Basel II and the Value of Bank Differentiation*

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Abstract

We investigate optimal capital requirements in a simple model of the banking sector in which banks need to invest in improved credit scoring systems to qualify for the use of the IRB-approach. Our main result is that regulators should use the Basel II Accord's distinction between the SA-approach and the IRB-approach to encourage large and sophisticated banks to keep their asset portfolios as safe as possible, while the financing of assets with high systematic risk should be concentrated in smaller banks that are regulated as SA-banks. We find that the regulator optimally induces large banks to increase their equity capital relative to SA-banks, leading to an additional increase in size di erentiation. We also analyze the moral hazard problem that banks misrepresent the risk of their loan portfolios under the IRB-approach. The regulator reacts to this problem by imposing stricter capital requirements, so that the advantage of bank di erentiation is reduced and a transition of all banks to IRB-status may emerge as optimal.

Key words: bank capital regulation, bank failure, risk-taking, Basel Accord, internal ratings.

JEL classification: K13, H41

1. Introduction

Motivation. The financial crisis since summer 2007 has vividly demonstrated the importance of moral hazard and forbearance in bank regulation. Many banks failed, and many others, including some of the largest banks in the world, only survived because of massive government intervention. As a result, the prudential regulation of banks and the recently enacted Basel II-Accord are under renewed scrutiny. Even prior to the financial crisis, numerous weaknesses became apparent, for example because the scope and complexity of bank assets and liabilities are much larger than only a decade ago, and because banks often engage in "regulatory arbitrage" to reduce capital requirements. The regulation of banks is set for another uncertain journey of public debate and attempts of reform, as the Basel Committee recognizes with recent announcements of a comprehensive overhaul focusing on tighter capital regulations.[1] In particular, the problem of systemically important large banks and the idea of targeting these banks specifically with higher equity demands and lower tolerance for risk-taking plays a prominent role in this discussion.[2]

Our paper contributes to this discussion by investigating the optimal system of capital requirements and its relation to the possible di erential treatment of banks and the market structure of the banking sector. We address the following questions: Is it necessary and desirable to regulate large banks di erently and to impose more stringent requirements on capital and risk-taking? And what are the consequences for the market structure and the stability of the banking sector if such a di erentiated approach is adopted? Our key finding is to demonstrate the advantages of capital requirements that di erentiate between banks according to their size and the development of their bank information and risk management systems: large and sophisticated banks should be encouraged to keep their asset portfolios as safe as possible, in order to minimize the potential risk they pose. The financing of risky assets should be concentrated and be quarantined in smaller

[1] The agenda includes higher risk weights for securitization and o -balance sheet activities, liquidity requirements and a second measure of capital adequacy based on leverage ratios, countercyclical capital bu ers, and higher capital requirements for large banks that pose systemic threats (according to the Basel Committee's "comprehensive response to the global banking crisis" press release on September 7, 2009).

[2] This is emphasized e.g. in recent remarks by Ben Bernanke: "We also are working with our domestic and international counterparts to develop capital and prudential requirements that take account of the systemic importance of large, complex firms whose failure would pose a significant threat to overall financial stability. Options under consideration include assessing a capital surcharge on these institutions." (Federal Reserve Bank of Boston Economic Conference, October 23, 2009); in the closing statement of the G-20 Pittsburgh summit; and in similar statements by the Basel Committee.

banks that, as we show, may be less sophisticated.

Since any advances in bank regulation are likely to be incremental, we deliberately take as our starting point the current Basel II Accord that aims at tailoring capital requirements more accurately to the true credit risk afforded by each individual bank loan or asset. Our focus on bank differentiation takes its inspiration from the two-layer approach of the Accord and its distinction between the Standard Approach (SA) and the Internal Ratings Based Approach (IRB) that emerged as a compromise to accommodate a heterogeneous banking landscape. Under the SA regulation which applies to banks with less highly developed scoring systems, certified credit rating agencies assign risk coefficients to bank loans and other bank assets, and the risk weight will be assumed to be 100% for commercial bank loans without an external rating. By contrast, under the IRB regulation, banks are authorized to undertake the risk classification of assets themselves according to their own credit scoring models.

An explicitly proclaimed objective of Basel II is to provide incentives so that banks improve their credit scoring systems in order to qualify for the IRB-approach. By contrast, a less straightforward and little investigated question is if the two layer-approach can be used to decrease the expected costs of banking failure by increasing the heterogeneity in the portfolio composition and the size of banks. Whereas the Basel II Accord so far has been studied mainly with respect to a single bank or in a partial equilibrium framework, we consider the impact of the two-layer approach on the market structure of the banking sector. In particular, our analysis explores the potential regulatory advantages of increased bank heterogeneity as a side effect of the two-layer system. Our main contribution is to show that the two-layer approach of the new Accord presents advantages for the stability of the banking sector since it allows to optimally differentiate the size and riskiness of banks. While this is a largely unintended consequence of the Accord's two-layer approach since the Basel Committee never explicitly envisioned it as a source of regulatory efficiency, our point is to highlight its potential usefulness, at a time when regulators are confronted with renewed scrutiny concerning the systemic risk threat posed by large banks.

The Basel Accord will effectively delegate the power to determine the risk weights for individual assets to the IRB-banks themselves. This raises genuine concerns about possible agency conflicts as banks can use this decentralization to reduce their overall required capital. Our second objective is thus to analyze the agency problem of misreporting portfolio risks in the context of two other incentive problems, bank risk-taking and banks' adoption of unregulated advanced scoring systems, and to study the possible interaction of these problems.

Analysis and Results. In our model, two banks choose between financing projects characterized by diversifiable risk (uncorrelated projects) and projects with substantial systematic risk (correlated projects). We assume that initially neither banks nor regulators can discriminate between uncorrelated and correlated projects. Banks, however, have the option to invest in an improved credit scoring system that enables them to screen between project types. With this investment, banks qualify for the use of the IRB-approach.

In our benchmark model, the regulator can perfectly verify the screening information obtained by banks. We also assume initially that the banks' equity endowment is fixed, so that banks need to reduce their lending in response to a tightening of capital standards. We then analyze three banking structures: two SA-banks, one IRB-bank, and two IRB-banks. With two SA-banks, banks are in identical situations, and hence will be equally large and will fund identical portfolios. As SA-banks cannot distinguish between projects, the frequency distribution of correlated and uncorrelated projects in the economy determines the composition of their portfolio. Depending on their equity and the social cost of banking failure, the regulator will either set very tight capital equity ratios to avoid any bankruptcy risk, or accommodate the risk of bank failure.

When there are two IRB-banks, they are also in identical situations and hence equally large, but they can screen between correlated and uncorrelated projects. The regulator then sets lower capital requirements for uncorrelated projects to encourage that all of them receive funding. As a consequence, the risk of bank failure will be smaller than with two SA-banks. Neglecting investment costs, two IRB-banks are superior to two SA-banks.

Our main result is that a banking sector with one IRB-bank is optimal as long as bank portfolios are observable. With one IRB-bank, the regulator can use the two-layer approach of Basel II to allocate **all** uncorrelated projects to the IRB-bank by setting a lower equity ratio for the IRB-bank which can discriminate project types. In addition, the regulator will limit the number of correlated projects for the IRB-bank to the maximum that it can cover with its own equity. While the IRB-bank then remains default-free, the SA-bank adopts a risky portfolio and is exposed to substantial failure risk. This allocation is optimal as there are economies of scale in the absorption of bank default risks: the more asymmetric the allocation of bankruptcy risk across banks, the lower the social cost of bank default. As a result, the regulator prefers to confine default risk to one bank and to keep that bank small.

We show that the expected cost from bank failure will be smaller when compared to two IRB-banks, and the total lending activity of banks will be higher. While our analysis confirms that SA-banks will suffer from the transition to Basel II by moving to smaller and lower-quality loan portfolios, it emphasizes the positive effects of this adjustment. Our analysis suggests that there should be a yardstick that separates between more and less advanced banks, based on the reality of a wide heterogeneity in banks' starting positions.

When tightening capital standards, bank regulators do not intend to reduce lending (although this is often the consequence) but rather to encourage banks to raise more equity. Consequently, we extend our model and consider that banks can raise equity at an increasing cost. If the banking sector faces positive bankruptcy risk in the optimal regulatory regime, we find that a differentiated banking sector with one IRB-bank and one SA-bank is still strictly superior to the two other regimes. The regulator induces the IRB-bank to increase its equity capital relative to the SA-bank, and the size differentiation between the two bank types will be even larger. If costs of bank failure are so high that the regulator prefers to keep both banks free of failure risk, then the marginal benefit of equity will be the same in all banks, and banks will be enticed to raise identical levels of equity. Overall, these findings show that our results hold and are even (weakly) strengthened when equity can be raised endogenously.

Under the IRB-approach, banks are themselves developing and administrating the tools that determine the risk weights and hence the capital requirements of their assets. Hence, in another extension, we consider the moral hazard problem that banks exploit their delegated responsibility by misrepresenting the risk of their loan portfolios. We consider the extreme case in which the regulator must fully rely on banks' reports and has no other information to check on the true risk of the banks' assets. Then, setting different capital requirements for the two project types is in fact meaningless as banks can misreport their portfolios in favor of projects with lower risk weight. We show that the tendency of banks to choose correlated projects, and hence to increase the riskiness of their loan portfolios, increases with their leverage, and we refer to this as risk-shifting. Anticipating this behavior, the regulator imposes stricter capital requirements that will curtail the lending capacity of IRB-banks. We hence find that moral hazard problems of IRB-banks will lead to substantially higher capital standards and smaller portfolios. This undermines the advantage of bank differentiation. As a consequence, two SA-banks can now be optimal if the propensity towards risk-shifting is important. Moreover, having two IRB-banks may also emerge as the optimal regime since together they have more "informed" equity than a single IRB-bank which ceteris paribus reduces their leverage and hence also their tendency to engage in risk-shifting.

In sum, our paper leads to a nuanced picture of the advantages of bank di erentiation. It allows to quarantine bank failure risk in smaller banks, implying lower expected bankruptcy losses and more lending compared with a situation in which all banks adopt IRB-status. But this e ect is reduced if banks can misreport the true portfolio risk and have high incentives for risk-shifting. Since the regulator needs to counteract by tightening capital ratios, a homogeneous banking sector can then be better than a di erentiated one. We also discuss the robustness of our analysis and its relationship to other important criticisms of risk-weighted capital standards, in particular concerning the problem of procyclicality and the role of bank competition.

Relation to the Literature. There is a vast literature on the optimal regulation of bank capital and on the relationship between bank capital and the propensity for risk-taking that is surveyed in Bhattacharya, Boot and Thakor (1998) and Allen (2004). Tighter capital requirements have been argued to increase risk-taking incentives (Thakor, 1996; Besanko and Kanatas, 1996) or to reduce them (Repullo, 2004). Other studies emphasize the role of bank profitability and bank competition, e.g. Matutes and Vives (1996) and Hellmann, Murdock and Stiglitz (2000). Boot and Greenbaum (1993) argue that excessive risk-taking can be mitigated by reputation e ects that in turn depend on imperfect competition.

The problem that banks may understate their portfolio risk when using internal ratings has been addressed by Morrison and White (2005), Dangl and Lehar (2004) and Pelizzon and Schaefer (2005) who argue that the benefit from internal ratings depends on the regulator's capacity to monitor and to deter banks from cheating. Gersbach and Wehrspohn (2001) argue that banks will underinvest in their scoring models because they will identify bad loans more often, which leads to higher capital requirements with internal ratings.

There is little previous work on the impact of Basel II on the riskiness of IRB-banks and SA-banks. Rime (2005) discusses the adverse e ect of Basel II on the loan quality of unsophisticated banks. Close in spirit is Repullo and Suarez (2004) who model a competitive banking sector where borrowers choose between IRB-banks and SA-banks. In their paper, IRB-banks will always specialize in riskier projects. The reason that our conclusions are so strikingly di erent from theirs is that, contrary to our analysis, they do not choose capital requirements to maximize social welfare and do not consider bank moral hazard, since their focus is on simulating the general equilibrium e ects of Basel II. Hakenes and Schnabel (2007) find, similar to our model, that the choice between the SA- and the IRB-approach may lead to more risk being adopted by less sophisticated banks, but they emphasize competition for liabilities among banks with di erent screening capabilities, whereas we focus on competition for assets.

Our concept of bank differentiation bears close similarity to Acharya and Yorulmazer (2007) and Acharya and Yorulmazer (2008). In their models, related to Acharya's (2001) analysis of the choice of systematic risk and regulatory consequences, the regulator opts ex post for bank mergers if few banks are failing, but with too many banks to fail, a bailout is ex post optimal. Ex ante, banks choose between high and low correlation with other banks, and while the regulator prefers bank differentiation, banks may herd in order to benefit from bailout insurance. They focus on ex post regulation and commitment against bailout whereas we consider bank capital requirements, optimal portfolio sizes and investment incentives, and explain that banks also may have incentives to differentiate.

In empirical work on the potential impact of the Basel II Accord, a substantial literature has focused on the likely procyclical effects, starting with Altman and Saunders (2001) who demonstrate that external ratings and hence the SA component of Basel II would likely to be procyclical. Peura and Jokivuolle (2004) find a comparable procyclical effect for internal rating systems. We relate our framework to this important discussion, and argue that the regulator should take full account of all economic effects of bank regulation and hence allow for time-varying capital standards that mitigate the procyclical effects of fixed requirements. The literature investigating the consistency of internal and external ratings to predict default risks includes Carey (2002) and Gropp and Richards (2001).

The paper is organized as follows. Section 2 introduces the model. Section 3 lays the groundwork by analyzing the case of two SA-banks and the portfolio choice of banks. Section 4 analyzes the scenario in which the banks' loan portfolio choice is observable to the regulator. Section 5 considers unobservable loan portfolios. In Section 6, we extend our benchmark model to allow for the optimal endogenous raising of bank equity. In Section 7, we discuss robustness questions and possible extensions relating to procyclicality and interest rate competition. Section 8 concludes.

2. The Benchmark Model

Banks and Projects. We consider a banking sector with two risk-neutral banks, Bank A and Bank B. Each bank chooses the size and the risk composition of its loan portfolio. Following standard arguments of the theory of financial intermediation (Diamond, 1984) and the bank regulation literature (Acharya, 2001; Acharya and Yorulmazer, 2008), we assume that banks can diversify their idiosyncratic risk, so that only the contribution of each loan to the systematic risk exposure of the bank matters. To make this point in a simple way, we assume that there are two di erent types of projects the banks can fund, and the choice between project types determines their risk exposure. Specifically, there is a continuum of projects in the economy with a total measure of one. Costs of all projects are normalized to one, i.e. funds worth n are needed to finance a portfolio of Lebesgue measure $n \leq 1$. Half of the projects contain only idiosyncratic risk that is fully diversifiable, and we call them **uncorrelated** projects. The other half of the projects is exposed to systematic risk that cannot be diversified, and we refer to them as **correlated** projects.

