IMPROVED MODELING OF DOUBLE DEFAULT EFFECTS IN BASEL II - AN ENDOGENOUS ASSET DROP MODEL WITHOUT ADDITIONAL CORRELATION

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Abstract. In 2005 the Internal Ratings Based (IRB) approach of 'Basel II' was enhanced by a 'treatment of double default effects' to account for credit risk mitigation techniques such as ordinary guarantees or credit derivatives. This paper reveals several severe problems of this approach and presents a new method to account for double default effects. This new *asset drop* technique can be applied within any structural model of portfolio credit risk. When formulated within the IRB approach of Basel II, it is very well suited for practical application as it does not pose extensive data requirements and economic capital can still be computed analytically.

Key words: Basel II, double default, IRB approach, regulatory capital, structural credit portfolio models

JEL Codes: G31, G28

1. Introduction

In 2005 the Basel Committee of Banking Supervision (BCBS) made an amendment (Basel Committee on Banking Supervision, 2005) to the original New Basel Accord of 2003 (Basel Committee on Banking Supervision, 2003) that deals with the treatment of hedged exposures in credit portfolios.[1] In the original New Basel Accord of 2003, banks are allowed to adopt a so-called *substitution approach* to hedged exposures. Roughly speaking, under this approach a bank can compute the

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Date: 30 October, 2009.

Most of this research was conducted while Eva Lütkebohmert was Juniorprofessor at the University of Bonn. Both authors acknowledge the financial support from the Bonn Graduate School of Economics.

[1] Meanwhile the amendment also has been incorporated in a revised version of the 2003 New Basel Accord, Basel Committee on Banking Supervision (2006). If not noted otherwise, this is the version we refer to with "Basel II".

risk-weighted assets for a hedged position as if the credit exposure was a direct exposure to the obligor's guarantor. Therefore, the bank may have only a small or even no benefit in terms of capital requirements from obtaining the protection. Since the 2005 amendment, for each hedged exposure the bank can choose between the substitution approach and the so-called *double default treatment.* The latter, inspired by Heitfield and Barger (2003), takes into account that the default of a hedged exposure only occurs if both the obligor and the guarantor default ("double default") and thus seems to be more sophisticated and realistic than the substitution approach.

As demonstrated by the recent global financial crisis, the importance of how to treat hedged exposures in credit portfolios is highly underestimated both in theory and in practice. This is also evidenced by the scarce literature on the double default treatment in Basel II and on modeling of double default effects in general.[2] Given that the former model sets a benchmark for the quantification of minimal capital requirements for hedged exposures of banks in the European Union, this seems to be unjustified. Furthermore, through the regulatory treatment of double default effects, the Basel Committee sets incentives for banks to obtain credit protection. There is no doubt that hedging exposures is a rather natural event than a rare exception. For example, granting loans and transferring the risk afterwards is a typical business for a bank. This can be done by use of numerous instruments (referred to as credit risk mitigation (CRM) techniques in Basel II) such as ordinary guarantees, collateral securitization and credit derivatives (in particular credit default swaps and bundled credit packages such as credit loan obligations), to name a few.[3] This is also why CRM techniques were discussed extensively in the new Basel accord of 2003 in the first place and why the Basel Committee chose to improve on the earlier version of Basel II by introducing the treatment of double default effects in 2005.

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[2] This is also evidenced by the elaborate literature review Grundke (2008). Grundke's empirical study is a notable exception that will be discussed thoroughly in this paper.

[3] The market for credit derivatives has grown rapidly in the years preceding the crisis. In the Mid-Year 2007 Market Survey Report of the International Swaps and Derivatives Association (ISDA), the notional amount of outstanding credit derivatives was estimated to be $45.46 trillion. See O'Kane (2008) for a comparison of several studies on the topic.

This paper begins in Section 2 with a review of the Internal Ratings Based (IRB) treatment of double default effects that reveals several severe problems of the approach. Most importantly, we argue that imposing additional correlation between obligors and guarantors is unsuitable to capture their essentially asymmetric relationship appropriately. We also show that this approach, in general, violates some of the assumptions of the Asymptotic Single Risk Factor (ASRF) model (see Gordy, 2003) which represents the mathematical basis of the IRB approach. Furthermore, it is implicitly assumed within the IRB treatment of double defaults that guarantors are external, i.e. there is no direct exposure to guarantors, and that different obligors in the portfolio are hedged by different guarantors. Section 3 is concerned with the robustness of the model underlying the IRB treatment of double defaults when applied under Pillar 2. In particular, we investigate whether this model can be applied in certain analytic approaches to quantify name concentration risks in credit portfolio as, for example, the granularity adjustment (GA) method of Gordy and Lütkebohmert (2007). The latter has recently been generalized by Ebert and Lütkebohmert (2009) to explicitly incorporate double default effects. Section 4 contains the major contribution of this paper, a new model to account for double default effects that can be used in all structural models of credit risk and, in particular, in the IRB approach of Basel II. The model does not exhibit any of the deficiencies mentioned in Section 2 for the IRB treatment of double defaults. Instead of modeling the relationship between an obligor and its guarantor through a dependency on an additional stochastic risk factor, we adjust the guarantor's default probability appropriately if the hedged obligor defaults. The model is endogenous as it actually quantifies the increase of the guarantor's default probability instead of exogenously imposing a numerical value as it is done in the IRB treatment of double default effects for the additional correlation parameters. The idea behind the model is to quantify the size of the downward jump of the guarantor's firm value process in case of the obligor's default which triggers the guarantee payment. We therefore call this approach an *asset drop* model. Another important feature of our proposed model is that practical application is straightforward since it does not require extensive data. Moreover, due to its simple analytic representation, economic capital can be computed almost instantaneously. The discussion and conclusions of the paper are given in Section 5.

