Correlation products :
A need for clients and a risk to manage for banks

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Gaël Riboulet - Natixis Equity Division

Correlation Markets: Client Motivations

- **Basket / Indices are popular, so are basket / index options**
  - Better Liquidity
  - Portfolio selection objective criteria, Calculation Agent Transparency
  - Largely promoted
  - Could be tailor made, thematic (SRI, Sustainable Investments, Emerging, Water …)

- **Risk profile customization**
  - Exposure to foreign risky assets without bearing the FX risk (Quanto Options)
  - Choice of the Numéraire

- **Tactical Bets**
  - Relative Value Bets (X will do better than Y)
  - Leverage : (neither X nor Y will go down)

- **Benefits of market diversification (Markowitz)**
  - Lower risk for a given expected return
  - Cheaper options

- **Price Optimization**
  - structured products distributors intermediate between dealers and end-users
  - They often need cheap options (particularly true today for 100% guaranteed capital)

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Correlation Products

Typical correlation products that investment banks sell:

- Quanto Products
- Basket Options
- WorstOf Reverse convertible, WorstOf Autocall (« First To Defaults» of the equity world)
- Rainbow Options, Himalaya, Cordillere
- Everest, Altiplanos, Podiums : WorstOf structures for retail aggregators (Germany)
- Fixed Best, Individual Up & Out Calls
- Individually Capped Calls, Cappucino : Basket structures for retails (Benelux, France)
- Worst Of Digits (Spain, Italy)
- ...

⇒ **Almost all of them leave the seller short correlation**

Typical correlation products that investment banks try to sell (but not always with success) :

- Compo options
- Dispersion (Best Minus Worst, Average distance to the Basket)
- Reverse Convertible on Basket
- BestOf Autocall
- ...

=> Those would be natural hedges of the previous ones

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« Correlation » Risk Management

Usual 1D – PnL explanation formula for delta hedged portfolio is easily extrapolated to a multi dimensional framework. Hence assuming :

- diffusion for the underlyings dynamics
- no rates, dividends, repo rates
- perfect delta hedge and (nothing else)

$$dP(X) = \frac{1}{2}\sum X_i X_j \frac{\partial^2 P}{\partial x_i \partial x_j}\left(\frac{d\langle X_i, X_j\rangle}{X_i X_j} - \rho_{ij}\sigma_i\sigma_j dt\right)$$

•As long as option is written on observable (continuous) underlyings, the price of the option is worth its replicating strategy and the **PnL variations are due to difference between implied and realised parameters** (does not apply to most of the credit approach, in particular copula ones)

•Banks trading books are **sensitive to the covariance** between underlyings (rather than correlation itself).

Short correlation positions are actually short covariance positions. They usually observe

$$\frac{\partial^2 P}{\partial x_i \partial x_j} \leq 0$$

=>losses when realised covariance exceed the implied parameters used for pricing, i.e

$$\frac{\Delta X_i \Delta X_j}{X_i X_j} \geq \rho_{ij}\sigma_i\sigma_j dt$$

=> Large correlated moves (both up and down) are painful. This is all the more true as dealers have to remark to the new implied parameters (covariance gets sold higher after large moves)

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« Correlation » Risk Management: 2nd order hedge

Thankfully, dealers do not stay passive !!

Correlation sensitivity and cross gammas are managed.

> ***"Banks now routinely offload the risks they accumulate through their structured products business – volatility, correlation and skew – by selling it on to hedge funds."***
> ***(Risk Mag – July 2007)***

Because of the biased structured product market towards long correlation products, the implied correlation trades significantly above its historical realisations (in average). Multi-strategy Hedge funds are willing to carry that risk and partly offset exotic desks positions.

Typical instruments traded to hedge correlation risk:

- Dispersion trades :
  - **Call Basket vs Basket Calls** (dealers' favourite because it hedges the covariance risk)
  - Index option vs components options
  - through Variance Swaps market (tedious to implement)
- Correl Swap: average of realised minus implied correlations

Drawbacks:

- Macro hedges : mismatch between traded bespoke portfolio and book exposures
- local hedges only : do not match the same risk profile dynamics