Uncorrelated projects yield a gross return (rate of return plus one) of $X > 2$ with probability $k \geq \frac{1}{2}$, and zero with probability $1-k$. The variable X includes all the benefits to project owners and to society of bank lending, including su cient cash flows,[3] but also non-monetary e ects and externalities for other stakeholders. As uncorrelated projects contain only idiosyncratic risk, the return of any measurable portfolio of n uncorrelated projects will be exactly knX. Each correlated project yields X with probability $\frac{1}{2}$ and nothing with probability $\frac{1}{2}$.[4]

To capture that correlated projects contribute to a bank's systematic risk exposure in a simple way, we assume that a portfolio of measure n of correlated projects yields tnX, where $t \in [0,1]$ is a uniformly distributed random variable. Thus, t indicates the realization of the systematic risk in the loan portfolio. A bank failure involves a real cost for society that consists of the disappearance of the organizational capital, know-how, proprietary knowledge of borrower relationships of the bank, and the disruption of the financing and payment flows for the bank's borrowers and lenders. We assume that social costs of bankruptcy are convex in the number of projects financed by a bank. In the wake of the bankruptcy of Lehman Brothers in 2008 and the renewed discussion on the adverse e ects of bank size, it seems natural that the social costs of bank failure rise more than proportionately in size because of contagion, systemic e ects and other negative ripple-on e ects to the economic system of the failure of large banks. We assume quadratic bankruptcy costs for convenience, and we denote the expected bankruptcy loss by $Z = zn^2 \cdot$Prob(bankruptcy). This assumption is nonessential, however, as our model can be extended to any weakly convex function including the linear case where $Z = zn$. We will discuss the robustness of this assumption in

[3] We assume that X contains cash flows larger than 2 that can be pledged to investors. This assumption is su cient to ensure that financing is always viable. Only X matters for our analysis, not the amount of cash flows.

[4] It would be possible to assume that X is di erent for correlated and for uncorrelated projects, but at the cost of a considerable more complex algebra that would add no new insights.

Section 7.

In the benchmark model, we assume that banks have fixed and identical equity endowments, $E_A = E_B = E$. In Section 6, we will relax this assumption and instead consider that banks can raise equity at an increasing cost.

Bank Regulation. Initially, both banks have low quality rating systems that cannot distinguish between uncorrelated and correlated projects. Banks with low rating systems are called **SA-banks** as they will be regulated according to the Standard Approach. Each bank $i \in \{A, B\}$ can, however, invest $C_i \geq 0$ to build a high quality internal rating system that allows to perfectly screen between uncorrelated and correlated projects.[5] We refer to them as **IRS-banks**, and we assume $C_A < C_B$ to capture heterogeneity among banks concerning their readiness to adopt internal rating systems. Under Basel II, an IRS-bank can decide to be regulated according to the Internal Ratings Based (IRB) Approach. Then, the bank will be called an **IRB-bank**.

We consider two di erent scenarios to analyze the impact of the potential agency problem between bank and regulator as a consequence of the bank's investment in an advanced information system and its resulting informational advantage compared to the regulator: under the **observable scenario**, the regulator can observe the loan portfolio of IRS-banks. This is not possible under the **unobservable scenario** in which the regulator is entirely dependent on the banks' reports.

Under the Basel II approach, the regulator can set di erent capital requirements for uncorrelated (b_S) and for correlated (b_R) projects. We assume $b_S \leq b_R$ to exclude the possibility that the regulator demands more capital for uncorrelated projects. Di erentiating between b_S and b_R, however, is only possible for IRB-banks. For SA-banks, neither banks nor external rating agencies can distinguish between projects so that the regulator needs to apply a uniform ratio b_U. Besides the capital requirements b_S, b_R, and b_U, the regulator can encourage or discourage investments in advanced rating systems by a subsidy $s > 0$ or a tax $s < 0$ for IRB-banks.

The Specificity of Bank Lending and Interest Rates. We assume that interest rates banks can charge from borrowers are determined by competition from non-banks (e.g. equity-based instruments, leasing, asset-backed loans, trade credit or factoring, and other financial institutions that are not regulated as banks). The main disadvantage of non-banks when compared with

[5] This choice captures the many di erent dimensions in which banks' expertise and their investments in organization and technology determine their capacity to adeaquately evaluate asset risks, and their potential informational advantage compared with bank regulators. For example, the dynamics of future investment and financing decisions must be evaluated, see Dangl and Zechner (2004).

banks is that they lack the monitoring skills that characterize bank lending. To capture this, we assume that the default probability of all projects will be $\frac{1}{2}$ when funded by a non-bank. That is, the success probability is strictly lower for uncorrelated projects, and since project owners do not observe their project type (only IRB-banks do), it follows that their expected success probability and hence their expected return is strictly lower when funded by a non-bank. Another distinctive feature is that only banks can acquire the capability to screen between uncorrelated and correlated projects, by investing in internal rating systems as discussed.

Also, non-banks do not have access to deposit insurance or a similar insurance scheme, so that they need to pay an interest rate that is fair in equilibrium, i.e. that compensates risk-neutral creditors for the equilibrium default risk. As a consequence, since non-bank funded projects will repay their lenders only half of the time, the interest rate factor at which non-banks expect to earn zero profits is $R = 2$. By contrast, depositors of banks are paid the risk-free interest rate normalized to 0, independently of the actual failure risk, because we assume that there is a deposit insurance scheme in place that e ectively covers all credit risk for bank depositors.[6]

We assume that banks are price-takers, and that they also o er loans at rate $R = 2$ to all projects.[7] Then, all projects prefer bank financing to non-bank financing because of the lower expected default probability for uncorrelated projects (project owners do not observe their type). Projects are indi erent whether to be financed by an IRS-bank or an SA-bank, but we assume that IRS-banks can use their informational advantage over SA-banks to attract their preferred projects.

Timing. Figure 1 illustrates the timing of the game which is as follows. At Time t_1, the regulator credibly commits to the vector b and to s. At Time t_2, banks decide whether to invest, and if they do, whether to request an IRB-license. At Time t_3, each project can make loan applications to one or several banks. The banks then simultaneously make yes/no-decisions on each loan application. Then projects reject or accept each of the financing o ers, with at most one acceptance decision. If a project receives two loan o ers, it prefers IRS-banks. With two identical banks, each bank is chosen with probability $\frac{1}{2}$. Finally, at Time t_4, Nature decides upon

[6] We assume that a bank's deposit insurance contribution does not depend on its current loan portfolio. This corresponds to the situation in most deposit insurance schemes. It also allows us to ignore any impact of the endogenous default risk on the bank's deposit insurance payment.

[7] In Section 7, we discuss how we can account for the fact that interest rates may vary in reaction to changes in the structure of the banking sector.

the success of projects. This also determines if a bank is insolvent or not.

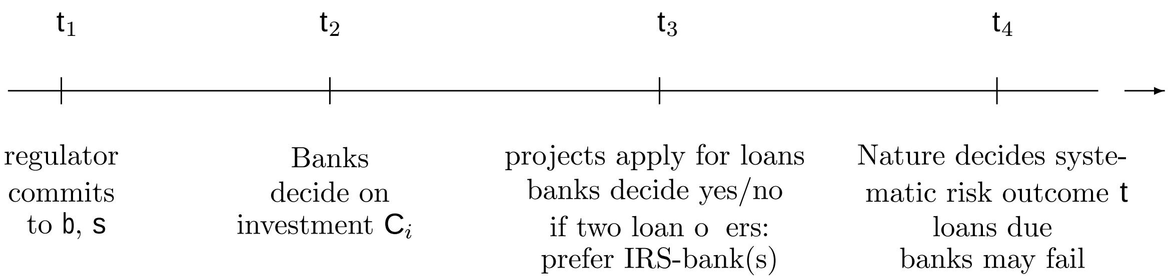

Figure 1: Time Line

Lending Volume. An essential consideration for the regulator is the optimal size of each bank and of the banking sector in total. As we will see, the optimal size of the two banks depends on their capital endowment E and on the relationship between the social benefits X to the bankruptcy loss parameter z. Crucially from the regulator's point of view, an increase in bank lending also increases the bank failure risk. In order to tie down this relationship, we first assume that $E < \frac{1-k}{2}$. This implies, as we will discuss, that if all projects are bank-financed then at least one bank faces insolvency risk; we exclude the opposite case since it would be uninteresting.

3. Elements of the Analysis

3.1. Two SA-banks

We start with the case where neither bank has invested into an IRS. Both banks are SA-banks that cannot distinguish between project types, and they are regulated by b_U ("uninformed") and can fund $n_u \leq \frac{E}{b_U}$ projects. This case is in many ways similar to the situation under Basel I rules, but not quite: under Basel I rules, the regulator cannot distinguish between project types, but banks might nevertheless invest in superior screening skills. Basel I with investment is akin to our case of unregulated IRS-investments discussed in Section 4.4.

Moving backwards, we first analyze profits and bankruptcy risks as a function of n_u (stage 4), and then turn to the regulator's choice of b_U (stage 1). Since banks cannot discriminate between uncorrelated and correlated projects, each loan portfolio consists of $\frac{n_u}{2}$ uncorrelated and $\frac{n_u}{2}$ correlated projects. For given n_u and E, the bank will avoid insolvency if t is large enough so that the bank's assets are sufficient to cover its liabilities:

$$\frac{n_u}{2}kR + \frac{n_u}{2}tR + E - n_u \geq 0. \tag{3.1}$$

Let $\tilde{t}_u$ denote the minimum value of t satisfying (3.1). Because t is uniformly distributed over the unit interval, $\tilde{t}_u$ conveniently embodies also the bankruptcy probability. The expected profit per bank (we will always look at gross profits that include bank equity E) is

$$\Pi_u = (1 - \tilde{t}_u)\left(\frac{n_u}{2}kR + \frac{n_u}{2}E\left[t \mid t > \tilde{t}_u\right]R + E - n_u\right). \tag{3.2}$$

This equation says that if the bank avoids insolvency, which it does with probability $1 - \tilde{t}_u$, then the return from $\frac{n_u}{2}$ uncorrelated projects is kR, and the expected return from $\frac{n_u}{2}$ correlated projects is $E\left[t \mid t > \tilde{t}_u\right]R = \frac{1+\tilde{t}_u}{2}R$, reflecting the conditional expectation of $t \geq \tilde{t}_u$. Furthermore, the bank owns E and invests n_u. Substituting $R = 2$ from now on, we get:

$$\Pi_u = (1 - \tilde{t}_u)\left[n_u k + n_u E\left[t \mid t > \tilde{t}_u\right] + E - n_u\right]. \tag{3.3}$$

The insolvency threshold $\tilde{t}_u$ is reached when the return to the bank, including its capital reserves E, is just enough to satisfy inequality (3.1),

$$\tilde{t}_u = \max\left\{\frac{2n_u(1-k) - 2E}{2n_u}, 0\right\}. \tag{3.4}$$

We focus on the case with positive bank failure risk which occurs if $n_u > \frac{E}{(1-k)}$. Substituting into the bank's profit function (3.2) and rearranging yields

$$\Pi_u = \frac{(2E + 2kn_u)^2}{8n_u}. \tag{3.5}$$

Because banks do not have to pay for the losses if they become bankrupt, Π_u is strictly increasing in n_u beyond the threshold when banks face failure risk, $n_u > \frac{E}{(1-k)}$. Hence, SA-banks fund as many projects as possible, meaning that in equilibrium $n_u = \frac{E}{b_u}$.

The regulator will determine b_u so that the optimal quantity of projects is funded, taking into account that bank failure is costly. As there are two banks, the regulator maximizes:

$$W_u = \max_{b_u}\left\{n_u kX + n_u\frac{1}{2}X - 2n_u - 2\tilde{t}(n_u)^2 z\right\} \tag{3.6}$$
$$\text{s.t. } n_u = \frac{E}{b_u}.$$

Substituting expression (3.4) into the objective function (3.6) and solving for the optimum leads the following proposition:

Proposition 1. *With two SA-banks, each bank funds* $n_u = \min\left\{\frac{X(1+2k)+4zE-4}{8z(1-k)}, \frac{1}{2}\right\}$ *projects, which the regulator implements by setting* $b_U = \frac{E}{n_u}$. *Each bank faces positive failure risk if* $z \leq \frac{X(1+2k)-4}{4E}$, *and it finances the maximal number of projects* $n_u = \frac{1}{2}$ *if* $z \leq \frac{X(1+2k)-4}{4(1-k-E)}$.

Proof. See Appendix.

Note that n_u is strictly increasing in k, E and X and strictly decreasing in z until it reaches the limit $n_u = \frac{1}{2}$. This is intuitive: The higher E and k, the lower is the bankruptcy risk for any number of projects given, and the higher X, the more valuable are projects. The higher z, the more important is it to avoid bankruptcy.

3.2. Choice of the Loan Portfolio

The portfolio choice of a bank that adopts an IRS is an important step of our analysis that we will use extensively below. We analyze whether the bank prefers uncorrelated projects, correlated projects, or a mixed loan portfolio.

When choosing its loan portfolio, the bank faces a trade-off between uncorrelated projects that offer better quality expressed by $k > \frac{1}{2}$ and correlated projects that offer the option to benefit from the limited liability effect. Consider an IRS-bank, say Bank A, that finances a given number of projects n^A, of which n_S^A are uncorrelated and $n^A - n_S^A$ are correlated. By analogy to the profit function (3.2) of a SA-bank, the IRS-bank's profit is then

$$\Pi^A = (1-\tilde{t}^A)\left(2kn_S^A + E\left[t \mid t > \tilde{t}^A\right] 2\left(n^A - n_S^A\right) + E - n^A\right), \tag{3.7}$$

where $\tilde{t}^A$ is the bankruptcy probability:

$$\tilde{t}^A = \max\left\{\frac{n^A - E - 2kn_S^A}{2\left(n^A - n_S^A\right)}, 0\right\}. \tag{3.8}$$

The bank maximizes its profit (3.7) subject to its equity constraint:

$$n_S^A b_S + \left(n^A - n_S^A\right) b_R \leq E. \tag{3.9}$$

We can then identify the following fundamental portfolio allocation rule:

Lemma 1. *An IRS-bank strictly prefers either uncorrelated or correlated projects. That is, it will finance the maximum feasible number* $\min\left\{\frac{1}{2}, \frac{E}{b_j}\right\}$ *of its preferred project type, and it will use its residual bank equity for projects of the other type.*

Proof. See Appendix.

The intuition for Lemma 1 is that the bank's profit Π^A is a **call option** on the true value of the loan portfolio, and the expected value of this call option exhibits the usual convex shape as a function of the risk choice. Thus, there is no interior solution, and the bank will prefer either uncorrelated or correlated projects. We will refer to the first case as **cherry-picking**, and to the second case as **risk-shifting**.

The Lemma and the convexity of returns on which it builds have two important implications: first, unless enough correlated projects are chosen so that there is positive bankruptcy risk, the bank will always prefer uncorrelated projects as $k > \frac{1}{2}$. Second, choosing either uncorrelated or correlated projects first yields strictly higher profits than the mixed allocation with two SA-banks, i.e. $\Pi^A > \Pi_u$ if the number of projects is identical.

4. Observable Loan Portfolios

We can now consider the optimal regulation if at least one bank becomes an IRB-bank. We start with the case in which the regulator can observe the loan portfolio, and we proceed as follows: we analyze the optimal capital requirements separately for the scenario with one IRB-bank and with two IRB-banks, and then compare the two allocations and determine the optimal size of the IRB-bank segment.

4.1. One IRB-Bank

When portfolios are observable and there is only one IRB-bank, the regulator can implement any project split by adjusting capital requirements (b_U, b_R, b_S) appropriately. Total expected bankruptcy costs are minimized by differentiating between the IRB-bank and the SA-bank - the IRB-bank funds all uncorrelated projects and the maximum number of correlated projects it can fund without bankruptcy risk. The number of projects funded by the SA-bank depends on the relation between the benefit of projects and the bankruptcy cost parameter. This leads to the following outcome:

Proposition 2. *With one IRB-bank and observable portfolios, the IRB-bank finances* $n^A = E + k$ *projects. Its portfolio comprises all uncorrelated projects* $\left(\frac{1}{2}\right)$ *and* $E + k - \frac{1}{2}$ *correlated projects. If* $z < \frac{X-2}{E}$*, then the SA-bank faces failure risk in equilibrium and funds* $n^B =$

$\min\left\{\frac{X}{2z} - \frac{1}{z} + \frac{1}{2}E, 1 - n^A\right\}$ *projects that are all correlated. If* $z \geq \frac{X-2}{E}$*, then the SA-bank is risk-free and funds* $n^B = E$ *correlated projects. The regulator implements this outcome by setting* $b_U = \frac{E}{n^B}$, $b_S = 0$ *and* $b_R = \frac{E}{E+k-\frac{1}{2}}$.

