Capital requirements, and taxpayer put option values for the major US banks*

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November 30, 2009

Abstract

Assets excluding cash plus short term investments and liabilities less debt are modeled as exponentials of two Lévy processes. The two Lévy processes are modeled as linear mixtures of four independent Lévy factors. Two of these factors drive assets and liabilities with positive correlations while two of them induce negative correlations. Equity is then a call option on the spread of assets over liabilities. We employ recently developed methods by Hurd and Zhou (2009) to value these spread options using a two dimensional Fourier inversion. The reserve capital required by the taxpayer is determined by making the aggregate risk acceptable to the general external economy at stress level 0.75 for the distortion *minmaxvar*. The compound spread option model is calibrated to equity option data at market close on year end to identify the joint law of risky assets and liabilities. We find GS and MS to have sufficient reserve capital while the other four banks studied are undercapitalized for reserves with the greatest shortfall occurring for WFC. The value of the limited liability put delivered by the taxpayer is also least for GS and MS with higher values for JPM and WFC and intermediate values for BAC and C.

*We thank participants at the joint Columbia NYU seminar, and the workshop on the Future of Risk Management at Courant Institute for their comments. We also thank Haluk Unal and Peter Carr for comments on an earlier version of this paper. Errors remain our responsibility.

1 Introduction

Large players in the financial markets of the world place the financial system, and the real economy, at risk when insufficiently capitalized. Furthermore, in the presence of limited liability forms of participation in markets the incentives to be insufficiently capitalized are exaggerated. It is shown in Madan (2009) that risk sensitive reserve capital requirements, set externally to the firm with a view to making the aggregate risk being held acceptable to the general economy or the taxpayer can counteract the adverse risk incentives of limited liability arrangements. This benefit associated with risk sensitive reserve capital does require that acceptable risks be sufficiently conservatively defined and yet not be so conservative as to stifle growth and innovation.

Acceptable risks and capital requirements are given precise operational definitions in Madan (2009) following on the earlier work of Artzner, Delbaen, Eber and Heath (1999), Carr, Geman and Madan (2001), Jaschke and Küchler (2001) and Cherny and Madan (2009). It is recognized that firms with zero debt may in modern financial markets access state contingent, random, and potentially unbounded liabilities. In the presence of limited liability structures the firms acquire an option to put the firm back to the general economy or the taxpayers. The purpose of the reserve capital requirement modeled here is to ensure that the value of this put option is not too large. Shareholder equity commonly included in Tier 1 capital is not part of the reserve capital that concerns us here as we are focused for this discussion on events when all equity and debt is destroyed and the firm is being put back to the general economy as an exercise of its limited liability clauses.

Equity holders have the option to put losses back to debt holders when asset values fall sufficiently and this put option is between the two financiers of the firm, equity and debt, and is not an asset of the firm. In the presence of potentially unbounded liabilities, however, there is also the additional option to put losses back to the taxpayer or the general economy when liabilities rise without bound. It is this second put option that we propose to value and this option is an asset of the firm whose value, though typically unreported, accrues to equity holders. It may be argued that all losses fall on voluntary counterparties and hence they are a concern for these specific counterparties and should be of no concern to the government or its financial regulators. As a counterargument we observe that the put option is provided by the government for free, and some measures should be undertaken to ensure that it is not too valuable. Otherwise the limited liability law will be exploited by astute parties to engage in activities that border on extortion. Furthermore, once an unwind is set into motion many involuntary counterparties will also be adversely affected.

Our strategy for determining both the reserve capital and the value of the taxpayer put hinges on calibrating the joint law of assets and liabilities, or the stochastic process for the future balance sheet. After this is done one may compute the value of the limited liability taxpayer put held by the firm. We perform such valuations to find the limited liability taxpayer put to be worth 294, 30, 3, 158, 220 and 156 billion dollars for JPM, MS, GS, BAC, WFC and

C respectively. As far as we are aware, this is the first such computation to be conducted.

These considerations require us to pay attention to two important new dimensions of financial modeling. The first is the joint modeling of the random evolution of risky assets and liabilities. The second is the definition of risks acceptable to the general economy. For the first we adopt a framework of correlated exponential Lévy models. For the second we work here with static notions of acceptability as developed in Artzner, Delbaen, Eber and Heath (1999) and the follow up references already cited. With these two extensions in place we follow the pioneering work of Black and Scholes (1973) and Merton (1973, 1974, 1977), building the view of equity now as a spread option with a strike adjusted for the initial reserve capital in place. For dynamic extensions of concepts of acceptable risks we cite Cheridito, Delbaen and Kupper (2004), Riedel (2004), and Roorda, Engwerda and Schumacher (2005).