2. Review and Discussion of the IRB Treatment of Double Defaults

Within the IRB approach of Pillar 1 in Basel II banks may choose between the simple *substitution approach* outlined in the Introduction and a double default approach where risk-weighted assets for exposures subject to double default are calculated as follows.[4] Assume the exposure to obligor n is hedged by guarantor g_n. Within the double default treatment in the IRB approach one first computes the unexpected loss (UL) capital requirement $\mathcal{K}_n$ for the hedged obligor n in the same way as the UL capital requirement for an unhedged exposure[5] with ELGD_n replaced by the loss given default ELGD_{g_n} of the guarantor. In the computation of the maturity adjustment the default probability is chosen as the minimum of the obligor's default probability PD_n and the guarantor's default probability PD_{g_n}. Then the UL capital requirement $\mathcal{K}_n^{DD}$ for the hedged exposure is calculated by multiplying $\mathcal{K}_n$ by an adjustment factor depending on the PD of the guarantor, namely

$$\mathcal{K}_n^{DD} = K_n \cdot (0.15 + 160 \cdot \mathrm{PD}_{g_n}). \tag{1}$$

Finally, the risk-weighted asset amount for the hedged exposure is computed in the same way as for unhedged exposures. Note that the multiplier $(0.15 + 160 \cdot \mathrm{PD}_{g_n})$ is derived as a linear approximation to the UL capital requirement for hedged exposures and the capital requirement for the unhedged exposure according to the usual IRB formula. Therefore, the ASRF framework, which also presents the basis for the computation of the risk weighted assets in the IRB approach, is used in an extended version to derive the exact conditional expected loss function for a hedged exposure. Specifically, it is assumed that the asset returns r_n (resp. r_{g_n}) of an obligor and its guarantor are no longer conditionally independent given the systematic risk factor X but also depend on an additional risk factor Z_{n,g_n} which

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[4] Compare Basel Committee on Banking Supervision (2006), paragraph 284.

[5] The latter is defined in paragraphs 272 and 273 of Basel Committee on Banking Supervision (2006).

only affects the obligor and its guarantor. More precisely,

$$r_n = \sqrt{\rho_n}X + \sqrt{1-\rho_n}\left(\sqrt{\psi_{ng_n}}Z_{n,g_n} + \sqrt{1-\psi_{n,g_n}}\epsilon_n\right), \tag{2}$$

where ρ_n is the asset correlation of obligor n, ψ_{n,g_n} is a factor specifying the sensitivity of obligor n to the factor Z_{n,g_n} and ϵ_n is the idiosyncratic risk factor of obligor n. By implicitly assuming that all hedges are perfect full hedges, guarantors are themselves not obligors in the portfolio and different obligors are hedged by different guarantors, the joint default probability of the obligor and its guarantor can be computed explicitly as[6]

$$\begin{aligned} &\mathbb{P}\left(\{\text{default of obligor n}\} \cap \{\text{default of guarantor}\, g_n\}\right) \\ = \quad &\Phi_2\left(\Phi^{-1}(\mathrm{PD}_n), \Phi^{-1}(\mathrm{PD}_{g_n}); \rho_{n,g_n}\right), \end{aligned} \tag{3}$$

where ρ_{n,g_n} is the correlation between obligor n and its guarantor g_n and $\Phi_2(\cdot,\cdot;\rho)$ denotes the cumulative distribution function of the bivariate normal distribution with correlation ρ. Therefore, the conditional expected loss function for a hedged exposure is given by

$$\begin{aligned} &\mathbb{E}\left[\mathbf{1}_{\{r_n \le c_n\}}\mathbf{1}_{\{r_{g_n} \le c_{g_n}\}}\,\mathrm{ELGD}_n\,\mathrm{ELGD}_{g_n}\,|X\right] = \mathrm{ELGD}_n\,\mathrm{ELGD}_{g_n}\cdot \\ &\cdot\Phi_2\left(\frac{\Phi^{-1}(\mathrm{PD}_n)-\sqrt{\rho_n}X}{\sqrt{1-\rho_n}}, \frac{\Phi^{-1}(\mathrm{PD}_{g_n})-\sqrt{\rho_{g_n}}X}{\sqrt{1-\rho_{g_n}}}; \frac{\rho_{n,g_n}-\sqrt{\rho_n\rho_{g_n}}}{\sqrt{(1-\rho_n)(1-\rho_{g_n})}}\right) \end{aligned} \tag{4}$$

for default thresholds c_n and c_{g_n} for obligor n and its guarantor g_n, respectively. One obtains the IRB risk weight function for a hedged exposure with effective maturity of one year by inserting $\Phi^{-1}(0.001)$ for X, subtracting the expected loss

$$\Phi_2(\Phi^{-1}(\mathrm{PD}_n), \Phi^{-1}(\mathrm{PD}_{g_n}); \rho_{n,g_n}) \cdot \mathrm{ELGD}_n\,\mathrm{ELGD}_{g_n} \tag{5}$$

and multiplying with 12.5 and 1.06. Since the expected loss should in general be rather small, in Basel Committee on Banking Supervision (2005) this term is set equal to zero. Moreover, double recovery effects are ignored by setting $\mathrm{ELGD}_{g_n} = 1$. Within the IRB treatment of double default effects, however, the linear approximation (1) of the exact conditional expected loss function (4) is used.[7]

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[6] For more details on the derivation see for example Grundke (2008), pp. 40-41.

[7] For a comprehensive and more detailed overview of the double default treatment we refer to Heitfield and Barger (2003) and Grundke (2008).

Let us now discuss the assumptions underlying this approach in more detail. First let us investigate how well correlation in general suits to model the dependency between a guarantor and an obligor. Positive correlation implies that the default of the obligor makes the default of the guarantor more likely. This seems very reasonable as the guarantor suffers from the guarantee payment, and if it is large it might even drag him into default. Vice versa, however, it seems neither theoretically nor empirically justified that the default of the guarantor implies a similar pain to the hedged obligor.[8] As correlation necessarily introduces a symmetric dependency between two random variables, it can never capture appropriately the asymmetric relationship that holds between a guarantor and an obligor.
Let us first consider a case where this criticism is not a problem. Suppose, first, there is no direct exposure to the guarantor and, second, the guarantor hedges exactly one position in the portfolio. In this case one is interested in the double default but otherwise not in the default of the guarantor. The unconditional dependence of the guarantor with the rest of the portfolio is ignored, but this can be compensated perfectly by choosing the additional correlation sufficiently high. Essentially, in this case the obligor and its guarantor (that interacts with the obligor and nobody else) constitute a conditionally independent unit in the portfolio. Then correlation (and also additional correlation) can be used reasonably to model the default dependency between the guarantor and its obligor and the default event of obligor 1 can be simply replaced with the less likely double default event.
The IRB treatment of double default effects simply makes no distinction, whether or not a guarantor is itself an obligor in the portfolio or if it guarantees for several obligors. The implicit approach undertaken for *any* hedging constellation is the one just explained.

If one of the two assumptions above is violated, an application of the IRB treatment of double default effects is no more rigorous. The effect of its application is that the interactions of the guarantor with the rest of the portfolio are ignored. To be more precise, if the guarantor itself is in the portfolio, it would be treated as

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[8] Essentially, the pain should be not larger than the default of any other firm in the economy whose default is reflected in the state of the systematic risk factor. Note also that the name of the guarantor in general will be unknown to the obligor.

any other obligor in the portfolio, in particular conditionally independent from its obligor. Its expected loss is computed as if it was not involved in a hedging relationship, i.e. with an unchanged default probability and a correlation parameter as used for obligors rather than guarantors. If a guarantor hedges several positions this problem becomes even more severe. Moreover, overly excessive contracting of the same guarantor is not reflected in the computation of economic capital.