Proof. See Appendix.

The key insight of Proposition 2 is that the regulator avoids all bankruptcy risk for the IRB-bank; its payoff will be just zero in the extreme systematic risk event $t = 0$. The SA-bank, by contrast, may have substantial failure risk even though it is the smaller bank, as a consequence of its correlated loan portfolio. This allocation is optimal as it shelters as many projects as possible from bankruptcy risk. The social cost of default is minimized if a relatively small lending volume is allocated to the riskier bank and a relatively large portfolio to the less risky bank.[8] If X is small compared to the bankruptcy risk parameter z, the regulator will keep both banks risk-free.

Proposition 2 expresses an insight that plays an important role in our analysis: because the bankruptcy probability increases at a decreasing rate when a bank adopts more correlated projects, there is an economies-of-scale effect in the allocation of bankruptcy risk, making an unequal allocation of overall failure risk optimal. The fact that bankruptcy costs are convex potentially makes the concentration of risk in one bank relatively more costly. Our analysis shows, however, that this effect is too weak to offset the economies-of-scale effect in risk allocation.

A major advantage of Basel II is that it allows to differentiate between two segments of the banking sector: the IRB-bank should have regulatory rates that entice it to finance low-risk projects, and which avoid the risk of default. The fact that IRB-banks pick predominantly uncorrelated projects whose risk is diversifiable will inevitably deteriorate the average quality of remaining projects available for SA-banks so that poorly skilled banks are becoming more fragile. But this asymmetry is just the consequence of the fact that all bank failure risk should be insulated in the SA-segment of the banking sector.

4.2. Two IRB-Banks

If both banks become IRB-banks and observe perfect signals on the quality of each project, there will be a symmetric equilibrium in which banks have the same project mix and identical profits,

[8] This allocation is only implementable if the IRB-bank indeed prefers to fund uncorrelated projects. With the capital requirements of Proposition 2, this is the case as the IRB-bank cannot fund more than $E + k - \frac{1}{2}$ correlated projects.

$\Pi^A = \Pi^B$. In this equilibrium, each project applies to both banks and receives a funding offer from both banks. By assumption, projects will randomly select one of the two banks, so that the allocation is symmetric.

There are three possible outcomes, the two boundary solutions in which the banking sector is risk-free or where all projects are financed, respectively, as well as an interior solution. Which of these outcomes is optimal from the regulator's point of view depends again on the relationship between z and X. The larger is z relative to X, the less projects will be funded, and the smaller is the interior solution. There exists a threshold $\hat{z}$ such that the regulator prefers to keep both banks risk-free if $z > \hat{z}$, as the following proposition shows:

Proposition 3. *With two IRB-banks and observable portfolios, portfolios and profits are equal, $\Pi^A = \Pi^B$. Each bank funds half of all uncorrelated projects and a number $\frac{n^R}{2}$ of all correlated projects, where*

(i) *all projects will be financed, $\frac{n^R}{2} = \frac{1}{4}$, if and only if $z < \frac{X-2}{4}$;*

(ii) *there exists $\hat{z}$ such that $\frac{n^R}{2} = \frac{k}{2} - \frac{1}{4} + E$ if and only if $z > \hat{z}$, where $\hat{z}$ is increasing in k and E. Both banks will be default-free;*

(iii) *for all intermediate values of z, there will be an interior solution, $\frac{n^R}{2} \in \left(\frac{k}{2} - \frac{1}{4} + E, \frac{1}{4}\right)$ and both banks will face default risk.*

Proof. See Appendix.

Following the logic expressed in Proposition 2, the regulator would prefer to shelter as many projects as possible from bankruptcy risk by allocating them to one bank that remains default-free. However, when both banks are IRB-banks, such an allocation with specialized loan portfolios is no longer feasible.

4.3. Comparison and Optimal Regulation

We can now compare the expected welfare with one IRB-bank, two IRB-banks and two SA-banks, respectively. We find:

Proposition 4. *With observable portfolios, welfare net of investment costs C_A and C_B, is highest with one IRB-bank, followed by two IRB-banks and two SA-banks. The number of projects funded is higher with one IRB-bank than with two IRB-banks.*

Proof. See Appendix.

The intuition for the optimality of one IRB-bank is that two IRB-banks will always have the same portfolio mix and size. Two IRB-banks are superior to two SA-banks as all uncorrelated projects will be financed. Note that investment costs do not influence the ranking between one IRB-bank and two IRB-banks as only one bank needs to invest in the first case. However, two SA-banks may be preferable to two IRB-banks if investment costs C_A and C_B are su ciently high. While the allocation with two SA-banks is inferior because not all uncorrelated projects are financed, it may still be better because of the savings in investment costs. Similarly, two SA-banks may be preferable to one IRB-banks if investment costs are too high at the bank that could become an IRB-bank more cheaply (Bank A).

Our finding that the regulator reduces average capital requirements if there is one IRB-bank, implying that total lending volume is higher, is a direct consequence of the value of bank di erentiation: since failure risk is better allocated by confining it within one bank, the regulator will allow average bank leverage to be higher.

We turn next to the question of implementation. This is important as the decision of banks to become IRB-banks and the regulator's preferred choice may not coincide. The objective functions of banks and regulator di er for two reasons: first, the regulator takes account of social costs of bank failure Z while banks do not; second, banks do not consider the externalities of their loan portfolio choice on the available loan quality of the other bank. The first di erence may lead to an **underinvestment** bias defined as any set of parameters that leads to $W_o^1 - W_u > C_A > \Pi_o^A(1) - \Pi_u$, i.e. a case in which the social benefit from investment is positive while Bank A would earn a negative net profit when investing (W_o^1 denotes the optimal welfare with one IRB-bank, $\Pi_o^A(1)$ the IRB-bank's profit in this case). The second di erence tends to induce **overinvestment** situations, i.e. cases where Bank A would invest although the social benefit is negative ($W_o^1 - W_u < C_A < \Pi_o^A(1) - \Pi_u$).

From the expressions derived earlier, it can easily be seen that both over- and underinvestment problems may arise. However, the regulator can always implement the optimal regime via subsidies ($s > 0$) and taxes ($s < 0$) for investments. Overinvestment can be deterred by a tax. With underinvestment, since the regulator prefers one IRB-bank to two IRB-banks, the optimal regulation not only needs to ensure that Bank A invests, but also that Bank B does not invest. This can be done by reducing subsidies when more than one IRB-banks invests. Thus, the optimal outcome can always be reached. We summarize in the following proposition:

Proposition 5. *With observable portfolios, the regulator can always uniquely implement the optimal outcome:*

(i) *If* $W_o^1 - W_u \geq C_A$, *only Bank* A *invests and finances* $n^A = E + k$ *projects. There is a subsidy if* $C_A > \Pi_0^A(1) - \Pi_u$.

(ii) *If* $W_o^1 - W_u < C_A$, *there will be two SA-banks. There is a tax on investment if* $C_A < \Pi_o^A(1) - \Pi_u$.

Proposition 5 confirms that the optimal allocation is always uniquely attainable if the portfolios of banks are observable.

4.4. Unregulated Investment

So far, we have neglected that one or two banks might invest in an IRS without requesting an IRB-license. We will refer to this situation in which the bank does not divulge its internal ratings information to the regulator as unregulated investments. The analysis of unregulated investments addresses the view that the major e ect of the IRB approach is to ensure that banks give regulators access to the information provided by their advanced internal rating systems. According to this view, banks invest in IRS-systems independently of regulation, and the Basel II regulation provides incentives for banks to share their internal rating information with regulators. In this sense, one insight of our model is that the objective of implementing full information sharing between banks and regulator is tantamount to analyzing the incentive constraints that banks submit their IRS-systems for IRB-approval.

Consider first the case in which the regulator prefers two SA-banks, but where one bank (Bank $i \in \{A, B\}$) may undertake unregulated investments (overinvestment problem). Then, Bank i's lending volume will be the one derived for two SA-banks as this is the banking structure the regulator assumes, and it will be weakly below $\frac{1}{2}$. From Lemma 1, we know that the bank strictly prefers either uncorrelated or correlated projects, and we denote the maximum profit in this case as $\hat{\Pi}^i$. If $\hat{\Pi}^i - \Pi_u > C_i$, the bank will invest even if $W_o^1 - W_u < C_i$. It can easily be shown that there exist cases where $\hat{\Pi}^A - \Pi_u > C_A$ and $W_o^1 - W_u < C_A$ hold simultaneously. In this case, the regulator cannot prevent the overinvestment by Bank A as unreported investments cannot be taxed. The best response is then to accommodate the investment and to set incentives to request an IRB-license. This dominates unregulated investment as it allows the regulator to di erentiate the banking sector as analyzed in Section 4.1. We thus find that investment without requesting an IRB-license can never be a subgame perfect strategy. The option of doing so, however, remains important: it is the reason why the equilibrium outcome will be one IRB-bank whenever $\hat{\Pi}^A - \Pi_u > C_A$, even if the regulator prefers to have two SA-banks because $W_o^1 - W_u < C_A$.

Second, recall that the regulator always prefers one IRB-bank to two IRB-banks. Bank B, however, will invest whenever $\hat{\Pi}^B(2) - \Pi^B(1) > C_B$, where $\Pi^B(1)$ denotes Bank B's profit when being the only SA-bank, and $\hat{\Pi}^B(2)$ its profit when undertaking an unregulated investment. Then, the number of projects it can fund is still determined by the capital requirements for SA-banks as the regulator wants to allocate all uncorrelated projects to the (single) IRB-bank A and assumes that Bank B is an SA-bank even though Bank B is in fact an IRS-bank. It can be shown that $\hat{\Pi}^B(2) - \Pi^B(1) > C_A$ and $W_o^1 - W_u < C_A$ may hold simultaneously, so that this case cannot be excluded. Again, the best the regulator can then do is to provide incentives so that Bank B also applies for IRB-status. We summarize:

Proposition 6. *With observable portfolios and the possibility of unregulated investments, the regulator implements one IRB-bank (two IRB-banks) whenever one (two) banks would otherwise invest without applying for approval. Unregulated investments do not occur in equilibrium.*

A comparison of Proposition 5 and Proposition 6 shows that the regulator can no longer avoid overinvestment when we allow for the possibility of unregulated investments. Hence, the regulator accomodates the situation by certifying these investments and granting IRB-bank status whenever banks would invest anyway.

5. Unobservable Loan Portfolios

We turn to the agency problems associated with improved rating capabilities. We consider the extreme case in which the regulator cannot observe the projects chosen by banks. In this case, banks have full discretion to manipulate their risk weights, and in particular to misreport correlated projects as uncorrelated ones. It follows that banks then will always report the project type that is subject to the lowest capital requirement. As a consequence, di erentiating between the requirements of uncorrelated and of correlated projects becomes meaningless, and only $b_I = \min(b_S, b_R)$ matters. Then, there are e ectively only two capital requirements we need to consider, namely $b_I = \min(b_S, b_R)$ as the relevant ratio for IRB-banks, and b_U for SA-banks.

5.1. One IRB-Bank

With observable portfolios, the regulator could prevent substitution between uncorrelated and correlated projects by setting b_S and b_R appropriately. This is impossible when portfolios are unobservable, so that we need to answer two questions: When will the IRB-bank engage in risk-shifting? And what is the optimal response of the regulator in terms of fixing the unique capital requirement b_I that is relevant for IRB-banks? Concerning the first question, we find:

Lemma 2. *With unobservable portfolios, there exists a unique threshold $\bar{n}^A$ such that an IRB-bank (say Bank A) chooses cherry-picking if $n \leq \bar{n}^A$ and risk-shifting if $n > \bar{n}^A$. The threshold $\bar{n}^A$ is strictly increasing in E and k over the interval $\bar{n}^A \in (0, E + k)$.*

Proof. See Appendix.

Lemma 2 shows that the IRB-bank's incentive to give priority to correlated projects is increasing in the total number of projects funded, i.e. in the leverage of the bank. The smaller the portfolio, the lower is the bankruptcy probability if correlated projects are chosen. And since the advantage of correlated projects is precisely that limited liability allows to keep the upside and hedges against the downside of their riskier cash flows, it follows that the higher expected project return of uncorrelated projects dominates for small portfolios, whereas the higher variance of correlated projects dominates for large portfolios. An increase in k means that uncorrelated projects are more attractive, and an increase in E means that the bank has more to lose in case of bankruptcy. We can then analyze the regulator's preferred allocation considering the constraint expressed in Lemma 2:

Proposition 7. *With one IRB-bank and unobservable portfolios, the regulator's optimal choice of n^A depends on the parameters E and k:*

(i) Region 1: *If $\bar{n}^A \geq E + k$, then the regulator sets $n^A = E + k$, Bank A opts for cherry-picking and will be risk-free. $\bar{n}^A \geq E + k$ holds if E and k are sufficiently large.*

(ii) Region 2: *There exists a threshold $n^+ < \frac{1}{2}$ such that if $\bar{n}^A \in (n^+, E + k)$, the regulator implements $n^A = \bar{n}^A$, Bank A opts for cherry-picking and will be risk-free. This case arises for intermediate values of E and k.*

(iii) Region 3*: If $\bar{n}^A < n^+$, the regulator does not avoid risk-shifting. Bank A funds only correlated projects, and Bank B funds a portfolio of projects from the remaining pool that is of identical size to Bank A's portfolio. This case arises if E and k are small.*

Proof. See Appendix.

Proposition 7 divides the parameter space into three regions. In Region 1, risk-shifting is no problem and we get the same solution as with observable portfolios. But outside this region, the agency problems caused by unobservable portfolios lead to higher bankruptcy costs since the regulator cannot implement the optimal allocation: In Region 2, the IRB-bank would engage in risk-shifting for $n^A = E + k$, and the best the regulator can do is to implement the maximum number of projects where risk-shifting is avoided.