For an operational definition of risks acceptable to the general economy we focus attention on the probability law of the risk being undertaken. Acceptability is then reduced to a positive expectation under a sufficiently concave distortion of the distribution function. For a risk with exposure to a terminal cash flow X with distribution function $F(x)$, one computes expectations under the distribution function $G(x) = \Psi(F(x))$, where $\Psi(u)$ is a concave distribution function on the unit interval. The greater the concavity of Ψ, the more conservative and smaller is the set of acceptable risks. Cherny and Madan (2009) proposed parameterized families of such distortions, $\Psi^{\gamma}(u)$, that get more concave as one increases γ and the set of acceptable risks decline as we raise γ with only arbitrages being acceptable as γ tends to infinity. The level of γ employed is then a measure of the level of stress being placed on the cash flow distribution to test for its acceptability at the level γ.

It is suggested in Madan (2009) that the level of γ may be calibrated by selecting the smallest value at which the sensitivity of capital required to risk taken, dominates the sensitivity of equity value to risk, given the presence of limited liability. For the parametric distortion *minmaxvar* studied in Madan (2009) the value of $\gamma = 0.75$ has been suggested and we shall employ this value here as a base case.

All the computations may be accomplished and the reserve capital required may be identified once we have described the probability law $F(x)$ of the terminal cash flow. For the terminal cash flow we consider the bank's balance sheet at year end. Our interest is then the description of the probability law of the balance sheet at year end. The risky assets exclude cash and short term investments that constitute the reserve capital, while the risky or potentially unbounded liabilities exclude debt. For each of the six banks we compute the ratio of required capital to existing reserve capital as at the end of 2008. We find these ratios to be 1.89, 0.55, -0.34, 1.97, 5.08, and 1.33 respectively for JPM, MS, GS, BAC, WFC, and C. The paper provides the details for this computation.

The options market via prices of equity options at a point of time provides a read on the risk neutral law of the underlying risks embedded in the bank's balance sheet. We have at any time between 30 to 100 price quotes for options on the underlying equity price. We generalize the Merton (1973, 1974, 1977) model of equity as a call option on the underlying assets to a call option on the spread between randomly evolving, correlated and risky, assets and liabilities. We then calibrate the joint law of assets and liabilities to the entire surface of options on equity, now viewed as compound spread options or as options on a spread option. This requires, for any parameter set, that we simulate forward the assets and liabilities. On each path we then apply the spread option pricing formula as developed in Hurd and Zhou (2009) and implemented for example in Madan (2009), to compute the price of the underlying equity at each equity option maturity on each path. The model prices for equity options, at their respective maturities and strikes, are then obtained by averaging equity option cash flows across the paths and discounting. These model prices are then compared to market prices and parameter values for the underlying asset liability dynamics are calibrated by minimizing the root mean square error between market and model prices across all quoted options. Once we have the estimated parameter values we may simulate the assets and liabilities to the annual horizon and calculate the required reserve capital required by evaluating the distorted expectation of the resulting cash flow viewed as being held by the general economy. We also simulate the balance sheet to a calibrated maturity. At the calibrated maturity the observed stock price equals the value of a call option on the spread of risky assets over risky liabilities with a strike given by debt less cash reserves. The value of the taxpayer put is computed as a put option on the spread of risky assets less risky liabilities for a strike given by the negative of cash reserves. The higher the cash reserves, the lower the put strike and the lower the value of the taxpayer put.

We recognize that acceptability is initially defined with respect to a physical measure and our base measure is a risk neutral one. In this regard we make the following observation. The reserve capital required is really a cash stake the general external economy or taxpayer requires the owners to put at risk with a view to controlling the value of the limited liability put they receive. From this perspective a mere positive physical expectation that falls short of earning the rewards for risk compensation will not meet external or first best approval. Hence we take for a base measure a risk neutral measure, and then reduce the cone of acceptable risks by adding other measures equivalent to this base measure, or equivalently distorting the base risk neutral probability law.

The outline of the rest of the paper is as follows. Section 2 briefly reviews the essentials for computing required reserve capital on behalf of the external economy as outlined in Madan (2009). Section 3 describes the model for equity as a spread option on the underlying risky assets and liabilities. The specific joint process for the correlated evolution of risky assets and liabilities is presented in Section 4. Section 5 describes the balance sheet and option data on the six banks employed in the study. Section 6 presents the details for calibrating the joint law of risky assets and liabilities using balance sheet and equity option data as at the end of 2008. Calibration results are presented in Section 7. Section 8 presents the results for computation of required reserve capital, the value of the taxpayer put and related variables of interest. Section 9 concludes.