Further note that the IRB treatment of double default effects is generally unsuited to deal with the above situations because of the additional correlation assumption. If the guarantor is itself in the portfolio, its default will significantly increase the default probability of the obligor, what, as mentioned before, is an unappreciated consequence. If on the other hand the guarantor hedges more than one obligor, say 3 hedges 1 and 2, then the default of 1 increases the guarantors default probability which itself increases the default probability of 2.[9] That is, 1 and 2 are no more conditionally independent because they share the same 'contagious' guarantor. In general, this seems to be very unreasonable as there need not be any business relationship between 1 and 2 or there even might be a negative relationship between them such that the default of 1 should actually decrease the default probability of obligor 2. Thus we conclude that the IRB treatment of double default effects can only be used reasonably if every obligor in the portfolio has a different guarantor and if there is no direct exposure to any of those guarantors.

There is also another, theoretical or mathematical reason why one should not impose additional correlation within the IRB approach. Suppose that a guarantor hedges several obligors or that a guarantor is internal in the sense that there is also direct exposure to the guarantor. In this case the additional correlation violates the conditional independence assumption, on which the ASRF model is based. Conditional independence, however, is required as the ASRF model relies on a law of large numbers. The asymptotic result in this situation only holds when the exposure shares of obligors that are correlated through more than the common risk factor are sufficiently small. Note further, that the same problem also occurs when we have partial hedging of a single obligor. This situation might be even more common as there might be many exposures to an obligor of which some are hedged

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[9] Note that this argument holds no matter whether there is direct exposure to the guarantor or not.

while others are not.[10]

Finally, let us also mention another deficiency of the IRB treatment of double default effects which is very relevant for practical applications. It concerns the parameter choice of the conditional correlation parameters. While not questioning the assumption of imposing additional correlation between an obligor and its guarantor in general, in a recent and long overdue empirical study, Grundke (2008) investigates the numerical values of the correlation parameters $\rho_{g_n} = 0.7$ and $\rho_{n,g_n} = 0.5$ set by the Basel Committee. To this purpose, he reviews empirical studies on default correlation and further initiates new simulation studies, which yield rather different results. While the empirical studies he considers imply the parameters to be chosen overly conservative, the simulation experiments "show that the assumed values are not unrealistic for capturing the intended effects".[11] He also notes that the appropriateness of the parameter choice actually depends, for example, on the size of the guarantor and the amount guaranteed. The IRB treatment of double default effects does not consider these quantities as the parameters are identical for all guarantors. Implicitly this means, for instance, that a small bank and a large insurance company would suffer equally if they had to fulfill a guarantee payment of $10 million.

3. Can the IRB 'Treatment of Double Default Effects' be Used Under Pillar 2?

In this section we show that besides the conceptual shortcomings of the IRB treatment of double default effects mentioned in the previous section, there are also some drawbacks when using this approach under Pillar 2. In particular, we discuss whether the treatment of double default effects that is introduced within the IRB approach under Pillar 1 is mathematically rigorous and robust enough to be applied within possible adjustments for capital requirements under Pillar 2. We will clarify our argument by examining the example of a granularity adjustment (GA).

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[10] See Ebert and Lütkebohmert (2009) for a detailed treatment of partial hedging and resulting interactions under Pillar 2 of Basel II.

[11] See p. 58 of Grundke (2008).

The ASRF model underlying the IRB approach applied in Pillar 1 of Basel II assumes that all idiosyncratic risk in the credit portfolio has been diversified away. A granularity adjustment quantifies the add-on to economic capital due to name concentration risk in credit portfolios.[12] To account for these types of risks is required under Pillar 2 of Basel II. Gordy and Lütkebohmert (2007) have derived a simple GA within a CreditRisk$^+$ framework which has been parameterized to achieve consistency with the IRB model. For a portfolio of N obligors it is given by the following expression

$$\begin{aligned} GA_{0,N} \;=\; & \frac{1}{2\mathcal{K}_{0,N}} \sum_{n=1}^{N} s_n^2 \Bigg[\delta \mathcal{C}_n(\mathcal{K}_n+\mathcal{R}_n) + \delta(\mathcal{K}_n+\mathcal{R}_n)^2 \cdot \frac{\mathrm{VLGD}_n^2}{\mathrm{ELGD}_n^2} \\ & \qquad -\mathcal{K}_n \left(\mathcal{C}_n + 2(\mathcal{K}_n+\mathcal{R}_n) \cdot \frac{\mathrm{VLGD}_n^2}{\mathrm{ELGD}_n^2} \right) \Bigg] \end{aligned} \tag{6}$$

where s_n denotes the exposure share of obligor n on total exposure and the quantities $\mathcal{K}_n$ and $\mathcal{R}_n$ denote the unexpected loss (UL) and the expected loss (EL) of obligor n, respectively. $\mathcal{K}_{0,N}$ is the weighted average of the $\mathcal{K}_n$ where the weight associated with $\mathcal{K}_n$ is s_n. The quantities ELGD_n and VLGD_n respectively denote expectation and volatility of the random loss given default of obligor n and $\mathcal{C}_n = (\mathrm{ELGD}_n^2 + \mathrm{VLGD}_n^2)/\,\mathrm{ELGD}_n$. The constant δ depends on model parameters such as the variance of the systematic risk factor X and can either be derived from the bank's data or it can be fixed by the regulators. This model, however, does not account for guarantees within a credit portfolio.

A possibility to treat double default effects within a GA is to use the above formula for the GA and insert for the unexpected loss requirement of a hedged exposure to obligor n, the corresponding expression derived within the IRB treatment of double defaults, i.e. $\mathcal{K}_n^{DD}$. We call such an approach a *two-step approach* and denote the corresponding two-step GA for a portfolio with one hedged and $N-1$ unhedged positions by $GA_{1,N-1}^{\text{2step}}$. In a first step one derives a method to quantify name concentration risks as if there were no double default effects. In a second step one sets up a double default model with the aim to derive new expressions for EL and UL for the hedged positions.[13] These expressions are then inserted in the *solution*

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[12] See Lütkebohmert (2009a) for more information on the granularity adjustment.

[13] Note that the EL reserve requirement is the same for hedged and unhedged exposures in the IRB treatment of double defaults.

of the model from the first step. Hence, roughly speaking, under a two-step approach the computation of EL and UL for the individual positions in the portfolio and the computation of economic capital are solved separately (rather than jointly) and then are put together naively. This, of course, implies a fairly easy derivation, however, with the shortcoming of missing any mathematical justification. This lack of mathematical rigor has direct economic implications. In a two-step approach, necessarily, any interaction of the guarantor with the rest of the portfolio (and also with other guarantors) is ignored. That is, even the common dependence induced by the systematic risk factor is ignored. As we explained in Section 2, this is the case in the IRB treatment of double default effects under Pillar 1, whenever guarantors are not external to the portfolio or not distinct for all obligors. Thus, the IRB treatment of double default effects can be regarded as relying on such a two-step approach in those cases.