In Region 3, the maximum number of projects Bank A could be allowed to fund without opting for risk-shifting is so small that the regulator prefers a larger portfolio with risk-shifting to the maximum portfolio implementable without risk-shifting. We can show that in principle it would be optimal to make the IRB-bank A the smaller bank, since its loan portfolio is dominated by correlated projects and its portfolio quality is worse than that of the SA-bank. However, allocating less projects to an IRB-bank than to a SA-bank is excluded by the assumption that $b_S \leq b_U$. Since portfolios are unobservable, the bank can declare all projects as uncorrelated ones, so that $n^A \geq n^B$, and naturally this constraint will be binding in the regulator's optimal policy.[9]

5.2. Two IRB-Banks

With two IRB-banks and unobservable portfolios, it is impossible to enforce di erent capital requirements for di erent types of projects, and risk-shifting incentives must be taken into account. Following the logic of Lemma 2, we can represent these risk-shifting incentives in the form of a threshold $\bar{n}$ (now valid for both banks) such that IRB-banks prefer cherry-picking if the maximum portfolio size they can realize is smaller than $\bar{n}$, and prefer risk-shifting otherwise. Deriving $\bar{n}$ is similar to Lemma 2 since the comparison of profits with uncorrelated and correlated projects is independent from the behavior of the other bank. Still, the two IRB-banks interact as they compete for the same projects - when they prefer uncorrelated projects, each bank can now fund only a maximum of $\frac{1}{4}$ of them. This competition explains why the allocation is di erent compared with the case of only one IRB-bank, and why $\bar{n} < \bar{n}^A$ (the risk-shifting threshold with one IRB-bank). The di erence is intuitive since less uncorrelated projects are available for each

[9] The assumption $b_S \leq b_U$ is realistic but from a theoretical point of view not always optimal. For some parameter values the regulator would prefer $b_S > b_U$ to curtail the possibility that an IRB-bank engages in risk-shifting. The IRB-bank would then be the smaller bank, and hence investment subsidies are typically needed. A full analysis of this case is available from the authors.

cherry-picking bank when there are two competing IRB-banks. lthough the details of the analysis are similar to the case of one IRB-bank. Again, $\bar{n}$ is increasing in E and k, i.e. the smaller are the equity endowment and the expected return of uncorrelated projects, the more severe is the constraint. We summarize the regulator's preferred outcome as follows:

Proposition 8. *With two IRB-banks and unobservable portfolios, the allocation will be symmetric. The regulator's optimal choice of portfolio size depends on the threshold $\bar{n}$ and the optimal allocation $\frac{n^R}{2}$ given in Proposition 3:*

(i) Region 1: *If $\bar{n} \geq \frac{n^R}{2} + \frac{1}{4}$, then the regulator sets $n = \frac{n^R}{2}$, both banks opt for cherry-picking and choose the optimal allocation $\frac{n^R}{2}$ given in Proposition 3. $\bar{n} \geq \frac{n^R}{2} + \frac{1}{4}$ holds if E and k are sufficiently large.*

(ii) Region 2: *There exists a threshold n^{++} such that if $\bar{n} \in \left[n^{++}, \frac{n^R}{2} + \frac{1}{4}\right)$, the regulator implements $n = \bar{n}$ for both banks, and banks opt for cherry-picking. This case arises for intermediate values of E and k.*

(iii) Region 3*: If $\bar{n} < n^{++}$, the regulator does not avoid risk-shifting.*

Proof. See Appendix.

The structure of the preferred allocation is similar to that of observable portfolios, and the logic behind Proposition 8 follows that of Proposition 3. The allocation is symmetric since a di erentiation between the two banks is not possible, and the total lending volume depends on the social attractiveness of lending, characterized by the relationship between X and z. However, since the regulator cannot enforce di erent capital requirements for di erent project types when portfolios are unobservable, risk-shifting incentives expressed by the threshold $\bar{n}$ must be taken into account. If $\bar{n}$ is not binding, clearly the optimal allocation is the same as with observable portfolios. Otherwise, a similar same case distinction as with one IRB-bank arises, and the regulator will accommodate risk-shifting if portfolio sizes would otherwise be to small.

5.3. Comparison and Optimal Regulation

The following proposition summarizes the comparison of the bankruptcy losses in the di erent regimes (subscript n for "non-observable"):

Proposition 9. *Welfare excluding investment costs, C_A and C_B, is highest with one IRB-bank if $\bar{n}^A$ is large (large values of E and k), and welfare is highest with two SA-banks if $\bar{n}^A$ is small*

(small values of E and k). For intermediate values of E and k, either one IRB-bank or two IRB-banks can be optimal.

Proof. See Appendix.

Part (i) of Proposition 9 shows that, neglecting investment costs, one IRB-bank is the dominant regime if risk-shifting incentives are low (large values of E and k), and two SA-banks dominate if risk-shifting incentives are important. By contrast, one IRB-bank is always superior with observable portfolios where risk-shifting is no concern (Proposition 4). This is the first important di erences between the cases with observable and unobservable portfolios: as the regulator can avoid risk-shifting no longer by di erentiating between the capital equity ratios for uncorrelated and correlated projects but only by reducing the total lending volume, improved rating systems lead to serious agency problems. The second di erence is that two IRB-banks may become optimal for an intermediate region.

The findings of Proposition 9 are illustrated in Figure 2 that plots the welfare (net of investment costs C_i) for the three regimes. If risk-shifting incentives impose a tight constraint on $\bar{n}$, then two SA-banks are optimal as the constraint only applies to IRB-banks. For less severe constraints, two IRB-banks are optimal. The reason is that two IRB-banks have twice as much equity as one IRB-bank, allowing them to lend more collectively while having lower bankruptcy costs. Finally, if risk-shifting is of little importance, one IRB-bank is again optimal, since this is the only regime that allows bank di erentiation.

Insert Figure 2 about here

Similar to the case with observable portfolios, it can again easily be seen that the regulator can implement the desired structure of the banking sector by subsidies or taxes when his objectives do not coincide with those of banks. As with observable portfolios, there may be over- or underinvestment incentives. As an example, consider Region 2: we know from Lemma 9 that in this case total bankruptcy costs may be lowest when there are two SA-banks. However, Bank A's profit is always higher when it invests ($\Pi_n^A(1) - \Pi_u > 0$) as $n^A > \frac{1}{2}$ by definition of Region 2, and taxes are needed to avoid investment. The logic follows closely the one with observable portfolios, and we can summarize our findings as:

Proposition 10. (i) *If* $W_u > \max\{W_n(1) - C_A, W_n(2) - C_A - C_B\}$, *the optimal outcome is two SA-banks. The regulator can uniquely implement this outcome, and there is a tax on investment if* $C_A < \Pi_n^A(1) - \Pi_u$.

(ii) *If* $W_n(1) - C_A \geq \max\{W_u, W_n(2) - C_A - C_B\}$, *the optimal outcome is that only Bank* A *invests. The regulator can uniquely implement this outcome. The regulator offers a subsidy if* $C_A > \Pi_n^A(1) - \Pi_u$, *and imposes a tax if* $C_B < \hat{\Pi}_n^B(2) - \Pi_n^B(1)$.

(iii) *If* $W_n(2) - C_A - C_B \geq \max\{W_u, W_n(1) - C_A\}$, *the optimal outcome is that both banks invest. The regulator can uniquely implement this outcome, and there is a subsidy if* $C_B > \Pi_n^B(2) - \Pi_n^B(1)$.

6. Endogenous Equity

6.1. Extended Model

In our benchmark model, we assume that equity levels are exogenously given and that both banks have the same equity endowment. Now, we extend the analysis by assuming that banks can raise equity on the market, at an increasing (quadratic) marginal cost. Specifically, we assume that profits with n_S uncorrelated and n_R correlated projects are

$$\Pi_{ee} = (1 - \tilde{t})\left[2n_S k + E\left[t \mid t > \tilde{t}\right] 2n_R + E - (n_S + n_R)\right] - \lambda E^2. \tag{6.1}$$

The only difference to the preceding sections is that equity of each bank is endogenous, and that banks have costs λE^2 of raising equity. We restrict attention to the case of observable portfolios. The optimal equity levels and the optimal portfolio compositions can then be implemented by adjusting the capital requirements appropriately. The regulator now faces the following trade-off: By choosing more restrictive capital requirements, banks are induced to raise more equity which reduces the bankruptcy risk for a given number of projects funded. This, however, will also increase total equity costs. Considering endogenous equity yields interesting insights concerning the size and the structure of the banking sector.

6.2. Analysis and Optimal Regulation

As the regulator can not differentiate between two SA-banks, both banks will have the same equity as in the benchmark model, and the same holds for two IRB-banks. With one IRB-bank, however, the regulator can implement different equity levels for the IRB-bank and the SA-bank.

Then, three new interesting questions emerge: First, how will total equity be divided between the IRB-bank and the SA-bank? Second, how is this division influenced by the bankruptcy risk of the SA-bank? And third, will total equity be higher with a banking sector without failure risk or if there is some failure risk?

We find that our main result that the optimal allocation consists of one large risk-free IRB-bank and one SA-bank that funds only correlated projects extends to endogenous equity. We leave the details of the analysis to the Appendix and summarize our findings as follows:

Proposition 11. *Suppose banks can raise equity E_i, $i \in \{A, B\}$ endogenously, at a quadratic cost of λE_i^2. Then the following equilibrium outcomes obtain:*

(i) *If there is one IRB-bank and one SA-bank of which the SA-bank faces failure risk, then the IRB-bank will be induced to raise a strictly higher level of equity, $E_A > E_B$. In all other cases, the two banks will raise the same amount of equity, $E_A = E_B$.*

(ii) *If $\lambda \leq \frac{z}{4}$, the regulator keeps both banks free of failure risk. Welfare net of investment costs is then the same with one IRB-bank and two IRB-banks, followed by two SA-banks.*

(iii) *If $\lambda > \frac{z}{4}$, the regulator will allow bank failure to occur with positive probability. Welfare net of investment costs is highest if there is one IRB-bank, followed by two IRB-banks and by two SA-banks. Only the SA-bank will face bankruptcy risk if there is one IRB-bank.*

(iv) *Taking investment costs into account, either one IRB-bank or two SA-banks can be optimal. The regulator can always implement the preferred allocation with taxes or subsidies.*

Part (i) of Proposition 11 provides interesting insights for the case in which there is only one IRB-bank. In this case, the marginal project funded by both the IRB-bank and the SA-bank is a correlated project. Hence, marginal benefits are identical. And if both banks are without default risk, then marginal bankruptcy costs are also identical (zero). It follows that marginal costs of raising equity must also be the same which can only be the case if equity levels are identical. If the SA-bank is optimally exposed to bankruptcy risk, however, then its equity must be lower than that of the IRB-bank: As the marginal social benefit of an additional project at the IRB-bank is higher because of the absence of failure risk, marginal equity costs must also be higher, and this implies that the IRB-bank receive incentives to raise more equity. Thus, our extension to endogenous equity leads to an important insight: We find that the new Basel Accord should be used to re-allocate equity from (risky) banks with low screening capabilities to (safe) banks with high screening capabilities. This reinforces the advantages of a di erentiated banking sector.

The remaining parts of the proposition confirm that the main findings in the benchmark model extend to the case of endogenous equity. We find that when there is only one IRB-bank, the regulator will avoid bank failure risk if and only if the cost of raising equity is low compared with the bankruptcy cost parameter z, $\lambda \leq \frac{z}{4}$. Part (ii) refers to this case: Under the same parameters, no bank optimally faces default risk if there is one IRB-bank, or if there are two IRB-banks. Welfare will be identical for the following reason: When there are two IRB-banks, both banks have the same equity and fund the same number of projects. But with one IRB-bank and no bankruptcy risk, we already know from part (i) that both banks will also have identical equity levels. And as the total number of projects that can be funded without bankruptcy risk depends only on the total bank equity in the economy (and not on its distribution), the allocation e ciency must be the same in both regimes. For reasons similar to those in the benchmark model, the case when there are two SA-banks is dominated since less uncorrelated projects are funded.

For part (iii), it is useful to recall that one IRB-bank was optimal when bankruptcy risk is positive even when banks had identical exogenous equity endowments. The possibility to raise equity di erentially increases welfare with one IRB-bank, and nothing changes if there are two SA-banks or two IRB-banks.

The implementation of the optimal outcome via taxes and subsidies is identical to the benchmark model.

7. Possible Extensions and Discussion

In this Section, we briefly discuss how our results would be a ected if we relaxed some of our key assumptions.

7.1. Robustness of Bankruptcy Costs

It seems natural that the social costs of bank failure are convex in size because of contagion or systemic e ects. Our analysis is carried out with quadratic bankruptcy costs. This greatly simplifies the formal tractability and makes it possible to characterize our results in closed form. It is interesting to ask whether our results depend on this assumption, and how they would change with a di erent functional form of bankruptcy costs. Fortunately, our main results do not depend on the functional form of bankruptcy losses. This is quite intuitive: convex bankruptcy costs work against the benefits of specializing in the absorption of systematic risks. Therefore, our results on the advantage of size di erentiation should also hold if we assume that bankruptcy costs are linear, and this is indeed what we find.

7.2. Procyclicality of Bank Capital Requirements

Perhaps no topic of discussion concerning the Basel II framework has attracted more attention than its stipulated procyclical e ects, since capital requirements conditional on estimated default probabilities tend to tighten in downturns. Our model does not explicitly account for cyclical e ects, but it is illuminating to relate our model to this discussion. In fact, our model has important ingredients for such a discussion: It distinguishes between the return to investors R and the social return X of funded projects, it incorporates the social cost of bank failure, and it endogenizes the socially optimal total lending volume both with exogenous and with endogenous equity.

A number of model parameters should be a ected by cyclical fluctuations. A downturn would increase the probability of projects having low cash flow returns, which could mean in our model that the support of the variable t that measures the joint cash flow realization of correlated projects shrinks, say from $t \in [0, 1]$ to a smaller interval $t \in [0, \bar{t}_d]$ with $\bar{t}_d < 1$. With constant risk weights, therefore, the capital requirements are procyclical. Cyclical fluctuations would also a ect the social costs of bank failure z, and the social return X of projects. Arguably, the costs of bank failure z increase in a downturn, but also the social return X since X includes all e ects to society of funding projects including external e ects, and these e ects are larger when economic activity and employment are receding.

In our model, the regulator would then capital requirements that are optimally time-varying, as most recently envisioned by the Basel Committee in its overhaul announcement. Our analysis shows that the optimal time-varying capital requirements depend on the relationship of X and z. If X increases more than z in a downturn, then the regulator wants to expand bank lending and would hence in fact act countercyclically by lowering capital requirements. Vice versa, if X increases less than z, then the regulator would optimally tighten capital requirements and act procyclically.

Thus, our model shows that only fixed capital requirements are necessarily procyclical. However, if capital requirements are time-varying and take all e ects into account, then the direction of the regulator's optimal reaction to economic fluctuations is not a foregone conclusion. We emphasize that the regulator in our model acts as maximizer of social welfare, and not on a narrower mandate to account for the stability of banks alone. This difference is crucial.

There are other effects that need to be taken into account, notably the cost of equity financing. It is plausible that the cost of raising bank equity, captured by the parameter λ, increases in downturns. Our analysis then shows that, with time-invariant capital requirements, the capital regulation would be procyclical. But again, the outcome with optimally time-varying capital requirements would be different. When determining the optimal response to cyclical variations in λ, the regulator considers the social costs of raising bank capital (i.e., the opportunity costs of crowding out other uses of capital), while private bank owners take into account the market costs of raising bank equity. The two criteria might differ, and in dramatic downturns with a heightened systemic risk for bank, it is plausible that market costs of raising bank equity are above social costs. Then, the optimal policy response in our model would be to subsidize bank equity, e.g. via guarantees or (co-)financing.

7.3. Bank Competition and Interest Rates

Banks are competing in our model, but for the sake of simplicity we do not consider that bank competition might extend into the determination of interest rates. It is possible to extend the model in this sense, by allowing banks to lower their interest rates in order to increase their market share. This possible extension reveals another interesting effect of bank differentiation, namely reduced interest rate competition.

In general, the academic and regulatory discussion on interest rate competition is marked by a trade-off. On one hand, less competition will lead to higher profit margins for banks and hence mitigate the failure risk and notably risk-taking incentives (e.g., Matutes and Vives, 1996; Hellmann, Murdock and Stiglitz, 2000). This theme has probably attracted most attention in the academic literature. But on the other hand, there are also two positive effects of interest rate competition: first, lower interest rates will generally attract more borrowers and hence increase economic activity. Second, bank competition might also impose discipline on banks and induce them to better manage their risks.

For the sake of the analysis, it is interesting to focus just on the first effect, the negative impact of interest rate competition on bank stability and risk-taking incentives. Consider our benchmark model of Section 2. Suppose banks have the possibility to lower interest rates if this allows them to attract market share, or to attract uncorrelated projects at the expense of their competitor. We focus on the case in which the regulator prefers that all projects be funded. In our model, this is the case if z is sufficiently small relative to X.[10] One can then show that the option of undercutting the competitor is only valuable if it allows a bank to either attract more uncorrelated projects, or to increase its failure risk (limited liability effect).