2 Required Reserve Capital

The set of risks viewed as random variables X on a probability space $(\Omega, \mathcal{F}, P)$ that are acceptable to the general economy are modeled as a cone containing the nonnegative cash flows, as the latter are always acceptable by virtue of being devoid of risk. It follows that (Artzner, Delbaen, Eber and Heath (1999)) there exists a convex set $\mathcal{M}$ of supporting probability measures $Q \in \mathcal{M}$ with the property that X is acceptable just if

$$E^Q[X] \geq 0, \text{ for all } Q \in \mathcal{M}, \text{ or} \tag{1}$$

$$\inf_{Q \in \mathcal{M}} E^Q[X] \geq 0. \tag{2}$$

Madan (2009) contrasts this condition with the condition for a positive α cash flow where one only requires such a condition for a single measure Q that respesents an equilibrium pricing measure. The class of acceptable cash flows is generally considerably smaller than positive α cash flows, and the acceptability requirement is thereby considerably more conservative.

It is critical to remark at this point that the base measure in the original formulations in Artzner, Delbaen, Eber and Heath (1999) and Cherny and Madan (2009) was the so called physical or true measure. From the perspective of generalizing risks acceptable to the general economy, a positive expectation under P that fails to earn adequate risk compensation may not be acceptable to the general external economy. One may wish to start in such a formulation with a base measure that is already some risk neutral measure. We then expand the set of test measures to add to this base risk neutral measure. In what follows we shall in practice work with a base risk neutral measure to start with that we continue to denote by P.

A risky cash flow may in general not be acceptable as it exposes the general economy to too much risk of loss. A business set up with limited liability and insufficient capital if permitted to proceed places such a risk on the general economy. A natural remedy is to seek to add capital in the amount C such that the capitalized firm with cash flow $C + X$ is acceptable. It then follows that the smallest such capital is

$$C = -\inf_{Q \in \mathcal{M}} E^Q[X]. \tag{3}$$

For an already acceptable cash flow satisfying condition (2), this capital required will be negative and one may remove cash and yet be acceptable. When X is not acceptable on its own, one may use equation (3) to compute the reserve capital that the external world demands to be put at risk.

The question that now arises is, "How do we compute this required reserve capital?". For this we turn to Cherny and Madan (2009). Suppose first that acceptability is to be defined completely by the probability law or distribution function $F(x)$ of the risk at hand. Cherny and Madan (2009) then describe the link between acceptability and concave distortions of the distribution function. Let $\Psi(u)$ be a concave distribution function on the unit interval and define acceptability as positive expectation under concave distortion of F by Ψ or the condition

$$\int_{-\infty}^{\infty} xd\Psi(F(x)) \geq 0. \tag{4}$$

The set of supporting measures $\mathcal{M}$ for this set of acceptable risks is all measures Q with density $Z = \frac{dQ}{dP}$ satisfying the condition

$$E^P\left[(Z-a)^+\right] \leq \Phi(a) = \sup_{u\in[0,1]} (\Psi(u) - ua)\,, \text{ for all } a \geq 0.$$

In summary, the condition (4) defines a valid cone of acceptable risks that depend on just a knowledge of the distribution function of the cash flow. We may observe on rewriting the integral in condition (4) as

$$\int_{-\infty}^{\infty} x\Psi'(F(x))f(x)dx,$$

that our expectation under concave distortion is also an expectation under a measure change. We note that large losses with $F(x)$ near zero are reweighted upwards by $\Psi'(F(x))$ as Ψ' decreases for any concave distortion. The more concave the distortion the higher the upward reweighting of losses and the more difficult it is to be acceptable.

Cherny and Madan (2009) go on to propose a sequence of concave distortions indexed by a real number γ that are increasingly more concave with a corresponding decreasing sequence of sets of acceptability. The recommended distortion that we employ in this paper is *minmaxvar* for which

$$\Psi^{\gamma}(u) = 1 - \left(1 - u^{\frac{1}{1+\gamma}}\right)^{1+\gamma}. \tag{5}$$

A simple computation yields the equation for capital required at level γ as

$$C = -\int_{-\infty}^{\infty} xd\Psi^{\gamma}(F(x)) \tag{6}$$

with a computation associated with a simulated set of cash flows sorted into increasing order as $x_1 \leq x_2 \leq \cdots x_N$ by

$$C \approx \sum_{j=1}^{N} x_j \left(\Psi^{\gamma}\left(\frac{j}{N}\right) - \Psi^{\gamma}\left(\frac{j-1}{N}\right)\right).$$

We abbreviate equation (6) by referring to the integral as a distorted expectation and write

$$C = -de\left(X, \gamma, 'minmaxvar'\right) \tag{7}$$

where the distortion is explicitly *minmaxvar* and the stress level is γ.

