks propagates in the market. From the point of view of i, the net value of capital less its interbank liabilities does not change overtime when contagion spreads: the value e'_i defined by $e'_i = e_i - \sum_j \omega_{ij}$

is unchanged throughout the process. Given that banks in D have failed, the solvency condition of bank i not in D is updated to:

$$z_i + e'_i < \sum_{j \in D} \omega_{ji}. \tag{2}$$

The right hand side is the sum of the liabilities to i stemming from the defaulted banks and which have been lost (only i's neighbors in D matter for i). The process of contagion is described as follows. Given the realized values for the z_i, we start with an initial set of 'active' nodes $D^0 = \{i, \text{ for which (1) fails.}\}$. At time t, we update the solvency condition (2) by taking $D = D^{t-1}$. The process stops at the step where D^t does not change.

This process fits in the class of 'threshold' models. The $z_i + e'_i$ is a (random) threshold and the ω_{ji} is the influence of j on i. A node not yet 'active' at step t becomes active in step t if the influence of its neighbors in step $t-1$ is larger than its threshold, here if the sum of their liabilities, $\sum_{j \in S} \omega_{ji}$ for node i is larger than the net asset $z_i + e'_i$.

Initial Conditions Initially, the values are set so that the overall result of the bank satisfies a condition similar to a value at risk requirement. Specifically, the level of capital, e_i, and the interbank assets and liabilities are set so that the probability of default is smaller than a certain level, $1 - \alpha$ (α can be taken to be 0.99% for example):

$$Proba(e_i + \tilde{z}_i + \sum_j \omega_{ji} - \sum_j \omega_{ij} > 0) = \alpha.$$

Denoting by F_i the distribution for z_i, this gives

$$F_i(-e_i - \sum_j \omega_{ji} + \sum_j \omega_{ij}) = 1 - \alpha.$$

Observe that i's interbank assets, the $\sum_j \omega_{ji}$, are not included in the computation of the risk. In other words, the value at risk is computed as if the risk only stems from $\tilde{z}_i$. An interesting extension is to account for the risk associated with the interbank relationships. In that case, the risk is endogenous: Given some priors F_i on default, say associated to the $\tilde{z}_i$, assign capital levels e_i. These levels in turn generate the risk of failure, hence a new distribution for the distribution of i's risk F_i, which depends on the e_k chosen by other banks (neighbors or not). Such a dynamics is affected by the information that banks have on others. We leave this point for further research.

4 Measuring External Effects

To account for the total loss on capital levels due to the failure of i, let us add to e_i the expected loss over all banks triggered by i's default. Formally, denoting by D^∞ the (random) final set of failed banked following i's default, we define

$$Ext(i) = \sum_{j\, in\, D^\infty} e_j Proba(D^\infty | i \text{ fails}) \tag{3}$$

The definition extends when i is replaced by a subset of initial defaulting nodes $A = D^0 :\rightarrow Ext(A)$.

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