Therefore, we obtain the following results: If both banks are SA-banks, they do not observe project types, and hence cannot attract uncorrelated projects at the expense of their competitor. Furthermore, since the regulator will keep capital requirements tight so that lowering interest rates does not allow to increase the market share, we obtain $R = 2$ in equilibrium as before. Next, suppose one bank is an IRB-bank. Since it is a monopolist in having accurate information on project quality, it is sufficient to undercut interest rates by a tiny amount in order to attract all uncorrelated projects. Again, we obtain $R = 2$ in equilibrium as in our benchmark model. However, the situation is different with two IRB-banks. Banks can then increase their share of uncorrelated projects by undercutting their rival's interest rate, and this will also allow them to increase their total lending volume. As a consequence, interest rate competition is unavoidable, and this leads to an increase in bank failure risk.

This analysis leads a second independent channel of the value of bank differentiation: if banks are heterogeneous in their screening capacities and their regulatory regime, interest rate competition is reduced, which from the regulator's point of view has the aforementioned advantage of reducing failure risk and risk-shifting. This is an interesting observation, even though we hasten to reiterate that this discussion is not a full-fledged analysis of bank competition, since we neglect the benefits on market discipline and lending volume.

8. Conclusion

Within the "fundamental trade-off between flexibility and regulatory standardization in bank regulation" (Von Thadden, 2004), the Basel II Accord marks a shift towards more flexibility.

[10] To carry out the analysis with endogenous interest rates, it is necessary to keep lending volumes fixed. Also, some additional assumptions on the extensive form game are needed since games with price competition among asymmetric players are sensitive to the timing assumptions. We assume that IRB-banks have an advantage in setting interest rates, capturing their possibility to take advantage of their better information. Formally, this is captured by giving IRB-banks a second-mover advantage. Details and formal proofs are contained from a working paper version.

We consider a model in which two banks can invest to improve their internal credit screening capacities, and in which the regulator determines capital requirements that maximize the welfare trade-off between the social benefits of bank lending and the expected cost of bank failure.

Our analysis reveals an original positive effect of Basel II's two-layer approach that differentiates the capital adequacy ratios between IRB-banks and SA-banks: by optimally adjusting the size and portfolio structure of the two segments of the banking sector, it allows to better exploit economies of scale in the allocation of systematic risks. IRB-banks should receive incentives to keep their loan portfolios safe and bank failure risks should be confined to SA-banks. While the two-layer approach originally was conceived as a transitory regime, our analysis reveals it as an attractive feature of the Basel II architecture. There is no need to strive for homogeneity among banks. Ultimately, bank differentiation could give rise to more rather than less investment in internal rating systems if the yardstick that separates between the two layers of the banking sector is adjusted dynamically to the available credit information technology. In this sense, our analysis also provides a theoretical rationale for recent policy proposals to differentiate capital requirements and regulatory oversight between systemically important banks and others.

If the regulator can observe the banks' portfolios, our results show that it is possible to reap the full differentiation advantage of Basel II. The regulator never wants both banks to invest in internal rating systems which would diminish the differentiation advantage. In this case, the regulator gives incentives to the IRB-bank to finance all uncorrelated projects and the maximum number of correlated projects that keeps it default-free. If moral hazard problems concerning the IRB-bank's true default risk are pervasive, however, the bank might opportunistically abuse its screening capabilities to fund correlated projects while misreporting its portfolio to the regulator. The regulator may then be unable to fully exploit the size differentiation effect, since the possibility of risk-shifting and misreporting portfolio risks introduces new restrictions on the maximal lending volume of the IRB-bank. This constraint reduces social welfare compared with the case of observable bank portfolios, and may imply that two IRB-banks is now the optimal regime. Our findings are strengthened if we allow banks to raise equity endogenously, since the IRB-bank will typically be induced to raise more equity if the banking sector is differentiated.

In this paper, we make the stylized assumption that it is always possible to keep a large IRB-bank free from any default risk. In reality, uncertainty is sufficiently pervasive, for example about the true correlations, that banks will never be entirely free of default risk, except if bank leverage were to be banned under some form of "narrow banking". Also, even large banks may be specialized in lending to a few industries, so that their risk correlation is higher because of the concentrated nature of their exposure. Still, su ciently tight capital regulations can reduce the bank default risk considerably.

Finally, risk weights under the Basel II Accord focus on individual project risk, whereas we argue, following Acharya (2001) and others, that the relevant measure of risk for diversified banks must include the all-important dimension of risk correlations. From the point of view of banking theory, Basel II's emphasis on individual default risks and its neglect of risk dependencies in the determination of risk weights is not fully satisfactory. Implicitly, our model incorporates the notion that is widely held in practice that identifying assets with higher default probabilities typically also leads to the identification of assets with higher contributions to systematic risk. Also, one might hope that the financial crisis will speed up progress towards a better integration of credit scoring models and risk management systems and a better representation of correlation risks, as many banks have demonstrated in their recent stress tests. Still, modeling the coexistence of credit scoring models and risk management systems remains an important area for future research.

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Appendix

Proof of Proposition 1. We are only left with deriving the quantities and thresholds for an interior solution. Substituting $\tilde{t}_u$, the welfare function can be written as:

$$W_u = n_u k X + n_u \frac{1}{2} X - 2n_u - 2\left(n_u (1-k) - E\right) n_u z.$$

Solving for the first-order condition yields:

$$n_u = \frac{X(1+2k) + 4zE - 4}{8z(1-k)},$$

which is increasing in E and k. An interior solution requires $n_u < 1/2$, which yields

$$z > \frac{X(1+2k) - 4}{4(1-k-E)}.$$

Finally, banks are risky if n_u exceeds the bankruptcy threshold:

$$\tilde{t}_u \geq 0 \Leftrightarrow n_u \geq \frac{E}{(1-k)},$$

or if:

$$z \leq \frac{X(1+2k) - 4}{4E}. \blacksquare$$

Proof of Lemma 1. We use the notation $n_R^A = n^A - n_S^A$ for the quantity of correlated projects the bank finances in addition to n_S^A uncorrelated projects. The profit Π^A of the bank can then be written as:

$$\Pi^A = (1 - \tilde{t}^A)\left(E\left[t \mid t \geq \tilde{t}^A\right] 2n_R^A + E - n_R^A + (2k-1)\, n_S^A\right), \tag{A.1}$$

which will be maximized by choosing a pair $\left(n_R^A, n_S^A\right)$, subject to the bank's equity constraint (3.9). For the bankruptcy threshold and the conditional expectation we get from (A.1):

$$\tilde{t}^A = \max\left\{\frac{n_R^A - E - (2k-1)\, n_S^A}{2n_R^A}, 0\right\}, \tag{A.2}$$

and

$$E\left[t \mid t \geq \tilde{t}^A\right] = \max\left\{\frac{1}{2}, \frac{3n_R^A - E - (2k-1)\, n_S^A}{4n_R^A}\right\}. \tag{A.3}$$

Eq. (A.2) implicitly defines a threshold value of $n_R^A = \bar{n} = E + (2k-1)\, n_S^A$ such that $\tilde{t}^A = 0$ for all $n_R^A \leq \bar{n}$ and $\tilde{t}^A > 0$ for all $n_R^A > \bar{n}$. Substituting for $\tilde{t}^A$ and $E\left[t \mid t \geq \tilde{t}^A\right]$ in Eq. (A.1) and rearranging, profits can be rewritten as:

$$\Pi^A = \begin{cases} E + (2k-1)\, n_S^A & \text{if } n_R^A \leq \bar{n} \\ \frac{1}{4n_R^A}\left(2n_R^A + E - n_R^A + (2k-1)\, n_S^A\right)^2 & \text{if } n_R^A > \bar{n} \end{cases}. \tag{A.4}$$

Inspection of (A.4) shows that Π^A must be continuous at the point $\bar{n}$. First, note that profits are increasing in n_S^A in both regions, i.e. for $n_R^A \leq \bar{n}$ and for $n_R^A > \bar{n}$. Furthermore, Π^A is independent of n_R^A for $n_R^A \leq \bar{n}$ as correlated projects just break even. Hence, uncorrelated projects are preferred for $n_R^A \leq \bar{n}$. For $n_R^A > \bar{n}$, the Kuhn-Tucker problem is well defined and becomes:

$$L = \left(\frac{1}{4n_R^A}\right)\left(2n_R^A + E - n_R^A + (2k-1)\, n_S^A\right)^2 - \lambda\left(n_R^A b_R + n_S^A b_S - E\right) - \mu_1\left(n_S^A - \frac{1}{2}\right) - \mu_2\left(n_R^A - \frac{1}{2}\right). \tag{A.5}$$

We obtain the necessary conditions:

$$\frac{\partial L}{\partial n_S^A} = \left(\frac{1}{2n_R^A}\right)\left(E - n_R^A + n_R^A R_R + n_S^A (2k-1)\right)(2k-1) - \lambda b_S - \mu_1 = 0 \tag{A.6}$$

$$\frac{\partial L}{\partial n_R^A} = \left(\frac{1}{4\left(n_R^A\right)^2}\left(\left(n_R^A\right)^2 - \left(E + (2k-1)\, n_S^A\right)^2\right)\right) - \lambda b_R - \mu_2 = 0. \tag{A.7}$$

From condition (A.6) it follows that either $\lambda > 0$ or $\mu_1 > 0$, or both. If $\lambda = 0$ (the equity constraint is slack), then $\mu_1 > 0$, i.e. the bank will finance $n_S^A = \frac{1}{2}$ safe projects. To determine the bank's optimal quantity n_R^A, consider the unconstrained problem (A.4). The second derivative of Π^A with respect to n_R^A gives:

$$\frac{\partial^2 \Pi^A}{\partial \left(n_R^A\right)^2} = \frac{1}{2\left(n_R^A\right)^3}\left(E + (2k-1)\, n_S^A\right)^2 > 0.$$

As Π^A is strictly convex in n_R^A, only boundary solutions can be optimal, i.e. either $n_R^A \leq \bar{n}$ or $n_R^A = \frac{1}{2}$. If $\lambda > 0$ (the equity constraint is binding), then the constraint (3.9) can be written as

$$n_S^A = \max\left\{\frac{E - n_R^A b_R}{b_S}, 0\right\}. \tag{A.8}$$

Substituting for n_S^A in (A.4) yields for values $n_R^A > \bar{n}$:

$$\Pi^A = \left(\frac{1}{4n_R^A}\right)\left(n_R^A + E + (2k-1)\cdot \max\left\{\frac{E - n_R^A b_R}{b_S}, 0\right\}\right)^2.$$

Moreover, Π^A is continuous and piecewise differentiable in n_R^A. Then, taking the second derivative of Π^A, if $n_S^A = \frac{E - n_R^A b_R}{b_S}$:

$$\left.\frac{\partial^2 \Pi^A}{\partial \left(n_R^A\right)^2}\right|_{n_R^A > \bar{n},\, n_R^A < \frac{E}{b_R}} = \frac{\left(E\left(1 + \frac{(2k-1)}{b_S}\right)\right)^2}{2\left(n_R^A\right)^3} > 0. \tag{A.9}$$

Moreover, if $n_R^A \geq \frac{E}{b_R}$ so that $n_S^A = 0$, from (A.8):

$$\left.\frac{\partial^2 \Pi^A}{\partial \left(n_R^A\right)^2}\right|_{n_R^A > \bar{n},\, n_R^A \geq \frac{E}{b_R}} = \frac{E^2}{2\left(n_R^A\right)^3} > 0. \tag{A.10}$$

Expressions (A.9) and (A.10) show that Π^A is piecewise strictly convex over the entire range $n_R^A > \bar{n}$. Only boundary points for n_R^A can be optimal. Hence, either $n_R^A \leq \bar{n}$ or $n_R^A = \frac{1}{2}$, or n_R^A is bounded by the budget constraint. Together with the result that the bank will always increase the number of safe projects until the budget constraint binds, Lemma 1 follows. ■

Proof of Proposition 2. We show that (i) the allocation given in Proposition 2 maximizes total welfare, and then (ii) that the allocation is implemented by the suggested capital requirements.
Part (i). For later reference, we prove this result more generally for the case $E_A \geq E_B$. Denote the IRB-bank A's total lending volume by n^A. In equilibrium, bank A will pick $n_S^A = \frac{1}{2}$ uncorrelated projects. Denoting the number of correlated projects as $n_R^A = n^A - \frac{1}{2}$, profits are:

$$\Pi^A = \left(1 - \tilde{t}^A\right)\left[k + E\left[t \mid t > \tilde{t}^A\right]\left(n^A - \frac{1}{2}\right) 2 + E_A - n^A\right], \tag{A.11}$$

from which we get

$$\tilde{t}^A = \max\left\{\frac{\left(n^A - E_A - k\right)}{2n^A - 1}, 0\right\}.$$

Suppose the total number of projects funded be both banks is $L > k + E_A + E_B$ so that the economy faces positive bankruptcy risk. We prove that allocating $n^A = k + E_A$ projects to the IRB-bank A and $L - n^A$ to the SA-bank B is optimal. As bank B has then only correlated projects, expected profits are:

$$\Pi_B = \left(1 - \tilde{t}^B\right)\left[2E\left(t|t > \tilde{t}^B\right)\left(L - n^A\right) - \left(L - n^A\right) + E_B\right] \tag{A.12}$$

where

$$\tilde{t}^B = \max\left\{\frac{L - n^A - E_B}{2\left(L - n^A\right)}, 0\right\}. \tag{A.13}$$

Welfare with L projects in total is, where W_o^1 expresses welfare in case of observable portfolios and one IRB-bank:

$$W_o^1 = \frac{1}{2}kX + \left(L - \frac{1}{2}\right)\frac{1}{2}X - L - \tilde{t}_A\left(n^A\right)^2 z - \tilde{t}_B\left(L - n^A\right)^2 z.$$

When varying n^A while keeping L fixed, the only variation in welfare is with respect to total expected bankruptcy costs. We first consider the case in which both banks face positive bankruptcy risk. We can then write bankruptcy costs as

$$\begin{aligned} Z_o\left(n^A\right) &= \tilde{t}^A\left(n^A\right)^2 z + \tilde{t}^B(L - n^A)^2 z \\ &= \left(\frac{\left(n^A - E_A - k\right)}{2n^A - 1}\left(n^A\right)^2 + \frac{L - n^A - E_B}{2}\left(L - n^A\right)\right) z. \end{aligned} \tag{A.14}$$

Taking the derivative of (A.14) w.r.t. n^A gives

$$\begin{aligned} \frac{\partial Z_o}{\partial n^A} &= \frac{\left(2\left(n^A - L\right) + \left(4n^A\left(E_A - E_B\right) + 8Ln^A + 4kn^A\right)\left(1 - n^A\right) + E_B - \left(n^A\right)^2\left(14 - 16n^A\right)\right)}{2\left(2n^A - 1\right)^2} z \\ &\equiv \frac{N}{2\left(2n^A - 1\right)^2} z, \end{aligned}$$

so that the sign depends on the numerator N. For the minimum $n^A \to \left(\frac{1}{2}\right)$, we get $N = \left(k + E_A - \frac{1}{2}\right)$, hence $\frac{\partial Z_o}{\partial n^A} > 0$. Next,

$$\frac{\partial N}{\partial n^A} = 2\left(2n^A - 1\right)\left(12n^A - 4L - 2k - 2E_A + 2E_B - 1\right) z,$$

which is strictly increasing in n^A. We hence consider the minimum n^A that allows for positive bankruptcy risk of Bank A, $n^A = (E_A + k)$. Substituting yields

$$\frac{\partial N}{\partial n}(n^A_{\min}) = 2\left(2k + 2E_A - 1\right)\left(10k + 10E_A + 2E_B - 4L - 1\right) z > 0,$$

as $k > \frac{1}{2}$ and $L \leq 1$. And as the denominator $2\left(2n^A - 1\right)^2 > 0$, $\frac{\partial Z_o}{\partial n^A} > 0 \ \forall n^A$ follows.