3 Equity as a Spread Option

We shall modify and extend the model formulated in Gray, Merton and Bodie (2008) that builds on the pioneering work of Merton (1973, 1974, 1977). We wish to account for the access to derivative markets that enables transformations of risk exposures and permits positions in a whole range of contingent liabilities. There is a component of assets that we shall call cash or money and denote by M, with total assets being $A+M$ where A is the random component of assets that may fluctuate in value.

On the liability side we also have a relatively fixed component like risky debt. In addition we allow for risky liabilities that are random and may rise in value, in principle without bound. The risky liabilities would include for example, short positions in stocks, the negative side of swap contracts, payouts on writing credit protections, payouts on selling options or short positions in variance swaps to mention a few possibilities. Hence we have in place of the Mertonian equation with random assets equalling equity plus risky debt, a more general equation

$$\begin{aligned} \mathit{Cash} + \mathit{Risky\ Assets} &= \mathit{Equity} + \mathit{Risky\ Debt} + \mathit{Risky\ Liabilities} \\ M(t) + A(t) &= J(t) + D(t) + L(t) \end{aligned}$$

The limited liability feature requires us to recognize that at debt maturity T with face value F, we have that

$$J(T) = (M(T) + A(T) - L(T) - D(T))^{+}.$$

while debt holders receive

$$D(T) = (M(T) + A(T) - L(T))^{+} \wedge F.$$

We recognize the relative nonrandomness of money by setting $M(t) = Me^{rt}$ for a continuously compounded interest rate of r, and we write

$$J(T) = (Me^{rT} + A(T) - L(T) - F)^{+}$$

Hence the equity and debt value initially, is

$$\begin{aligned} J &= E_0^Q \left[e^{-rT} \left(A(T) - L(T) - (F - Me^{rT}) \right)^{+} \right] \\ D &= E_0^Q \left[e^{-rT} \left(\left(Me^{rT} + A(T) - L(T) \right)^{+} \wedge F \right) \right] \end{aligned}$$

The value of the limited liability firm at debt maturity is

$$\left(Me^{rT} + A(T) - L(T) \right)^{+}$$

When $L(T)$ is zero and $A(T) \geq 0$ we may as well ignore M as we always have a positive value and there are no capital requirements to be imposed by the general economy. Under such assumptions the option to put the firm to the general economy is worthless by construction and the external world has no interest in managing the value of this put via the imposition of reserve capital requirements ensuring that the owners have some stake or exposure to the liabilities they generate. With the presence of random liabilities this is no longer the case as we now have state contingent and potentially unbounded liabilities. The value of the firm is now the value of a call struck at $-Me^{rT}$,

$$J + D = E_0^Q\left[e^{-rT}\left(Me^{rT} + A(T) - L(T)\right)^+\right] \geq M + A(0) - L(0)$$

with the excess being the value of the taxpayer put option, P, on $A(T) - L(T)$ at strike $-Me^{rT}$, or

$$P = E_0^Q\left[e^{-rT}\left(-Me^{rT} - (A(T) - L(T)\right)^+\right]. \tag{8}$$

External regulators must set capital requirements as per equation (6), at levels such that the required capital M^* is

$$M^* = -de\left(A(1) - L(1), \gamma,' minmaxvar'\right),$$

where we employ an annual horizon for the determination of reserve capital requirements.

We may compare this level with what was held by banks in terms of cash or cash equivalent reserves once we have the law of the risk held which is $A(T) - L(T)$. We report in addition to the required reserve capital the value of the limited liability put held by the firm computed in accordance with equation (8). The maturity employed is the calibrated maturity that makes the spread option price at this maturity match the observed market stock price.

The risk neutral law of $A(T) - L(T)$ may be modeled as the difference of two exponential Lévy processes that we may simulate forward in time. On this path space we may evaluate the path space of equity prices computed as a spread option with payoff

$$J(t) = E_t\left[e^{-r(T-t)}\left(A(T) - L(T) - (F - Me^{rT})\right)^+\right], \tag{9}$$

and we use these paths to construct equity option prices $w(K,t)$, for strike K and maturity t as

$$w(K,t) = e^{-rt}E_0^Q\left[(J(t) - K)^+\right].$$

We determine the parameters of the joint and correlated risky asset and liability value process to best fit the surface of the equity option surface as seen on the option markets. We then compute the value M^* at a level γ that mitigates adverse risk incentives for equity holders and compare this required reserve capital with the level of M obtained from balance sheets to determine which banks were undercapitalized or overcapitalized from the perspective of risk exposure of the external economy. We also compute the value of the taxpayer put as per equation (8). The value of M^* is determined at an annual maturity. For the taxpayer put value we use the maturity provided by equation (9) at time $t = 0$.