In the following, we will illustrate the main shortcomings of this two-step approach compared to the mathematically rigorous approach to quantify granularity in the presence of double defaults introduced in Ebert and Lütkebohmert (2009). The latter method represents a *bottom-up* approach as it incorporates double default effects in the model for the portfolio loss distribution and not just at the end. In this model a fully rigorous derivation is feasible. By comparing this rigorous treatment of double default effects within the simple two-step approach for the GA, we can explicitly measure the error when following a two-step approach in the GA computation. To see our argument, it is sufficient to consider a portfolio with one fully hedged position only.[14] Thus, let obligor 1 be fully hedged by obligor 2 and assume that all other obligors are unhedged. Denote the rigorous GA of Ebert and Lütkebohmert (2009) by $GA_{1,N-1}$.[15] We compare this bottom-up derivation of the GA where the double default model entered the definition of the portfolio loss with the simple two-step approach. The relative error in the benefit due to hedging

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[14] The result in Ebert and Lütkebohmert (2009) is derived for an arbitrary number of hedged positions and (multiple) partial hedging. This induces additional dependencies in the portfolio what might even strengthen our results.

[15] For the explicit formula for $GA_{1,N-1}$ we refer to the original paper of Ebert and Lütkebohmert (2009).

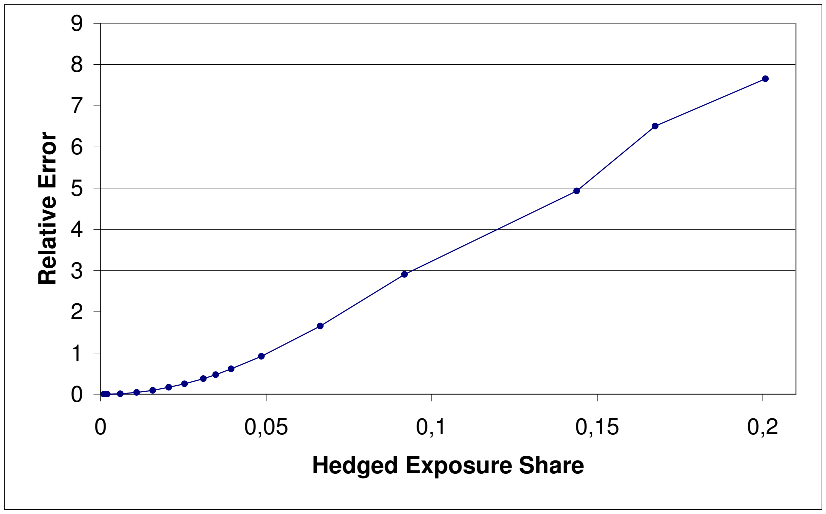
obligor 1 when applying the two-step approach in the GA computation is given by

$$e := \frac{GA^{2\text{step}}_{1,N-1} - GA_{1,N-1}}{GA_{1,N-1}} \tag{7}$$

and can be computed analytically. As the resulting expression is quite lengthy we omit it here and instead provide some numerical results on the behavior of this error.

Example 1 (Behavior of Relative Error)**.** Consider a homogeneous portfolio consisting of 1000 loans each of size 1. Assume all obligors have a PD of 1% and ELGD of 45%. To study the behavior of the relative error given in equation (7) from the two-step approach compared with the exact GA of Ebert and Lütkebohmert (2009), we assume that the exposure to obligor 1 is fully hedged by obligor 2. Let the PD of the guarantor, obligor 2, equal 0.1%. We then increase concentration in the portfolio by increasing the nominal of the hedged obligor 1 gradually to 250. Accordingly, its exposure share s_1 increases from 1/1000 to 1/5. Figure 1 below shows the effect of increasing name concentration risk on the relative error in the GA computation.

Figure 1. Relative error of two-step GA compared to exact GA

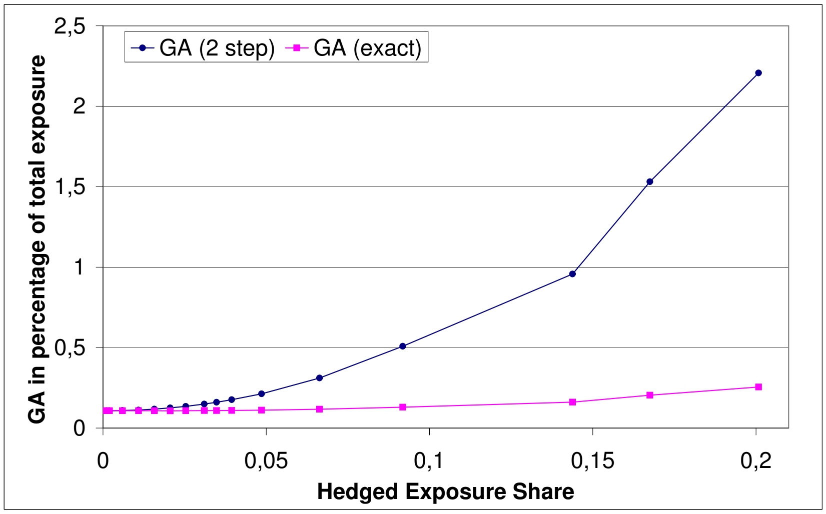


When only a small portion of the portfolio is hedged and the portfolio is almost homogeneous the error is negligible. However, when concentration increases in the portfolio and larger parts of the portfolio are hedged, the relative error between the exact GA and the two-step GA increases almost linearly with the portion of total exposure that is hedged. When 10% of the total portfolio exposure are hedged by a single guarantor, the two-step GA exceeds the exact GA computed within the approach of Ebert and Lütkebohmert (2009) already by a factor 2.9.

Figure 2 shows the GA in percent of total exposure computed with the bottom-up approach of Ebert and Lütkebohmert (2009) and with the two-step approach.

When the portfolio is almost homogeneous the error between both approaches is negligible while it increases with increasing concentration in the portfolio.

FIGURE 2. Exact GA and two-step GA for increasing name concentration



In this example the systematic risk factor X is assumed to be Gamma distributed with mean 1 and variance $1/\xi$ where $\xi = 0.125$. Moreover, the volatility of the LGD variable is modeled as $\mathrm{VLGD}_n = 1/2 \cdot \sqrt{\mathrm{ELGD}_n \cdot (1 - \mathrm{ELGD}_n)}$. Value-at-Risk is computed at the 99.9 percentile level.