Next, we need to consider the case where only Bank B may go bankrupt. Then, total bankruptcy costs are

$$Z_o\left(n^A\right) = \tilde{t}^B(L - n^A)^2 z = \left(\frac{\left(L - n^A - E_B\right)}{2}\left(L - n^A\right)\right) z. \tag{A.15}$$

Thus, $\frac{\partial Z_o(n^A)}{\partial n^A} = \frac{1}{2}z\,(2n - 2L + E_B) < 0$ as there were no bankruptcy risk otherwise. Since $\frac{\partial Z}{\partial n^A} > 0$ if $n^A > k + E_A$ and $\frac{\partial Z}{\partial n^A} < 0$ if $n^A < k + E_A$, it is optimal to implement $n^A = k + E_A$.

Recalling that bank A is risk-free and finances $n^A = k + E$ projects, and defining the number of correlated projects financed by bank B as $n_R^B = L - k - E$, welfare can be written as:

$$W_o^1 = \frac{1}{2}kX + \left(n_R^B + k + E - \frac{1}{2}\right)\frac{1}{2}X - \left(k + E + n_R^B\right) - \max\left\{\frac{n_R^B - E}{2n_R^B}, 0\right\}\left(n_R^B\right)^2 z,$$

which gives the following derivative:

$$\frac{\partial W_o^1}{\partial n_R^B} = \begin{cases} \frac{1}{2}X - 1 > 0 & \text{if } n_R^B \leq E \\ \frac{1}{2}X - 1 - n_R^B z + \frac{1}{2}zE & \text{if } n_R^B > E \end{cases}.$$

Moreover, for all $n_R^B > E$, W_o^1 is strictly concave in n_R^B, and the optimum is given by the first-order condition:

$$\left(n_R^B\right)^* = \frac{1}{z}\left(\frac{X}{2} - 1\right) + \frac{E}{2}.$$

Hence, bank B will be allowed to finance at least E projects. Moreover, it is optimal that bank B finances $n_R^B > E$ projects and faces positive bankruptcy risk if

$$\left.\frac{\partial W_o^1}{\partial n_R^B}\right|_{n_R^B \searrow E} = \frac{1}{2}X - 1 - Ez + \frac{E}{2}z > 0 \Leftrightarrow X > 2 + zE.$$

The total number of projects funded is

$$k + E + \left(n_R^B\right)^* = k + E + \frac{1}{z}\left(\frac{X}{2} - 1\right) + \frac{E}{2},$$

which gives an interior solution if $k + \frac{3}{2}E + \frac{1}{z}\left(\frac{X}{2} - 1\right) < 1$, i.e. if

$$z > \frac{X - 2}{2\left(1 - \left(k + \frac{3}{2}E\right)\right)}.$$

Part (ii). Suppose the regulator chooses $b_S = 0$, $b_R = \frac{E}{E+k-\frac{1}{2}}$ and $b_U = \frac{E}{n^B}$ as stated in the Proposition. Then, the IRB-bank needs no equity to fund all uncorrelated projects, and can fund exactly $E + k - \frac{1}{2}$ correlated projects. Funding more correlated projects is impossible as b_R is binding. For the SA-bank, there are no uncorrelated projects left, and it can fund n^B correlated projects. ■

Proof of Proposition 3. Since banks are identical and face the same capital requirements, the outcome will be symmetric. Each IRB-bank will prefer to finance as many uncorrelated projects as possible regardless of the number of correlated projects. Moreover, it is straightforward to show that profits are increasing in the number of correlated projects if $\tilde{t}_o(2) > 0$ (Subscript "o" denotes observable portfolios and "(2)" denotes two IRB-banks). It follows that they will also finance as many correlated projects as possible. Let $\frac{n^R}{2}$ be the number of correlated projects financed by each bank in the symmetric outcome. As the regulator will choose b_R so that banks will at least exhaust their riskless lending capacity, we know that $n^R \in [k - \frac{1}{2} + 2E, \frac{1}{2}]$. We use the notations $n^{\min} = k - \frac{1}{2} + 2E$, and $n^{\max} = \frac{1}{2}$.

We denote social welfare with two IRB-banks by W_o^2, for which we obtain:

$$\begin{aligned} W_o^2\left(n^R\right) &= \frac{1}{2}kX + n^R\frac{1}{2}X - n^R - \frac{1}{2} - 2\tilde{t}_o^2\left(\frac{1}{4} + \frac{n^R}{2}\right)^2 z \\ &= \frac{1}{2}kX + n^R\frac{1}{2}X - n^R - \frac{1}{2} - \left(\frac{1}{2n^R}\left(1 + 2n^R - 2k - 4E\right)\right)\left(\frac{1}{4} + \frac{n^R}{2}\right)^2 z. \qquad \text{(A.16)} \end{aligned}$$

We next derive the general form of the welfare function $W_o^2\left(n^R\right)$ over the critical interval $\left(k - \frac{1}{2} + 2E, \frac{1}{2}\right)$. The first derivative is

$$\frac{\partial W_o}{\partial\left(n^R\right)} = \frac{\partial}{\partial n}\left(\frac{1}{2}kX + n\frac{1}{2}X - n - \frac{1}{2} - \left(\frac{1}{2n}\left(1 + 2n - 2k - 4E\right)\right)\left(\frac{1}{4} + \frac{n}{2}\right)^2 z\right),$$

and the second derivative is

$$\frac{\partial^2 W_o^2}{\partial\left(n^R\right)^2} = \frac{z\left(2k + 4E - 1 - 8\left(n^R\right)^3\right)}{16\left(n^R\right)^3}.$$

Evaluated at the lower bound $n^{\min} = \left(k - \frac{1}{2} + 2E\right) \leq \frac{1}{2}$,

$$\left.\frac{\partial^2 W_o^2}{\partial\left(n^R\right)^2}\right|_{n^R = k - \frac{1}{2} + 2E} = 2z\left(k + 2E\right)\frac{1 - k - 2E}{\left(2k + 4E - 1\right)^2} > 0.$$

Evaluated at the upper bound, $n^{\max} = \frac{1}{2}$, gives

$$\left.\frac{\partial^2 W_o^2}{\partial\left(n^R\right)^2}\right|_{n^R = \frac{1}{2}} = -z\left(1 - k - 2E\right) < 0.$$

Moreover, $\frac{\partial^3 W_o^2}{\partial\left(n^R\right)^3} = \frac{3}{16n^4}z\left(1 - 2k - 4E\right) < 0$ in the critical interval. It follows that $W_o^2\left(n^R\right)$ will be convex for small n^R, concave for large n^R, and have a unique inflection point within the interval $n^R \in [k - \frac{1}{2} + 2E, \frac{1}{2}]$.

We derive next the conditions under which the upper boundary solution, $n^R = n^{\max} = \frac{1}{2}$, is optimal. Given the characteristics of $W_o^2\left(n^R\right)$ just derived, a necessary and sufficient condition is that (i) $W_o^2(n^R)\big|_{n^R = k - \frac{1}{2} + 2E} < W_o^2(n^R)\big|_{n^R = \frac{1}{2}}$ and that (ii) $\left.\frac{\partial W_o^2}{\partial n^R}\right|_{n^R = \frac{1}{2}} > 0$. We obtain

$$\begin{aligned} W_o^2(n^R)\big|_{n^R = k - \frac{1}{2} + 2E} &= X\left(k + E - \frac{1}{4}\right) - \left(k + 2E\right), \\ W_o^2(n^R)\big|_{n^R = \frac{1}{2}} &= \frac{1}{2}kX + \frac{1}{4}X - 2\left(1 - 2E - k\right)z - 1. \end{aligned}$$

Hence $W_o^2(n^R)\big|_{n^R = k - \frac{1}{2} + 2E} < W_o^2(n^R)\big|_{n^R = \frac{1}{2}}$ if:

$$X\left(k + E - \frac{1}{4}\right) < \frac{1}{2}kX + \frac{1}{4}X - 2\left(1 - 2E - k\right)z + k - 1 + 2E,$$

or if

$$z < \frac{1}{4}\left(X - 2\right). \tag{A.17}$$

Moreover,

$$\begin{aligned} \left.\frac{\partial W_o^2}{\partial n^R}\right|_{n^R = \frac{1}{2}} &= \frac{1}{2}X - 1 - \frac{1}{8}z2\left(2k + 1 + 4E - 4E - 2k + 2 - 1\right) \\ &= \frac{1}{2}X - 1 - \frac{1}{2}z, \end{aligned}$$

and hence $\left.\frac{\partial W_o^2}{\partial n^R} > 0\right|_{n^R = \frac{1}{2}} \Leftrightarrow z < X - 2$. Thus, $z < \frac{1}{4}\left(X - 2\right)$ is the necessary and sufficient condition.

In all other cases, to show that the existence of an internal optimum depends on z/X. We first show that expected bankruptcy costs are increasing in n^R:

$$\frac{\partial Z_o^2}{\partial n^R} = \frac{1}{8(n^R)^2}\left(\left(2n^R+1\right)^3(2k-1+4E)\left(1-2n^R\right)+8\left(n^R\right)^2\right)z > 0, \tag{A.18}$$

where we used the fact that $n^R \leq \frac{1}{2}$.

Next, the unique inflection point n^{infl} can be determined as:

$$\begin{aligned}
\frac{\partial^2 W_o^2}{\partial (n^R)^2} &= 0 \Leftrightarrow \frac{1}{16n^3}z\left(-8n^3+2k+4E-1\right) = 0 \\
&\Leftrightarrow \left(-8n^3+2k+4E-1\right) = 0 \\
&\Leftrightarrow n^{\text{infl}} = \sqrt[3]{\frac{1}{4}k+\frac{1}{2}E-\frac{1}{8}} = \frac{1}{2}\sqrt[3]{2k+4E-1},
\end{aligned}$$

which always has an interior solution and is independent of z. It is straightforward to verify that $\frac{\partial W_o^2}{\partial n^R} > 0$ at the inflection point. It follows that, for all $z > \frac{X-2}{4}$, there will be a local maximum over the interval $\left(n^{\text{infl}}, n^{\max}\right)$. Define this maximum as

$$W^{\sup} = \max_{n^R \in (n^{\min}, n^{\max})} W_o^2\left(n^R\right).$$

Next, we consider variations of z for given X. It follows from (A.16) that:

$$\frac{\partial W_o^2\left(n^R\right)}{\partial z} < 0 \ \forall n^R \in \left(n^{\min}, n^{\max}\right).$$

Therefore, as we vary z, $W^{\sup}$ will decrease continuously, and we write this relationship in functional form as $W^{\sup}(z)$. Next, we define the difference function between this maximum and the welfare at the lower boundary point $W_o^2\left(n^{\min}\right) = X\left(k+E-\frac{1}{4}\right)-(k+2E)$ as

$$\Delta(z) = W^{\sup}(z) - X\left(k+E-\frac{1}{4}\right)-(k+2E).$$

Also,

$$\lim_{z\to 0}\Delta(z) > 0,$$

since $\lim_{z\to 0} W_o^2\left(n^R\right) = \frac{1}{2}kX + n^R\frac{1}{2}X - \left(\frac{1}{2}+n^R\right) > W_o^2\left(n^{\min}\right) = X\left(k+E-\frac{1}{4}\right)-(k+2E)$, for all $n^R > n^{\min}$. Moreover, (A.17) implies that

$$\Delta(z) < 0, \text{ for all } z > \frac{X-2}{4}.$$

Since $\Delta(z)$ is a continuous function, it follows from the intermediate value theorem that there exists $z \in \left(0, \frac{X-2}{4}\right)$ such that $\Delta(z) = 0$, and this z defines the unique crossing point.■

Proof of Proposition 4. We adopt the notation $E_A = E_B = E$. If both banks are SA-banks, then our notation is that each bank finances n_u projects, half of which are uncorrelated and half correlated. Total welfare is:

$$W_u = n_u X\left(k+\frac{1}{2}\right) - 2n_u - 2\mathfrak{e}_u (n_u)^2 z = n_u X\left(k+\frac{1}{2}\right) - 2n_u - (2n_u(1-k)-2E)\,n_u z.$$

With one IRB-bank, the IRB-bank A funds $n_S^A = \frac{1}{2}$ uncorrelated and n_R^A correlated projects. Further, bank B finances n^B correlated projects. Total welfare is

$$W_o^1 = \frac{1}{2}kX + \left(n_R^A + n^B\right)\frac{1}{2}X - \left(\frac{1}{2} + n_R^A + n^B\right) - \tilde{t}_o^1\left(n^B\right)^2 z. \tag{A.19}$$

With two IRB-banks, there is a symmetric equilibrium and we denote the volume of each bank's portfolio of uncorrelated projects by $\frac{n_S}{2} = \frac{1}{4}$ and the number of correlated projects by $\frac{n_R}{2}$. Social welfare is then

$$W_o^2 = \frac{1}{2}kX + n_R\frac{1}{2}X - \frac{1}{2} - n_R - 2\tilde{t}_o^2\left(\frac{1}{4} + \frac{n_R}{2}\right)^2 z. \tag{A.20}$$

We compare the three regimes pairwise. First, compare two SA-banks to one IRB-bank. If it is optimal to implement no bankruptcy risk with two SA-banks, then the regime with one IRB-bank is trivially superior as the number of projects that can be implemented without risk is higher ($2E + k > \frac{2E}{1-k}$ as $E < \frac{1-k}{2}$) and the average return is higher since all uncorrelated projects are funded.

With positive bankruptcy risk, let n_u^* be the optimal number of projects financed by each of two SA-banks. Suppose hypothetically that n^B is adjusted such that $n_S^A + n_R^A + n^B = 2n_u^*$ which means that, with one IRB-bank, the regulator (inefficiently) implements the number of projects that is optimal with 2 SA-banks. For any n_u^*, we have with two SA-banks $\tilde{t}_u = \frac{2n_u(1-k)-2E}{2n_u}$ and hence expected bankruptcy costs of

$$Z_u = 2\left(\left(\frac{2n_u(1-k)-2E}{2n_u}\right)n_u^2 z\right) = \left(2n_u z\left(n_u(1-k)-E\right)\right).$$

If the same total number of projects is implemented in the regime with one IRB-bank, then the IRB-bank funds $E + k$ projects and the SA-bank hence $n^R = (2n_u - (E+k))$ projects which are all risky. This yields a bankruptcy threshold of

$$\tilde{t}_u = \frac{((2n_u - (E+k)) - E)}{2(2n_u - (E+k))}.$$

with total expected bankruptcy costs of

$$\begin{aligned} Z_o^1 &= \tilde{t}\left(n^R\right)^2 z = \left(\frac{((2n_u - (E+k)) - E)}{2(2n_u - (E+k))}\right)(2n_u - (E+k))^2 z \\ &= \left(\frac{((2n_u - (E+k)) - E)}{2}\right)(2n_u - (E+k))z. \end{aligned}$$

Comparing the two bankruptcy costs gives

$$\begin{aligned} \Delta Z &\equiv Z_u - Z_o^1 = \left(2n_u z\left(n_u(1-k)-E\right)\right) - \left(\frac{((2n_u - (E+k)) - E)}{2}\right)(2n_u - (E+k))z \\ &= -\frac{\left(3kE - 2n_uE + 2E^2 + 4kn_u^2 - 4kn_u + k^2\right)}{2}z. \end{aligned}$$

The numerator N is decreasing in n_u since $\frac{\partial N}{\partial n_u} = 8kn_u - 2E - 4k < 0$ as $n_u < \frac{1}{2}$. To prove that $\Delta Z > 0$ always holds, we thus need to consider the minimum n_u that leads to positive bankruptcy risk in the regime with one IRB-bank which is $n_u = \left(\frac{2E+k}{2}\right)$. Then, $N = -2kE + 4kE^2 + 4k^2E - k^2 + k^3$ which is increasing in E. Substituting the maximum $E = \left(\frac{1-k}{2}\right)$ in N yields $N = 0$. Hence, the numerator is always negative if there is positive bankruptcy risk which proves that $\Delta Z = Z_u - Z_o^1 > 0$.