4 Net Asset Value Process

Let us take the risk neutral risky asset and the risky liability as exponential Lévy processes with

$$\begin{aligned} A(t) &= A(0)\exp\left(X(t) + (r+\omega_X)t\right) \\ L(t) &= L(0)\exp\left(Y(t) + (r+\omega_Y)t\right) \end{aligned}$$

where we now allow for a rich dependence in these processes. If we take a linear mixture of just two independent Lévy processes we get jumps occurring on two rays from the origin. If the independent processes are variance gamma VG processes for example then we have a VG process running in log space on a particular ray from the origin with the asymmetry parameter on this ray being the skewness parameter of the VG. The VG uses three parameters for each ray which is two sided. Given that we operate in a two sided way for each independent Lévy process, we need to cover 180 degrees of possible directions of motion. We take 4 VG processes with 12 parameters placed at the degrees 30, 60, 120, and 150. This gives us two rays with a positive relation between assets and liability movements and two rays with a negative dependence. We shall let the calibration determine the relative variance placed on each of the four rays. For the four angles η_j, $j = 1, \cdots 4$ we have the jumps in assets and liabilities as

$$\begin{aligned} x_j &= u_j \cos(\eta_j) \\ y_i &= u_j \sin(\eta_j) \end{aligned}$$

where u_i is the jump in the j^{th} VG process with parameters $\sigma_j, \nu_j, \theta_j$. We then have that

$$\begin{bmatrix} X(t) \\ Y(t) \end{bmatrix} = \begin{bmatrix} \cos(\eta_1) & \cos(\eta_2) & \cos(\eta_3) & \cos(\eta_4) \\ \sin(\eta_1) & \sin(\eta_2) & \sin(\eta_3) & \sin(\eta_4) \end{bmatrix} \begin{bmatrix} U_1(t) \\ U_2(t) \\ U_3(t) \\ U_4(t) \end{bmatrix}$$

and our joint law is the linear mixture of 4 independent VG Lévy processes with a prespecified mixing matrix. The joint characteristic function is

$$\begin{aligned} E\left[\exp\left(iuX(t) + ivY(t)\right)\right] &= \prod_{j=1}^{4} \left(\frac{1}{1 - i(u\cos(\eta_j) + v\sin(\eta_j))\theta_j\nu_j + \frac{\sigma_j^2\nu_j}{2}(u\cos(\eta_j) + v\sin(\eta_j))^2} \right)^{\frac{t}{\nu_j}} \\ &= \phi(u,v) \end{aligned}$$

The value of

$$\begin{aligned} \omega_X &= \sum_{j=1}^{4} \frac{1}{\nu_j} \ln\left(1 - \cos(\eta_j)\theta_j\nu_j - \frac{\sigma_j^2\nu_j\cos^2(\eta_j)}{2}\right) \\ \omega_Y &= \sum_{j=1}^{4} \frac{1}{\nu_j} \ln\left(1 - \sin(\eta_j)\theta_j\nu_j - \frac{\sigma_j^2\nu_j\sin^2(\eta_j)}{2}\right) \end{aligned}$$

and the characteristic function of the logarithm of assets and liabilities is

$$E\left[e^{iu\ln(A(t))+iv\ln(L(t))}\right] = \phi(u,v)\exp(iu\ln(A(0))+iv\ln(L(0))+iu(r+\omega_X)t+iv(r+\omega_Y)t)$$

Our equity value at any date t given a simulation of $A(t), L(t)$ is the price of a spread option with some strike and maturity using this joint characteristic function with initial values $A(t), L(t)$ and time to maturity $T-t$. For the initial value of risky assets and risky liabilities excluding debt, we take these magnitudes from the balance sheet but permit some option market adjustment factor to match the stock price. The adjustment factor is calibrated by equating the value of equity computed as a spread option at the strike of debt less initial cash equivalent reserves with the initial stock price at market close on the calibration date.

5 Balance Sheet and Option Data

For the balance sheet we access compustat data from Wharton Research Data Service. For each of the six banks we obtained data for the year end 2008. We first take data on cash plus short term investments, the variable CHE in compustat and we shall use this value for our initial cash equivalent reserve or the variable M in our calibration procedures. For risky assets, A, we take total assets, AT in compustat less CHE. For risky liabilities, L, we take all liabilities less long term debt ($DLTT$) plus debt in current liabilities (DLC). For the level of debt, D, we take $DLTT$ plus DLC. In addition we need the number of shares outstanding, N, and the stock price, S. The data is presented in Table 1.