The exact GA of Ebert and Lütkebohmert (2009) is formulated within an extended CreditRisk$^+$ framework and afterwards parameterized to achieve consistency with the IRB approach. Note that it is in principle possible to formulate an exact GA within the mathematical model underlying the IRB treatment of double defaults. The resulting expression, however, is very complex and hardly tractable. It involves, in particular, the computation of several cumulative bivariate normal distribution functions which by itself is already a nontrivial task. Moreover, it can only be analytically formulated in the case where each obligor is hedged by a different external guarantor. Thus, such an approach would not be very well suited for practical applications at all.

Before concluding this section let us point out that the mathematical model underlying the IRB treatment of double default effects is also not robust enough to be used within the saddle-point approximation method for the quantification of name concentration risk.[16] It has been shown in Lütkebohmert (2009b) that the equation for the saddle-point equivalent to the GA can neither be solved analytically in the framework underlying the IRB treatment of double defaults nor can it be solved with numerical standard software within reasonable time and effort. While it might be in principle possible to compute the saddle-point GA in this way, however, this approach is not competitive anymore to standard Monte Carlo simulation.

We conclude that the methodology used in the IRB approach to account for double default effects is not very robust towards application in other contexts. In particular, it can not be successfully applied in the computation of a granularity adjustment under Pillar 2 of Basel II.

4. The Asset Drop Technique as an Alternative Approach

In this section we will present an alternative method to account for double default effects in credit portfolios that does not rely on additional correlation between obligor and guarantor. It does capture their asymmetric relationship, i.e. that the guarantor should suffer much more from the obligor's default (triggering the guarantee payment) than vice versa. Further, our method distinguishes the case where there is direct exposure to the guarantor from the case where it is external to the portfolio. Furthermore, we properly treat the situation where a guarantor hedges several obligors.

Instead of modeling the relationship between guarantor and obligor through a dependency on an additional stochastic risk factor, we adjust the guarantor's default probability appropriately if the obligor defaults. Our model is endogenous as it actually quantifies the increase of the guarantor's default probability instead of exogenously imposing numerical values as it is done in case of the additional correlation parameters ρ_{n,g_n} in the IRB treatment of double default effects. The increase in the guarantor's default probability in our new approach depends on the size of the guarantee payment as well as on the size of the guarantor measured in terms of its asset value. The method is very well suited for practical applications as it does not pose any extensive data requirements. Moreover, due to its simple analytic

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[16] In case of a conditional independence framework Gordy (2002) has shown that tail percentiles of the loss distribution can be successfully calculated by applying the saddle-point approximation method.

representation of economic capital when incorporated in the IRB model, it can be computed almost instantaneously.

Within a structural model of default, the guarantee payment that occurs to the guarantor corresponds to a downward jump in its firm value process or, equivalently, in the firm's asset return. This causes the unconditional default probability to increase by a factor $(1 + \lambda_{n,g_n})$. This qualitative observation can be found in Grundke (2008), p. 53.[17] To illustrate the idea of our approach, let us first consider the simple case where obligor 1 is hedged by a guarantor, g_1, which is external to the portfolio. That is, the guarantor is itself not an obligor in the portfolio. We want to quantify the impact of obligor 1's default on the guarantors unconditional default probability. In the current situation the default of the guarantor is only of interest if obligor 1 defaults as well, because if solely the guarantor defaults there is no loss as there is no direct exposure to the guarantor. Thus, our goal is to compute the guarantor's (increased) default probability when the hedged obligor already has defaulted such that the guarantee payment has been triggered. The loss due to the guarantee payment may cause the guarantor's default or may make it more likely. For simplicity and for consistency with the IRB approach we illustrate the method within the model of Merton (1974). However, in principle our new approach can also be applied in more sophisticated structural credit risk models.

In the IRB approach we consider a two-period model with a 1-year horizon where time t is today and T refers to one year in the future. Our input parameters are the initial firm value $V_{g_1}(t)$ of the guarantor g_1, i.e. the firm's value at time t taken e.g. from the balance sheet or inferred from the current stock price, as well as an estimate of its volatility σ_{g_1}. We further need the (non-portfolio specific) default probability PD_{g_1}, that could be obtained from a rating agency, and the risk-free

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[17] In order to assess the conservativeness of the parameter choices for the additional correlation in the treatment of double default effects in the IRB approach, Grundke (2008) shows that the additional correlation approximately translates into an increase of 100% in the guarantor's PD. In principle, one could use Grundke's calculation to obtain individual additional correlation parameters from our estimate of λ_{n,g_n}.

interest rate r. In Merton's model under the risk-neutral measure, we have

$$\mathrm{PD}_{g_1} = \mathbb{P}(V_{g_1}(T) < B_{g_1}) = 1 - \Phi\left(\frac{\ln(V_{g_1}(t)/B_{g_1}) + (r - \frac{1}{2}\sigma_{g_1}^2)(T-t)}{\sigma_{g_1}\sqrt{T-t}}\right) \tag{8}$$

where B_{g_1} is the guarantor's debt value. From this one can compute the default threshold of guarantor g_1 implied by Merton's model as

$$B_{g_1} = V_{g_1}(t) \cdot \exp\left(-\Phi^{-1}(1 - \mathrm{PD}_{g_1}) \cdot \sigma_{g_1}\sqrt{T-t} + \left(r - \frac{1}{2}\sigma_{g_1}^2\right)(T-t)\right). \tag{9}$$

Denote by $\hat{E}_{1,g_1}$ the nominal that g_1 guarantees for obligor 1. If obligor 1 defaults, this corresponds to a downward jump of size $\hat{E}_{1,g_1}$ in the guarantor's firm value process V_{g_1}.[18] According to Merton's model, the guarantor defaults with the increased probability PD'_{g_1}, when the guarantee payment has been triggered. This can, thus, be computed as

$$\mathrm{PD}'_{g_1} = \mathbb{P}(V_{g_1}(T) - \hat{E}_{1,g_1} < B_{g_1}) = \mathbb{P}(V_{g_1}(T) < B_{g_1} + \hat{E}_{1,g_1}) \tag{10}$$

which in Merton's model can be reformulated as

$$\mathrm{PD}'_{g_1} = 1 - \Phi\left(\frac{\ln\left(\frac{V_{g_1}(t)}{B_{g_1} + \hat{E}_{1,g_1}}\right) + (r - \frac{1}{2}\sigma_{g_1}^2)(T-t)}{\sigma_{g_1}\sqrt{T-t}}\right). \tag{11}$$

Thus, we can compute the increased PD'_{g_1} of the guarantor due to the obligor's default using equations (9) and (11).[19] This then provides an analytic formula for the unconditional default growth rate λ_{1,g_1} defined via

$$\mathrm{PD}'_{g_1} = \mathrm{PD}_{g_1} \cdot (1 + \lambda_{1,g_1}) \tag{12}$$

such that

$$\lambda_{1,g_1} = \frac{\mathrm{PD}'_{g_1} - \mathrm{PD}_{g_1}}{\mathrm{PD}_{g_1}}. \tag{13}$$

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[18] Note that at this point it can be seen that the model is, in principle, capable to capture also other dependencies such as business-to-business relationships. For example, if it is known that the guarantor also has a direct claim of E_{1,g_1} to obligor 1, it might be reasonable to continue the computation with the higher asset drop $\hat{E}_{1,g_1} + E_{1,g_1}$.