Second, compare two SA-banks with two IRB-banks. Suppose again that n_R is chosen such that $n_S^A + n_R = 2n_u^*$. Then, the only difference between the two regimes is that more uncorrelated projects are financed with two IRB-banks which trivially leads to higher welfare.

Third, compare one IRB-bank to two IRB-banks. We will show that, even if the regulator implements the number of projects that is optimal with two IRB-banks in both cases, bankruptcy costs with one IRB-bank will still be lower. This proves that welfare is higher as only bankruptcy costs matter when the number of projects is the same. Define n as the number of projects funded by each of two IRB-banks. Then, the difference in bankruptcy costs is

$$\Delta Z \equiv Z_o^1 - Z_o^2 = \left(\frac{((2n-(E+k))-E)}{2}\right)(2n-(E+k))\,z - 2\left(\frac{2n-k-2E}{4n-1}\right)n^2 z.$$

which can be simplified as

$$\Delta Z = \frac{1}{2}z\left(-k-E+4n\left(E+k-n+\frac{1}{2}\right)\right)\frac{k-2n+2E}{4n-1}.$$

We know that $n > \frac{1}{4}$ as all uncorrelated projects will be financed, and hence $4n-1>0$. Furthermore, with positive bankruptcy risk, we have $k-2n+2E<0$ so that the sign depends on

$$S \equiv \left(-k-E+4n_u\left(E+k-n_u+\frac{1}{2}\right)\right).$$

We need to prove that $S > 0$ for all n. S is decreasing in n as $\frac{\partial S}{dn} = 4k-8n+4E+2 < 0$ for $n < \frac{1}{2}$. Hence, we need to consider the minimum $n = \left(\frac{2E+k}{2}\right)$ that leads to positive bankruptcy risk. Then $S(n=\frac{1}{2}) = \left(-k-E+4n\left(E+k-n+\frac{1}{2}\right)\right) = k^2+2Ek+E > 0$ which proves that $\Delta Z = Z_o^1 - Z_o^2 < 0$. ■

Proof of Lemma 2. Define the total number of projects the IRB-bank can fund as n^A, and assume first that $n^A < \frac{1}{2}$. Then, profits with uncorrelated projects only are $\Pi_S^A = n^A(2k-1)+E$ while profits with only correlated projects are $\Pi_R = \frac{1}{4n^A}\left(n^A+E\right)^2$.[11] The difference is

$$\Delta\Pi^A \equiv \Pi_R^A - \Pi_S^A = \frac{1}{4n^A}\left(n^A+E\right)^2 - \left(n^A(2k-1)+E\right)$$

which yields

$$\frac{\partial\left(\Delta\Pi^A\right)}{\partial n^A} = \frac{1}{4(n^A)^2}\left(5\left(n^A\right)^2 - E^2 - 8k\left(n^A\right)^2\right),$$

$$\frac{\partial^2\left(\Delta\Pi^A\right)}{\partial (n^A)^2} = \frac{E^2}{2(n^A)^3} > 0.$$

Since $\frac{\partial^2(\Delta\Pi^A)}{\partial(n^A)^2} > 0$, and as both $\Delta\Pi^A < 0$ and $\frac{\partial(\Delta\Pi^A)}{\partial n^A} < 0$ for $n^A \to 0$, there can be at most a single switching point $\bar{n}^A < \frac{1}{2}$. If so, $\Delta\Pi^A = 0$ yields $\bar{n}^A = \frac{E}{\sqrt{5-8k}}$. For $k > \frac{5}{8}$, there is no risk-shifting as $\Delta\Pi^A < 0$ $\forall n^A > E$ where $n^A > E$ is required for positive bankruptcy risk. Furthermore, $\frac{\partial n^A}{\partial E} = \frac{1}{\sqrt{5-8k}} > 0$ and $\frac{\partial n^A}{\partial k} = \frac{4E}{(5-8k)^{\frac{3}{2}}} > 0$ if $\bar{n}^A < \frac{1}{2}$ exists at all.

Next, consider $n^A \in (\frac{1}{2}, E+k)$. Then, profits without risk-shifting are $\Pi_S^A = \left(k+E-\frac{1}{2}\right)$ regardless of the number of correlated projects which just break even in expectation if there is no bankruptcy risk. For profits with risk-shifting, we get

$$\Pi_R^A = \frac{\left(1+2k\left(n^A-\frac{1}{2}\right)+E-n^A\right)^2}{2}. \tag{A.21}$$

Hence,

$$\Delta\Pi^A \equiv \Pi_R^A - \Pi_S^A = \frac{\left(1+2k\left(n^A-\frac{1}{2}\right)+E-n^A\right)^2}{2} - \left(k+E-\frac{1}{2}\right)$$

[11] Recall from Lemma 1 that the bank funds either only uncorrelated or correlated projects for $n^A < \frac{1}{2}$.

and thus

$$\begin{aligned}\frac{\partial \Delta\Pi^A}{\partial n^A} &= (2k-1)\left(E + n^A(2k-1) + (1-k)\right) > 0,\\ \frac{\partial \Delta\Pi^A}{\partial (n^A)^2} &= (2k-1)^2 > 0.\end{aligned} \tag{A.22}$$

which proves that there can at most be one threshold. If $\bar{n}^A \in (\frac{1}{2}, E+k)$ exists, then we get from $\Delta\Pi^A = 0$

$$\bar{n}^A = \frac{(2E+2k-1)^{\frac{1}{2}} - E + k - 1}{(2k-1)}.$$

It remains to show that $\bar{n}^A$ is monotonic in k and E:

$$\frac{\partial \bar{n}^A}{\partial k} = \frac{1}{2k-1}\left(\frac{1}{\sqrt{2k+2E-1}} + 1\right) + \frac{2}{(2k-1)^2}\left(E - k - \sqrt{2k+2E-1} + 1\right).$$

A sufficient condition for $\frac{\partial n^A}{\partial k} > 0$ is

$$A \equiv \frac{\left(\sqrt{2k+2E-1} - 4E - 2k + 2E\sqrt{2k+2E-1} + 1\right)}{\sqrt{2k+2E-1}} > 0.$$

This is positive if the numerator is positive. After rearranging, it can easily be seen that this is the case if and only if

$$2\left(2k-1+2E(1-2E)\right)(1-k-E) > 0,$$

which always holds. Thus, $\bar{n}^A$ is monotonic in k. Moreover,

$$\frac{\partial \bar{n}^A}{\partial E} = \frac{1}{2k-1}\left(\frac{1}{\sqrt{2k+2E-1}} - 1\right) > 0.$$

Finally, note that we can ignore the case $\bar{n}^A > k + E$ as the regulator will never implement $n^A > E + k$ anyway. ■

Proof of Proposition 7. Part (i). We already know that $n^A = E + k$ is optimal in the class of cherry-picking allocations, and this allocation is feasible in Region 1 by definition.

Parts (ii) and (iii). In the region of part (ii), $n^A = E + k$ without risk-shifting is not feasible as $\bar{n}^A < E + k$. We first show that the regulator sets $n^A = \bar{n}^A$ if $\bar{n}^A \geq \frac{1}{2}$. $n^A = \bar{n}^A$ allocates all uncorrelated projects and $\bar{n}^A - \frac{1}{2}$ correlated projects to Bank A and as many correlated projects the regulator wants to Bank B. With risk-shifting, the maximum number of uncorrelated projects he can implement is weakly smaller than the maximum number of correlated projects as $n^A \geq n^B$. And as the regulator strictly prefers uncorrelated projects, the claim follows.

Thus we only need to consider the case $\bar{n}^A < \frac{1}{2}$. We are left with proving that there exists a unique $n^+ < \frac{1}{2}$ such that the regulator allows for risk shifting if and only if $\bar{n}^A < n^+$. Straightforward calculations yield that, when $n^A < \frac{1}{2}$, Bank A receives when cherry-picking,

$$\Pi_S^A = 2n^A k - n^A + E,$$

and when risk-shifting,

$$\Pi_R^A = \frac{\left(n^A + E\right)^2}{4n^A}.$$

It follows that

$$\bar{n}^A = E\frac{1+2\sqrt{2k-1}}{5-8k}, \tag{A.23}$$

and that $\bar{n}^A$ is strictly increasing in k.

We then consider the regulator's choices if $\bar{n}^A < \frac{1}{2}$. When risk-shifting is adopted, Bank B's portfolio will always contain more uncorrelated projects than that of Bank A, so that the constraint $n^B \leq n^A$ will be binding in the optimum. Thus, the regulator has two options: (i) either to implement cherry-picking with $n^B \leq n^A \leq \bar{n}^A$ or (ii) to implement risk-shifting with $n^B = n^A > \bar{n}^A$.

We consider cherry-picking first. Then, welfare is strictly concave in n^B for any given n^A since

$$\begin{aligned} W_S \;=\; & \left(\bar{n}^A + n^B\left(\frac{\frac{1}{2}-\bar{n}^A}{1-\bar{n}^A}\right)\right)kX + n^B\left(\frac{\frac{1}{2}}{1-\bar{n}^A}\right)\frac{1}{2}X - \bar{n}^A - n^B \\ & - \left(\frac{2}{n^B}\left[1-\bar{n}^A\right]\left[n^B-E\right] - 4k\left(\frac{1}{2}-\bar{n}^A\right)\right)\left[n^B\right]^2 z, \end{aligned} \tag{A.24}$$

from which we obtain:

$$\frac{\partial^2 W_S}{\partial\left(n^B\right)^2} = -4z\left(1-\bar{n}^A-2k\left(\frac{1}{2}-\bar{n}^A\right)\right) < 0.$$

Next, consider any sequence of optimal allocations under parameters (k, E) such that $\bar{n}^A \to 0$ (note from Eq. (A.23) that any sequence such that $E \to 0$ has the property $\lim \bar{n}^A \to 0$). Then, (A.24) immediately shows that:

$$\lim_{n^A \to 0} W_S = 0.$$

Next, we consider risk-shifting. We denote the welfare attainable in the optimum in this case by W_R. If $\bar{n}^A < \frac{1}{2} \to 0$, and $n^A = n^B < \frac{1}{2}$, then we obtain

$$W_R = \left(n^A + n^A\left(\frac{\frac{1}{2}}{1-\bar{n}^A}\right)\right)kX + n^A\left(\frac{\frac{1}{2}-\bar{n}^A}{1-\bar{n}^A}\right)\frac{1}{2}X - 2n^A - \tilde{t}^A\left[n^A\right]^2 z - \tilde{t}^B\left[n^A\right]^2 z, \tag{A.25}$$

where

$$\tilde{t}^A = \frac{n^A - E}{2n^A}. \tag{A.26}$$

Consider a sequence such that $\bar{n}^A \to 0$, implying that $E \to 0$. At the limit, only Bank A faces bankruptcy risk, so that we can rewrite (A.25) as:

$$W_R = \left(n^A + \frac{n^A}{2}\right)kX + \frac{n^A}{2}\frac{1}{2}X - 2n^A - \frac{\left[n^A\right]^2}{2}z,$$

and hence there clearly exists $n^A > \bar{n}^A \to 0$ such that W_R is strictly positive. This shows that $W_S < W_R$ for $\bar{n}^A$ sufficiently small.

Denote the optimal quantities with risk-shifting by n^*_R, where n^*_R must satisfy $n^*_R = n^B = n^A$. Furthermore, W_R is independent of $\bar{n}^A$. Consider then any vector of parameter values (X, z, k, E) such that $\bar{n}^A = n^*_R$, and consider a symmetric solution with cherry-picking, such that $n^B = n^A = \bar{n}^A$ $(= n^*_R)$. Denote the welfare obtained under this allocation by $\tilde{W}_S$. We then compare this allocation to an allocation with identical total quantities $\bar{n}^A = n^*_R$ with risk-shifting, which yields W_R. In the case of cherry-picking, there will be $\bar{n}^A + \bar{n}^A\frac{\frac{1}{2}-n^A}{1-n^A}$ uncorrelated and $\bar{n}^A\frac{\frac{1}{2}}{1-n^A}$ correlated projects financed. With risk–shifting, there will be $\bar{n}^A\frac{\frac{1}{2}}{1-n^A}$ uncorrelated and $\bar{n}^A + \bar{n}^A\frac{\frac{1}{2}-n^A}{1-n^A}$ correlated projects financed. Since $\bar{n}^A + \bar{n}^A\frac{\frac{1}{2}-n^A}{1-n^A} > \bar{n}^A\frac{\frac{1}{2}}{1-n^A}$, it follows that cherry-picking yields higher expected return and lower expected bankruptcy costs, hence $\tilde{W}_S > W_R$. Moreover, when $\bar{n}^A = n^*_R$,

$W_S > \tilde{W}_S$ as $\tilde{W}_S$ is obtained under the constraint $n^A = n^B = \bar{n}^A$, whereas W_S is obtained under the same n^A and an optimal n^B. Hence we have $W_S > W_R$ at this point, implying that there must be an intermediate value n^+ for which we have $W_S = W_R$.

We then proceed by case distinction.

I. Consider the case where at the point $\bar{n}^A = n^*_R$, the optimal n^B satisfies $n^B = \bar{n}^A$. It then immediately follows that in this case $\frac{\partial W_S}{\partial n^A} > 0$ since the welfare can be separated in the contribution of each bank, and is increasing when Bank B is constrained at $n^B = \bar{n}^A$.

II. Consider the case where at the point $\bar{n}^A = n^*_R$, the optimal n^B satisfies $n^B < \bar{n}^A$. Then, consider the allocation where $n^A = \bar{n}^A$ and $n^B = \hat{n}^B$ are chosen, where we define $\hat{n}^B$ as the largest risk-free allocation, $\hat{n}^B \equiv \arg\max_{n^B} \{\Pi_B \mid \text{Bank B has no bankruptcy risk}\}$, and denote the welfare obtained under this allocation by $\hat{W}_S$. Note that by keeping n^B at this maximum risk-free allocation, when increasing $\bar{n}^A$, we will have $\frac{\partial \hat{W}_S}{\partial n^A} > 0$ as the allocation contains strictly more uncorrelated projects with $k > \frac{1}{2}$, and bankruptcy costs are still zero.

II.1. Consider the subcase $\hat{W}_S > W_R$. Since the welfare when n^B is optimized satisfies $W_S \geq \hat{W}_S$, it follows that $W_S > W_R$, and thus existence of n^+.