TABLE 1
Balance Sheet on 6 Banks at end of 2008

	M	A	L	D	N	S
		in millions of dollars			millions	dollars
JPM	368149	1806903	1009277	633474	3732	31.59
MS	210519	448293	181159	392266	1047	15.16
GS	244425	640122	298546	498416	443	82.24
BAC	124905	1693038	882997	632946	5017	13.93
WFC	72092	1237547	781402	375232	4228	29.86
C	325681	1612789	769572	720317	5450	6.88

The other data we shall bring to bear on the study of capital required is the option surface at year end. Here we have over a hundred options trading at any time. We present along with the calibration results later the graphs of the market option prices used in the calibration along with the fitted prices from the compound spread option model. We take option maturities below 1.5 years.

6 Calibration Details

For each of the six banks we take data on equity option prices at the date of the balance sheet statement and we describe details for JPM. The level of risky assets was 1806903 and risky liabilities were at 1009277. The number of shares was 3732. We define $A(0)$, $L(0)$ to be risky assets and liabilities on a per share basis at 484.1647 and 270.4386 respectively. The total debt was at 633474 and the value of M was 368149 and this gives us a strike on a per share basis of $(633474 - 368149)/3732 = 71.0945$. Technically the strike should be future valued to the maturity but given the low rates and relatively short maturities involved we ignored this adjustment to the strike. The stock price was 31.59.

We take as parameters the maturity of equity as a spread option on the underlying spread of assets over liabilities and the 12 VG parameters on the four rays on which we run our mixture of VG processes. The first step is to solve for ξ such that the value of the spread option starting at asset level $A(0) * (1 - \xi)$, and liability level $L(0) * (1 + \xi)$ equals the observed market stock price of 31.59. This is done for the chosen set of VG parameters and ξ is the option market adjustment factor. The next step is to generate paths of assets and liabilities daily for 1.5 years and we generated 10000 such paths. Then we use the spread option pricing algorithm to compute a grid of prices of equity as a spread option at all the maturities for which we have equity option data. This grid is used to interpolate equity values for each of the maturities and all the 10000 paths. Given the interpolated equity values we compute the prices of equity options at all the traded strikes and maturities for which we have option data. We then form the mean square error between observed market option prices and the model computed option prices. This procedure gives a single value for the objective function that is minimized by an optimizer over the 13 dimensions of the 12 VG parameters and the maturity of the equity as a spread option. The entire calibration for a single name on a single day takes a few hours.

7 Calibration Results

We report the results in the order JPM, MS, GS, BAC, WFC, and C. The estimated maturities for equity as spread option were close to 5 years and are explicitly 4.4726, 4.9890, 5.0036, 5.0025, 4.9893, and 4.9991. We may in future calibrations just set this at 5 years and calibrate the other parameters. We report the VG parameters for the four angles in four separate tables.

$TABLE$ 2
VG 30^0

	σ	ν	θ
JPM	0.0955	0.1558	−0.0178
MS	0.0476	0.1491	−0.0593
GS	0.0018	0.1509	−0.0434
BAC	0.0289	0.1490	−0.0474
WFC	0.0385	0.1594	−0.0476
C	0.0553	0.1501	−0.0505

$TABLE$ 3
VG 60^0

	σ	ν	θ
JPM	0.4018	0.0810	−0.8448
MS	0.1422	0.0843	−0.1927
GS	0.1605	0.0937	−0.1935
BAC	0.0958	0.0744	−0.1792
WFC	0.0735	0.0875	−0.2037
C	0.1990	0.1007	−0.2001

$TABLE$ 4
VG 120^0

	σ	ν	θ
JPM	0.0968	0.1778	0.2967
MS	0.1699	0.2693	0.3217
GS	0.0761	0.2133	0.2092
BAC	0.0016	0.2331	0.2757
WFC	0.1088	0.2564	0.3439
C	0.1098	0.1992	0.2016

$TABLE$ 5
VG 150^0

	σ	ν	θ
JPM	0.0116	0.3524	0.0175
MS	0.0240	0.2003	0.0522
GS	0.0117	0.2002	0.0072
BAC	0.0671	0.2175	0.0737
WFC	0.0105	0.2023	0.0614
C	0.0598	0.1999	0.0209

We observe that skewness is negative on the positive angles and positive on the negative angles. Hence down jumps are more likely in directions where they move together, while up jumps are more likely when they move in opposite directions.

We present in Figures (1), (2), (3), (4), (5), and (6) the observed and fitted option prices.