[19] Thus the actual computation makes use of the same parameter values that are required by the Merton model when applied to corporate bond pricing. See for example Eom et al. (2004) how these can be obtained. However, we do not need to estimate the default threshold B_{g_1} which might pose the only serious obstacle, in particular, if the firm's debt structure is complicated. It can be computed using equation (9) which essentially calibrates Merton's model to the firm's observed rating category. See Carey and Hrycay (2001) for an overview of the standard methods used to obtain unconditional PD's over an one-year horizon.

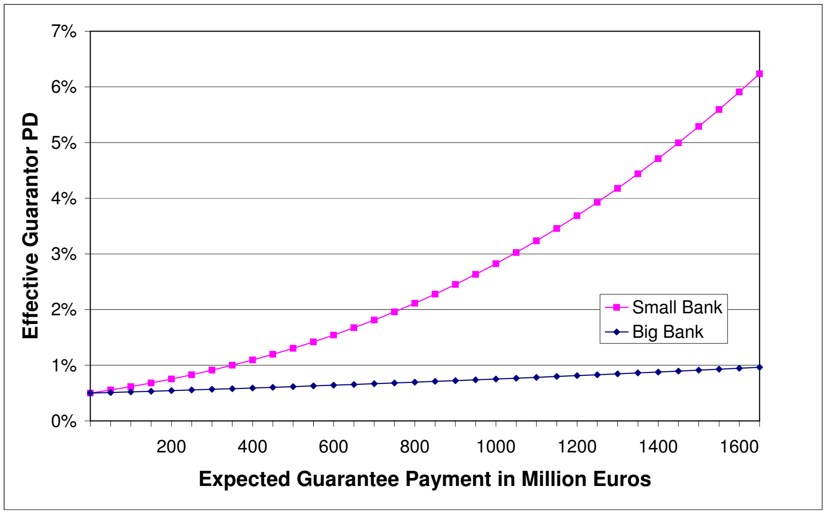
It is now straightforward, how to incorporate this approach in the IRB model for the computation of economic capital. Specifically, to compute the IRB capital charges for the hedged exposure to obligor 1, one simply inserts the double default probability $\mathrm{PD}'_{g_1}\,\mathrm{PD}_1$ instead of PD_1 in the formula for the IRB risk weight functions. Moreover, to respect double recovery effects ELGD_1 can be multiplied by ELGD_{g_1}.

Remark 1. By taking derivatives in equations (9) and (11) it can be shown that PD'_g is convex in the guarantee nominal. This convexity sets an incentive for banks to use several distinct guarantors for various loans. If, for example, there are two identical loans and two guarantors with exactly the same characteristics, the overall increase in default probability is smaller if each guarantor is contracted for one of the loans compared to when one guarantor is chosen to guarantee both loans. Thus also the bank's economic capital will be smaller if it diversifies its *guarantor risk.* In particular, as will be shown explicitly in Example 2, overly excessive contracting of the same guarantor will significantly increase economic capital. This definitely is an appreciated consequence from a regulatory point of view. However, the effect is not reflected in the current treatment of double default effects within the IRB approach. Under this approach economic capital does not depend on whether a hundred loans are hedged by one single guarantor or whether every loan is hedged by one out of a hundred different guarantors.

Example 2 (Computation of effective PD with the asset drop technique)**.** Consider two medium-sized banks, g_1 and g_2, which according to their balance sheets have total asset values of $V_{g_1}(t) = 50$ and $V_{g_2}(t) = 10$ billion Euros, respectively. Both firm value volatilities are estimated to be $\sigma^2_{g_1} = \sigma^2_{g_2} = 30\%$. Assume both to have the same rating which translates into an unconditional default probability of $\mathrm{PD}_{g_1} = \mathrm{PD}_{g_2} = 0.5\%$. The market's risk free interest rate is $r = 0.02\%$. Assume a 1-year time horizon such that $T - t = 1$ in our case. Using formula (9) we can compute the implicit default threshold for the larger bank in the Merton model and obtain $B_{g_1} = 22.517.068$ billion Euros. Likewise, for the smaller bank we obtain $B_{g_2} = 4.502.414$ billion Euros. Figure 3 shows the effective default probabilities PD'_{g_1} and PD'_{g_2} of the two banks, computed with the asset drop technique according to equation (11), as a function of the expected guarantee payment $\hat{E}_{1,g_1} \equiv \hat{E}_{1,g_2}$ due to the hedged obligor 1's default. For example, when the expected guarantee payment is 400 million Euros, the effective default probability of the smaller bank would be $\mathrm{PD}'_{g_2} = 1.09\%$, which corresponds to an increase by a factor $(1+\lambda_{1,g_2}) = 2.19$, i.e. $\lambda_{1,g_2} = 1.19$. This means, that a bank B which has no direct exposure to g_2 and which buys protection from the latter for its 400 million exposure to obligor 1, will use this increased default probability when computing its economic capital due to obligor 1. This makes sense because for the computation of B's economic capital, g_2's default is only of interest when obligor 1 already has defaulted. As for the larger bank the guarantee payment corresponds to a less significant loss, its effective default probability would only increase by a factor $(1 + \lambda_{1,g_1}) = 1.18$ to $\mathrm{PD}'_{g_1} = 0.59\%$, i.e. $\lambda_{1,g_1} = 0.18$.

Note also that the relationship is convex as already mentioned in Remark 1. Also note from equations (9) and (11) that the increase in PD is scale invariant with respect to the firm size and the loan nominal. Thus, for example, a true global player

FIGURE 3. Effective PD computed with the asset drop technique



with 100 times the firm size of the large bank considered here could guarantee 100 times as much as the large bank while suffering from the same increase in PD .