II.2. Consider the subcase $\hat{W}_S < W_R$. Again, we have $\frac{\partial \hat{W}_S}{\partial n^A} > 0$ and strict concavity when varying n^B while keeping n^A constant, $\frac{\partial^2 W_S}{\partial (n^B)^2} < 0$. As $W_S > W_R$ if $\bar{n}^A = n^*_R$, it follows that n^+ exists. ■

Proof of Proposition 8. We know that $n^A = n^B = n$ and that profits and allocations will be identical for both banks. Also, the optimal allocation from the regulator's point of view is given by Proposition 3. Part (i). In Region 1, the constraint $\bar{n}$ is not binding relative to the optimal allocations characterized in Proposition 3. Parts (ii) and (iii). In the region of part (ii), $n^A = \frac{1}{4} + \frac{n_R}{2}$ without risk-shifting is not feasible as $\bar{n} < \frac{1}{4} + \frac{n_R}{2}$. Denote by W_S the welfare if both banks opt for cherry-picking (if $n \leq \bar{n}$), and by W_R the welfare if both banks opt for risk-shifting (if $n > \bar{n}$). Writing out W_S and W_R, it follows that both welfare functions are continuous functions in the parameters (X, z, k, E). Since at any vector of (X, z, k, E) that imply $\bar{n} = \frac{1}{4} + \frac{n_R}{2}$ we have $W_S > W_R$, it follows from continuity of W_S and W_R that there must be a neighborhood of values (X, z, k, E) around any point at which $\bar{n} = \frac{1}{4} + \frac{n_R}{2}$ such that we still have $W_S > W_R$. Hence, we have shown that if n^{++} exists, $n^{++} < \frac{1}{4} + \frac{n_R}{2}$.

Finally, we need to prove that there exists a unique n^{++} such that the regulator allows for risk shifting if and only if $\bar{n} < n^{++}$. The proof is very similar to that of Proposition 7, and details are omitted. We compare the regulator's two options: (i) cherry-picking with $n^B = n^A \leq \bar{n}$ or (ii) risk-shifting with $n^B = n^A = n^*_R > \bar{n}$. We denote the optimal quantities when risk-shifting is adopted, yielding welfare W_R, by n^*_R. Next, considering a sequence such that $\bar{n} \to 0$, we find again that $\lim_{n^A \to 0} W_S = 0$. On the other hand, along this sequence, we must have $\lim_{n^A \to 0} W_R > 0$, which shows that $W_S < W_R$ for $\bar{n}$ su ciently small. We consider then the point $\bar{n} = n^*_R$, and with the same case distinctions and auxiliary functions $\tilde{W}_S$ and $\hat{W}_S$ as in the proof of Proposition 7, we can show that the intersection n^{++} such that $W_S = W_R$ exists. ■

Proof of Proposition 9. We know from Propositions 7 and 8 that for k and E su ciently large, the optimal allocation is feasible with one IRB-bank (if $\bar{n}^A > k + E$) and with two IRB-banks (if $\bar{n} > \frac{1}{4} + \frac{n_R}{2}$). Hence, the unobservability of portfolios makes no di erence, and if follows from Proposition 4 that one IRB-bank is optimal.

Next, we know from Propositions 7 and 8 that for k and E su ciently small, the feasible allocations under cherry-picking are constrained to $\lim \bar{n}^A \to 0$ and $\lim \bar{n} \to 0$, implying that $W_S \to 0$ in either case. Since $W_u > 0$, two SA-banks are superior to either alternative under cherry-picking. Then, consider the optimal allocation under risk-shifting, yielding W^1_R (one IRB-bank) or W^2_R (two IRB-banks). Denote the optimal allocation by $n^*_R(1)$ and $n^*_R(2)$, respectively. Consider then an allocation with two SA-banks with the same number of total projects financed. In this allocation, there are strictly more uncorrelated projects and strictly less correlated projects than under the two alternatives with risk-shifting. Two IRB-banks are then always dominated, and we only need to compare bankruptcy costs with two SA-banks to those with one IRB-bank.

We are thus left with showing that one IRB-bank with risk-shifting cannot be better. Consider the optimal allocation with one IRB-bank and risk-shifting. As shown in Proposition 7, we then have $n^A = n^B$. Denote the maximum welfare by W_R. Using $n_R^A = n^A = n^B$, we get:

$$W_R = \left(\frac{\frac{1}{2}}{1-n_R^A} n_R^A k + \frac{\frac{1}{2}-n_R^A}{1-n_R^A} n_R^A \frac{1}{2} + n_R^A \frac{1}{2}\right) X - 2n_R^A - \frac{n_R^A - E}{2} n_R^A z - \frac{\left(n_R^A - E\right)\left(1-n_R^A\right) - n_R^A k}{\left(1-2n_R^A\right)} n_R^A z.$$

If the same number of projects is funded with two SA-banks ($n^A = n^B = n_R^A$), we get

$$W_u = n_R^A k X + n_R^A \frac{1}{2} X - 2n_R^A - \left(2n_R^A (1-k) - 2E\right) n_R^A z.$$

We will show that $W_u > W_R$. This is the case if

$$\begin{aligned} & n_R^A k X + n_R^A \frac{1}{2} X - \left(\left(2n_R^A\right)(1-k) - 2E\right) n_R^A z \\ > & \left(\frac{\frac{1}{2}}{1-n_R^A} n_R^A k + \frac{\frac{1}{2}-n_R^A}{1-n_R^A} n_R^A \frac{1}{2} + n_R^A \frac{1}{2}\right) X - \frac{n_R^A - E}{2} n_R^A z - \frac{\left(n_R^A - E\right)\left(1-n_R^A\right) - n_R^A k}{\left(1-2n_R^A\right)} n_R^A z \end{aligned}$$

or if:

$$\left(\frac{1-n_R^A-\frac{1}{2}}{1-n_R^A}\right) n_R^A k X + \left(\frac{\frac{1}{2}-n_R^A}{1-n_R^A}\right) n_R^A \frac{1}{2} X > \left(2n_R^A (1-k) - 2E\right) n_R^A z - \frac{n_R^A - E}{2} n_R^A z - \frac{\left(n_R^A - E\right)\left(1-n_R^A\right) - n_R^A k}{\left(1-2n_R^A\right)} n_R^A z.$$

It is immediate that the LHS is positive. We will then show that the RHS is negative. This is the case if:

$$\left(\left(2n_R^A (1-k) - 2E\right) - \frac{n_R^A - E}{2} - \frac{\left(n_R^A - E\right)\left(1-n_R^A\right) - n_R^A k}{\left(1-2n_R^A\right)}\right) n_R^A z < 0,$$

or if:

$$\frac{3}{2} n_R^A - \frac{3}{2} E - 2k n_R^A - \frac{\left(n_R^A - E\right)\left(1-n_R^A\right) - n_R^A k}{\left(1-2n_R^A\right)} < 0.$$

A sufficient condition is:

$$-\frac{1}{2} E + 2n_R^A E + (4k-2)\left(n_R^A\right)^2 - \left(3k - \frac{1}{2}\right) n_R^A < 0$$

which is true, given that $n_R^A \leq \frac{1}{2}$ ad $k \in \left[\frac{1}{2}, 1\right]$. ■

Proof of Proposition 11. Part (i). Equities are identical for two SA-banks and two IRB-banks, respectively, as the regulator cannot differentiate between two identical banks. With one IRB-bank, we know from Lemma 1 that its is optimal to keep the IRB-bank risk-free for any $\mathbf{E} \equiv (E_A, E_B)$ given. The maximum number of correlated projects the IRB-bank A can fund without bankruptcy risk is $n_R^A = \left(k + E_A - \frac{1}{2}\right)$ which yields welfare

$$W_A = \frac{1}{2} k X + n_R^A X - \lambda E_A^2 = \frac{1}{4} X \left(4k + 2E_A - 1\right) - \lambda E_A^2.$$

Maximizing w.r.t. equity E_A yields optimal values of

$$E_A^* = \left(\frac{X}{4\lambda}\right), \quad \left(n_R^A\right)^* = \left(\frac{(X - 2\lambda + 4k\lambda)}{4\lambda}\right), \quad W_A^* = \frac{\left(X^2 + 4X\lambda(4k-1)\right)}{16\lambda}.$$

Assume first that the SA-bank is also kept risk-free. Then, it can fund $n^B = E_B$ correlated projects which yields welfare of (superscript "S" denotes a safe banking sector)

$$W_B^S = \frac{1}{2}XE_B - \lambda E_B^2,$$

with optimal values of

$$\left(E_B^S\right)^* = \left(n_B^S\right)^* = \left(\frac{4X}{\lambda}\right), \left(W_B^S\right)^* = \frac{X^2}{16\lambda}.$$

This shows that $E_A^* = \left(E_B^S\right)^* = \left(\frac{4X}{\lambda}\right)$ if the SA-bank is risk-free.

If bank B has bankruptcy risk, then (superscript "R" denotes a risky banking sector)

$$W_B^R = n^B\frac{1}{2}X - \tilde{t}^B\left(n^B\right)^2 z - \lambda E_B^2,$$

where $\tilde{t}_B = \left(\frac{(n^B - E_B)}{2n^B}\right)$. Inserting in W_B^R and maximizing with respect to n_B and E_B leads optimal values of

$$\left(E_B^R\right)^* = \left(\frac{X}{8\lambda - z}\right), \left(n_B^R\right)^* = \left(\frac{4X\lambda}{8z\lambda - z^2}\right), \left(W_B^R\right)^* = \left(\frac{X^2\lambda}{z(8\lambda - z)}\right).$$

It follows that $E_A^* - \left(E_B^R\right)^* = \frac{4X}{\lambda} - \left(\frac{X}{8\lambda - z}\right) = \frac{X(31\lambda - 4z)}{\lambda(8\lambda - z)} > 0$ as positive bankruptcy risk requires that $\lambda > \frac{z}{4}$ which finishes the proof of Part (i).

Parts (ii) and (iii). We first prove that the banking sector with one IRB-bank is risk free if and only if $\lambda < \frac{z}{4}$:

$$\left(W^S\right)^* - \left(W^R\right)^* = \frac{X^2}{16z\lambda}\frac{(z - 4\lambda)^2}{z - 8\lambda}$$

which is positive if and only if $\lambda < \frac{8}{z}$. However, inspection of

$$\tilde{t}_B = \left(\frac{(n_B - E_B)}{2n_B}\right) = \left(\frac{\left(\frac{4X\lambda}{8z\lambda - z^2}\right) - \left(\frac{X}{8\lambda - z}\right)}{2\left(\frac{4X\lambda}{8z\lambda - z^2}\right)}\right) = \left(\frac{1}{8\lambda}(4\lambda - z)\right),$$

shows that the bankruptcy risk is only positive iff $\lambda > \frac{z}{4}$ which is hence the relevant condition. Next, consider welfare with two IRB-banks and no bankruptcy risk. Each IRB-bank will fund $\frac{1}{4}$ uncorrelated projects, so that the maximum number of correlated projects per bank without bankruptcy risk is $n_R^S = \left(E + \frac{1}{2}k - \frac{1}{4}\right)$. Then, social welfare per bank is (subscript "2" denotes two IRB-banks)

$$W_2^S = \frac{1}{4}kX + n_R\frac{1}{2}X - \lambda E^2 = \frac{1}{8}X(4k + 4E - 1) - \lambda E^2,$$

and maximizing with respect to equity leads to

$$\left(E_2^S\right)^* = \frac{X}{4\lambda}, \left(n^S(2)\right)^* = \left(\frac{X + \lambda(2k - 1)}{4\lambda}\right), \left(W_2^S\right)^* = \frac{X(X - 2\lambda + 8k\lambda)}{8\lambda},$$

so that equities and social welfare are identical to the case with one IRB-bank. Furthermore, note that welfare with positive bankruptcy risk and two IRB-banks is lower than with one IRB-bank and positive bankruptcy risk when equities are identical. It follows that this is also the case when equities are optimally differentiated with one IRB-bank. This proves Part (iii). It also implies that, when no bankruptcy risk is optimal with one IRB-bank, it will also be optimal with two IRB-banks which finishes the proof of Part (ii). That two SA-banks are always inferior is obvious. ■

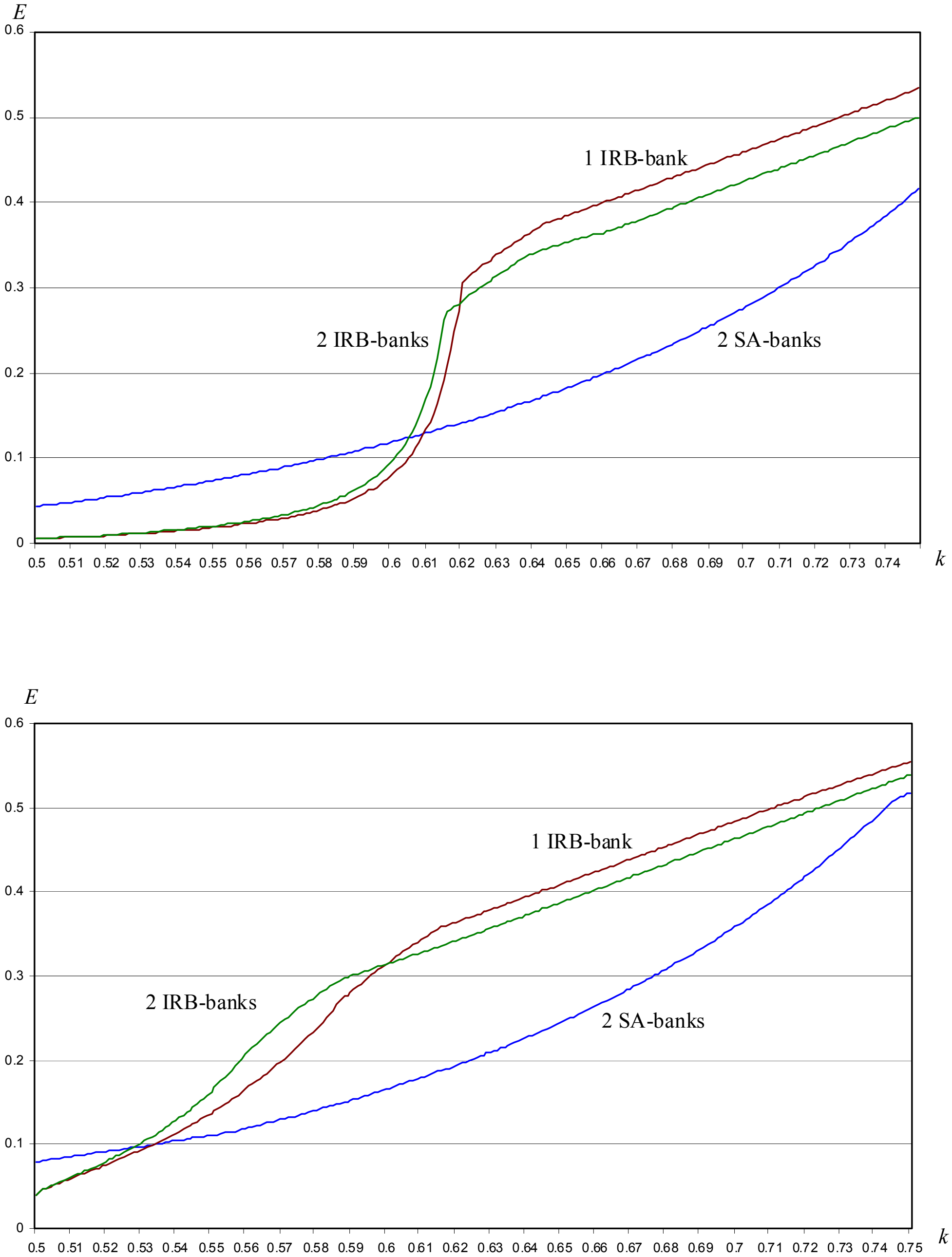

Figure 2: Welfare with unobservable portfolios, under three different bank structure regimes.

This figure shows the social welfare with unobservable portfolios net of investment costs C_i, under the three bank structure regimes of 2 SA-banks, 2 IRB-banks, and 1 IRB-bank, 1 SA-bank. The graph above depicts the outcome for the case of low equity levels, $E = 0.01$; The graph below for relatively high equity levels, $E = 0.08$. The common parameter choices in both graphs are: $X = 2.5$; $z = 1$; and k varies from 0.5 to 0.75.