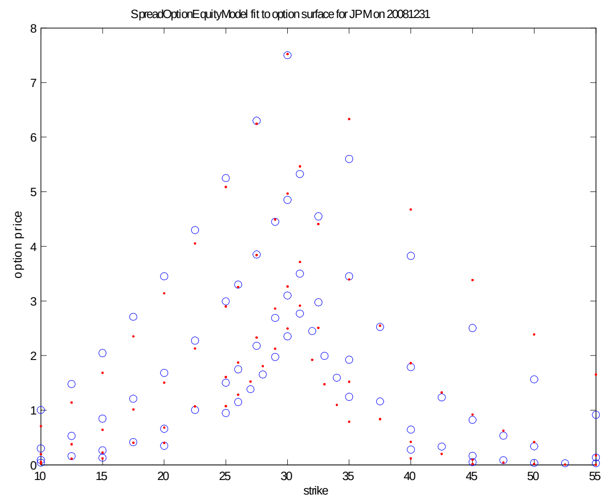

Figure 1: Market Prices and Model Prices for JPM.

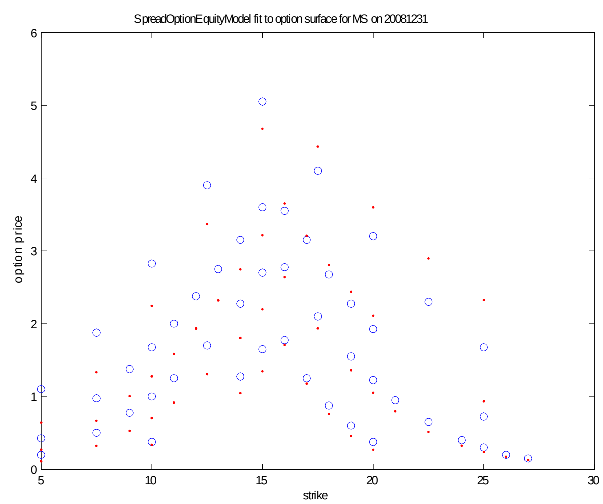

Figure 2: Market Prices and Model Prices for MS.

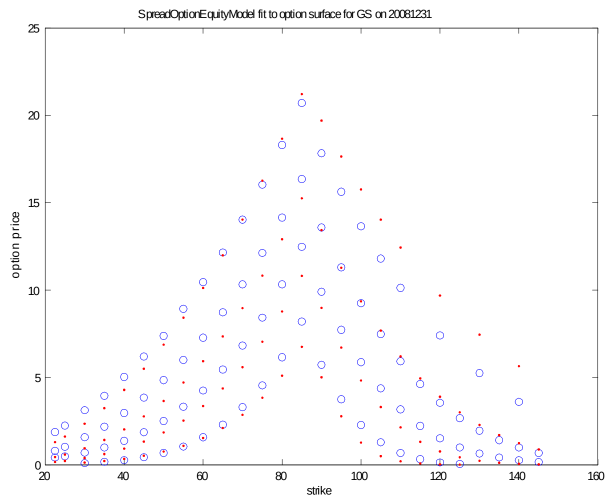

Figure 3: Market Prices and Model Prices for GS.

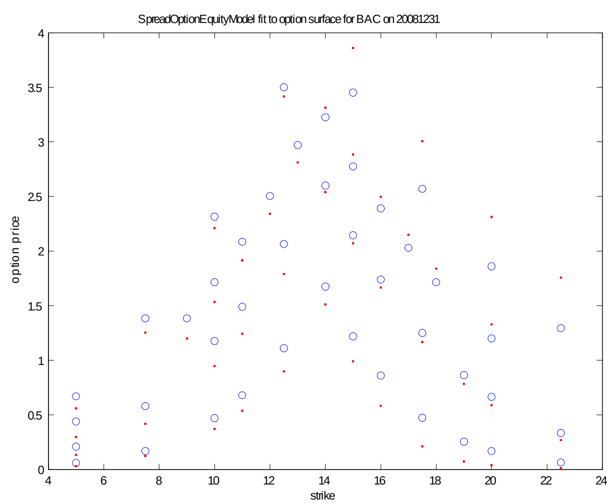

Figure 4: Market Prices and Model Prices for BAC.