Let us now consider the more complicated case where there is direct exposure to the guarantor. Denote the exposure share of obligor 1 by s_1 and assume that it is fully hedged by guarantor g_1. Denote the direct exposure share to the guarantor by s_{g_1}. In this case we also have to focus on the default of the guarantor itself, i.e. a loss also occurs if the guarantor defaults and the hedged obligor survives. In this situation, in a sense, there are two appropriate default probabilities of the guarantor. If obligor 1 has already defaulted, the default probability of the guarantor is given by PD'_{g_1} . Otherwise it is given by PD_{g_1} . To compute the contribution to economic capital of the hedged obligor and its guarantor we have to compute the conditional expected loss of both. The probability distribution of the joint loss variable L_{1,g_1} of obligor 1 and its guarantor g_1, is given by

(14)

$$\mathbb{P}(L_{1,g_1} = l) = \begin{cases} \mathrm{PD}'_{g_1}\,\mathrm{PD}_1 & \text{for } l = s_1\,\mathrm{ELGD}_1\,\mathrm{ELGD}_{g_1} \\ & \qquad +s_{g_1}\,\mathrm{ELGD}_{g_1} \\ \mathrm{PD}_{g_1}(1-\mathrm{PD}_1) & \text{for } l = s_{g_1}\,\mathrm{ELGD}_{g_1} \\ (1-\mathrm{PD}'_{g_1})\,\mathrm{PD}_1 + (1-\mathrm{PD}_{g_1})(1-\mathrm{PD}_1) & \text{for } l = 0. \end{cases}$$

Note that the increased unconditional default probability PD'_{g_1} occurs together with PD_1 (i.e. with the probability that obligor 1 defaults), as in these situations the guarantee payment is triggered. The original default probabilities occur in the complementary case such that indeed probabilities sum up to one. The first case corresponds to the situation where both the obligor and the guarantor default (i.e. to the double default case). Here also double recovery effects are respected as s_1 is multiplied by ELGD_1 and ELGD_{g_1}. In the second case only the guarantor defaults such that only the direct exposure to g_1 is lost. The third case comprises the hedging case, i.e. the obligor defaults and the guarantor succeeds in delivering the guarantee payment (although its default probability has increased) and the case where both the guarantor and the obligor survive. Thus in this third case no loss occurs. The expected loss can be computed as

$$\begin{aligned}
\mathbb{E}[L_{1,g_1}] &= \mathrm{PD}'_{g_1}\,\mathrm{PD}_1(s_{g_1}\,\mathrm{ELGD}_{g_1} + s_1\,\mathrm{ELGD}_1\,\mathrm{ELGD}_{g_1}) \\
&\quad + \mathrm{PD}_{g_1}(1-\mathrm{PD}_1)s_{g_1}\,\mathrm{ELGD}_{g_1} \\
&= s_{g_1}\,\mathrm{ELGD}_{g_1}\big(\mathrm{PD}_{g_1} + \mathrm{PD}_1\cdot(\mathrm{PD}'_{g_1} - \mathrm{PD}_{g_1})\big) \\
&\quad + s_1\,\mathrm{ELGD}_1\,\mathrm{ELGD}_{g_1}\,\mathrm{PD}'_{g_1}\,\mathrm{PD}_1 \\
&= s_{g_1}\,\mathrm{ELGD}_{g_1}\,\mathrm{PD}_{g_1}(1+\lambda_{1,g_1}\,\mathrm{PD}_1) + s_1\,\mathrm{ELGD}_1\,\mathrm{ELGD}_{g_1}\,\mathrm{PD}'_{g_1}\,\mathrm{PD}_1\,.
\end{aligned}$$

This can be reformulated as

$$\mathbb{E}[L_{1,g_1}] = s_{g_1}\,\mathrm{ELGD}_{g_1}\,\overline{\mathrm{PD}}_{g_1} + s_1\,\mathrm{ELGD}_1\,\mathrm{ELGD}_{g_1}\,\mathrm{PD}_1\,\mathrm{PD}'_{g_1} \tag{15}$$

where $\overline{\mathrm{PD}}_{g_1} := \mathrm{PD}_{g_1}(1+\mathrm{PD}_1\,\lambda_{1,g_1})$. Note the difference between $\overline{\mathrm{PD}}_{g_1}$ and PD'_{g_1}. The former is the expected default probability of the guarantor whereas the latter is its default probability conditional on obligor 1's default. The second term in equation (15) is the expected loss due to obligor 1 that only occurs in the situation of double default. This term is the same as in the case where the guarantor is external. The first term in equation (15) is the expected loss due to obligor 2 whose default probability increases if it has to exercise its guarantee payment. That is, the expected loss due to an obligor increases if it is involved in a hedging activity because its expected PD increases. This fact is ignored in the treatment of double default effects in the IRB approach since guarantors are implicitly treated as external.[20]

The derivation of economic capital for the hedged exposure and its guarantor is obtained as follows. The conditional expected loss can be obtained as in the model underlying the IRB treatment of double default effects when there is no additional correlation, i.e. $\rho_{1,g_1} = 0$. As the asset returns of all obligors and guarantors in the portfolio only depend on the systematic risk factor X, we still have a conditional independence framework. Thus, we do not violate the assumptions of the ASRF model underlying the IRB approach. Hence, the cumulative distribution function of the bivariate normal distribution in equation (4) reduces in our setting to

(16)

$$\begin{aligned}\mathbb{E}[L_{1,g_1}|X] &= s_1 \,\mathrm{ELGD}_1 \,\mathrm{ELGD}_{g_1} \,\mathbb{E}[\mathbb{1}_{\{r_1<c_1\}}\mathbb{1}_{\{r_{g_1}<c'_{g_1}\}}|X] \\ &\quad + s_{g_1} \,\mathrm{ELGD}_{g_1} \,\mathbb{E}[\mathbb{1}_{\{r_{g_1}<c_{g_1}\}}\mathbb{1}_{\{r_1\geq c_1\}} + \mathbb{1}_{\{r_{g_1}<c'_{g_1}\}}\mathbb{1}_{\{r_1<c_1\}}|X] \\ &= s_1 \,\mathrm{ELGD}_1 \,\mathrm{ELGD}_{g_1} \,\mathrm{PD}_1(X)\,\mathrm{PD}'_{g_1}(X) \\ &\quad + s_{g_1} \,\mathrm{ELGD}_{g_1} \,\mathrm{PD}_{g_1}(X)(1-\mathrm{PD}_1(X)) + \mathrm{PD}'_{g_1}(X)\,\mathrm{PD}_1(X).\end{aligned}$$

Thus, with the arguments as above we obtain

$$\mathbb{E}[L_{1,g_1}|X] = s_{g_1}\,\mathrm{ELGD}_{g_1}\,\overline{\mathrm{PD}}_{g_1}(X) + s_1\,\mathrm{ELGD}_1\,\mathrm{ELGD}_{g_1}\,\mathrm{PD}_1(X)\,\mathrm{PD}'_{g_1}(X) \tag{17}$$

where

$$\overline{\mathrm{PD}}_{g_1}(X) = \mathrm{PD}_{g_1}(X)\cdot(1+\lambda_{1,g_1}\,\mathrm{PD}_1(X)) \tag{18}$$

and

$$\lambda_{1,g_1}(X) = \frac{\mathrm{PD}'_{g_1}(X) - \mathrm{PD}_{g_1}(X)}{\mathrm{PD}_{g_1}(X)}. \tag{19}$$

Moreover, the conditional default probability is given as in the IRB model by

$$\mathrm{PD}_n(X) = \Phi\left(\frac{\Phi^{-1}(\mathrm{PD}_n) - \sqrt{\rho_n}X}{\sqrt{1-\rho_n}}\right) \tag{20}$$

for $n = 1$ or g_1 and analogously for $\mathrm{PD}'_{g_1}(X)$.

Partial hedging and the case where a guarantor hedges multiple obligors in a portfolio can be approached with the same technique just presented and the results

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[20] Note, again, that under the IRB approach it would not be reasonable to take into account direct exposure to a guarantor as the additional correlation would induce an unrealistic dependency between obligor and guarantor.

are straightforward. See also footnote 17 for the treatment of business-to-business dependencies as well as Ebert and Lütkebohmert (2009) for a detailed treatment of these situations under Pillar 2 of Basel II.

Example 3. Consider a portfolio with $N = 110$ obligors. The first $n = 1, \ldots, 10$ loans in the portfolio are hedged by guarantors $101, \ldots, 110$, who also act as obligors in the portfolio. Assume the exposures to equal $\mathrm{EAD}_n = 1$ for all $n = 1, \ldots, 110$. The PDs are assumed to be 1% for $n = 1, \ldots, 100$ and 0.1% for the guarantors $n = 101, \ldots, 110$. Suppose the ELGDs are 45% for $n = 1, \ldots, 100$ and 100% for guarantors $n = 101, \ldots, 110$.[21] Value-at-Risk is again computed at the 99.9% percentile level. The IRB treatment of double default effects yields an economic capital of 5.45% of total exposure.[22] This is only slightly below the value obtained when neglecting double default effects entirely which equals 5.63%. Denoting by x_q the percentile at level q of the systematic risk factor X, with the asset drop technique we calculated the IRB capital as

$$\begin{aligned}
&\sum_{n=1}^{10} s_n \,\mathrm{ELGD}_n \,\mathrm{ELGD}_{g_n} \left[\mathrm{PD}_n(x_q)\,\mathrm{PD}_{g_n}(x_q)(1+\lambda_{n,g_n}(x_q)) - \mathrm{PD}_n \,\mathrm{PD}_{g_n}(1+\lambda_{n,g_n})\right] \\
&+ \sum_{n=11}^{100} s_n \,\mathrm{ELGD}_n \left(\mathrm{PD}_n(x_q) - \mathrm{PD}_n\right) \\
&+ \sum_{n=1}^{10} s_{g_n} \,\mathrm{ELGD}_{g_n} \left[\mathrm{PD}_{g_n}(x_q)\cdot(1+\mathrm{PD}_n(x_q)\cdot\lambda_{n,g_n}(x_q)) - \mathrm{PD}_{g_n}(1+\mathrm{PD}_n \cdot\lambda_{n,g_n})\right].
\end{aligned} \tag{21}$$

Table 1 shows the influence of the parameter lambda on the IRB capital computed within the asset drop approach. Here we chose a constant level of lambda for all hedged obligors in the portfolio. With increasing lambda the IRB capital increases. This is very intuitive as higher values of lambda mean that the expected default probabilities of the guarantors increase. This obviously results in higher capital requirements. For $\lambda = 2.5$ both methods lead to the same value for economic capital.

Table 1. Influence of the parameter λ on economic capital (EC)

λ	0.0	0.2	0.4	0.6	0.8	1.0
EC (in % of total EAD)	4.78	4.88	4.95	5.01	5.06	5.12

Note also that the IRB capital computed in this way is significantly lower for values of λ in $[0, 1]$ than the one obtained from the double default treatment of Basel II. Hence, within this approach financial institutions are much more rewarded for taking credit protections as under Basel II. For $\lambda = 3.5$ the IRB capital computed via the asset drop technique equals 5.63% and thus agrees with the IRB capital when

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[21] We chose the ELGDs of the guarantors as 100% to achieve consistency with the IRB treatment of double defaults where double recovery effects are neglected.

[22] This computation is based on the approximation in equation (1) as this is the one applied in practice.

guarantees are neglected in the calculation of risk weighted assets. Hence, the parameter λ can also be interpreted as in indicator of default contagion effects. When λ is empirically fitted or (if data permits) if λ is computed as in equation (13) with PD' obtained from (11), it indicates whether hedging benefits or contagion effects prevail in the credit portfolio. This is also interesting in light of the current financial crisis as λ can in this way be used as a warning system for default contagion.

5. Conclusion

In this paper we pointed out several severe problems of the treatment of double default effects applied under Pillar 1 in the IRB approach of Basel II and discussed its applicability under Pillar 2. Our main criticism is that it relies on the assumption of additional correlation between obligors and guarantors. Thus, it fails to model their asymmetric dependence structure appropriately, that is, that the guarantor should suffer much more from the obligor's default triggering the guarantee payment than vice versa. The particular choice for the additional correlation parameter is the same for all obligors and guarantors and it remains entirely unclear how specific guarantor and obligor characteristics could be reflected in this parameter. Further, all guarantors are treated as distinct for different obligors and external to the portfolio. That is, if there is direct exposure to a guarantor or if several obligors have the same guarantor, then the additional dependencies and concentrations in the credit portfolio implicitly are ignored. Thus also overly excessive contracting of the same guarantor is not reflected in the computation of economic capital. Due to the additional correlation assumption an appropriate treatment of these cases necessarily would violate the conditional independence framework of the ASRF model that underpins the IRB approach. Finally, we showed that the model is not robust enough to be applied under Pillar 2 of Basel II.

To overcome these deficiencies, we proposed a new approach to account for double default effects that can be applied in any model of portfolio credit risk and, in particular, under the IRB approach of Basel II. It is easily applicable in terms of data requirements and computational time. Despite of its simplicity it does not show any of the above mentioned shortcomings. The model endogenously quantifies the impact of the guarantee payment on the guarantor's unconditional default probability. Within a structural model of portfolio credit risk the guarantor's loss due to the guarantee payment corresponds to a downward jump in its firm value process.

This is the main idea behind our novel approach and also explains why we say that the model relies on an *asset drop technique*. This new technique could also be applied to model other dependencies within a conditional independence framework, as for example default contagion effects through business-to-business dependencies.

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