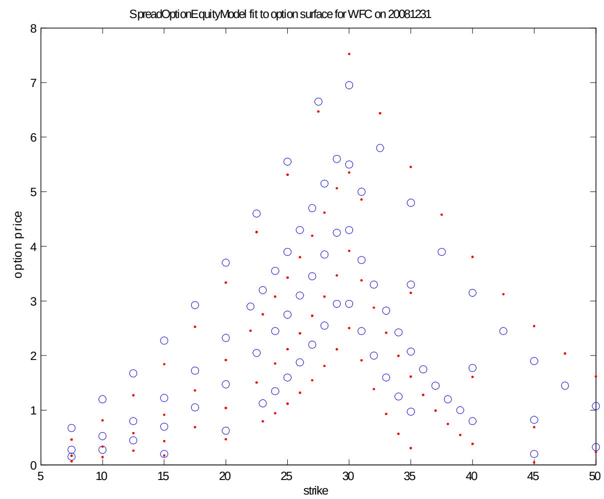

Figure 5: Market Prices and Model Prices for WFC.

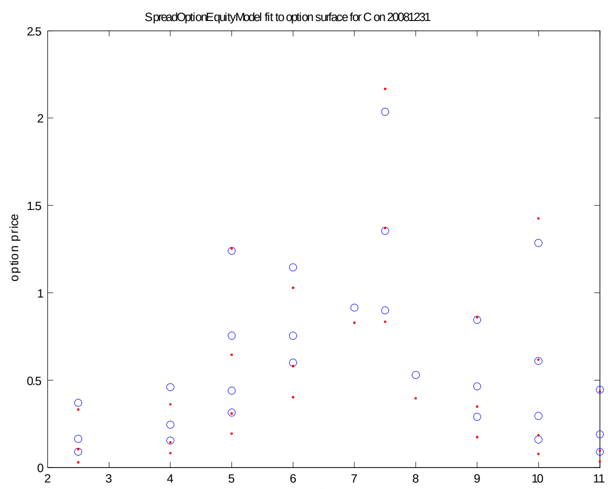

Figure 6: Market Prices and Model Prices for C.

8 Computations of Required Reserve Capital and the value of the Firm's Limited Liability Put

We present in Table 6 the computed externally required reserve capital at the stress level of 0.75 that was recommended in Madan (2009) for the distortion *minmaxvar*. Also presented are the level of cash equivalent capital held, the value of the limited liability put held by the firm, the ratio of required reserve capital to cash equivalent capital held and the option adjustment factor.

TABLE 6

In Billions of Dollars

	Reserve Capital Required	Reserve Capital Held	Limited Liability Put Value	Required to Actual Ratio	Adjustment Factor
JPM	698.039	368.149	293.96	1.8961	0.3154
MS	116.273	210.519	29.75	0.5523	0.4113
GS	−83.840	244.425	3.37	−0.3430	0.1796
BAC	246.065	124.905	158.17	1.9700	0.2840
WFC	366.832	72.092	220.14	5.0884	0.2107
C	434.596	325.681	156.21	1.3344	0.3984

We observe that GS could remove all cash plus some more as its required reserve capital is negative. All the others should hold cash equivalent reserves but MS could reduce its cash equivalent holdings by around 50% while the others need to add capital with WFC facing the biggest shortfall. The shortfall in the case of WFC may just be a consequence of having recently taken over Wachovia. The value of the limited liability put is low for GS and MS with much higher values for JPM and WFC and intermediate values for BAC and C.

9 Conclusion

We model assets excluding cash plus short term investments and liabilities excluding debt as two positive random processes. They are taken as exponentials of two Lévy processes that are modeled as linear mixtures of four independent Lévy processes that may be viewed as factors. Two of these factors drive assets and liabilities with positive correlations while the other two of them induce negative correlations. As a consequence equity is a call option on the spread of risky assets over risky liabilities. We employ recently developed methods by Hurd and Zhou (2009) to value these spread options using a two dimensional Fourier inversion.

The capital required is computed as suggested in Madan (2009), with a view to making the aggregate risk of assets less liabilities acceptable to the external world that has sold the firm a put option via the limited liability arrangement for the firm. The distortion employed is *minmaxvar* and the stress level is

0.75. To work out the capital required and the value of the limited liability put option, we need to determine the law of assets and liabilities by calibrating our model for the joint evolution of assets and liabilities.

For the calibration we use the equity option surface, simulating first the paths of assets and liabilities, then transforming them to paths of equity values using the model of equity viewed as spread option on the underlying joint asset liability process. The resulting equity value path space permits the computation of equity option prices for the traded strikes and maturities that are then compared to market prices to calibrate the joint asset liability process.

The calibrated process is then used to determine externally required reserve capital and implicit taxpayer put option values. We find GS and MS to be sufficiently capitalized with also lower taxpayer put option values, while the other four banks are undercapitalized with the greatest shortfall occurring for WFC. The taxpayer put option values for the other banks are also quite substantial.

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