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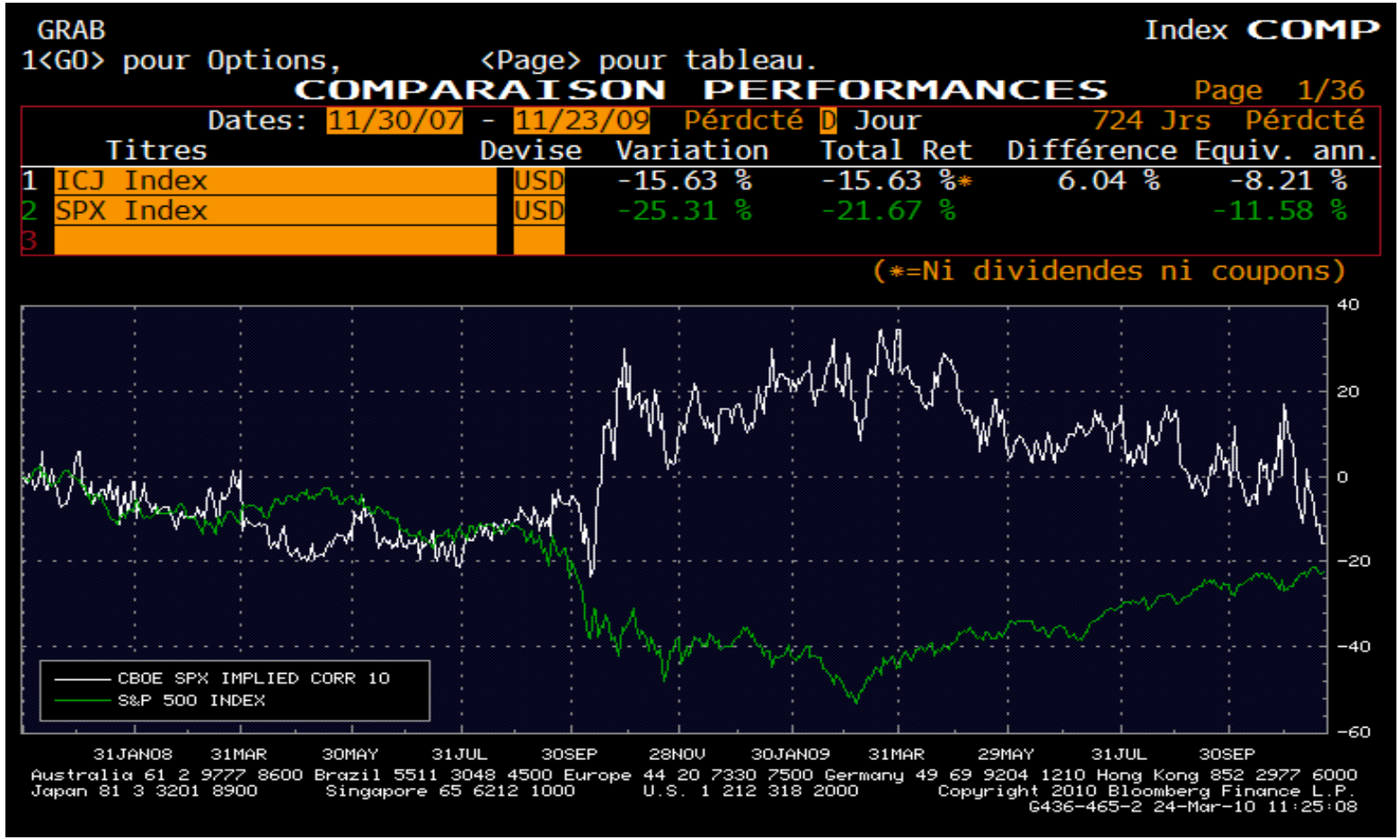
« Correlation » Dynamics and Correlation Skew

Equity correlation is anti-correlated to the Equity markets



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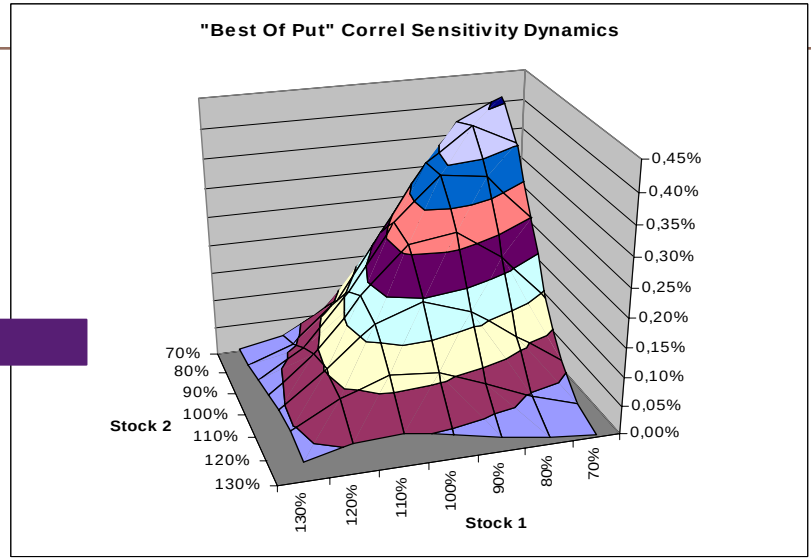
Correlation Skew Products

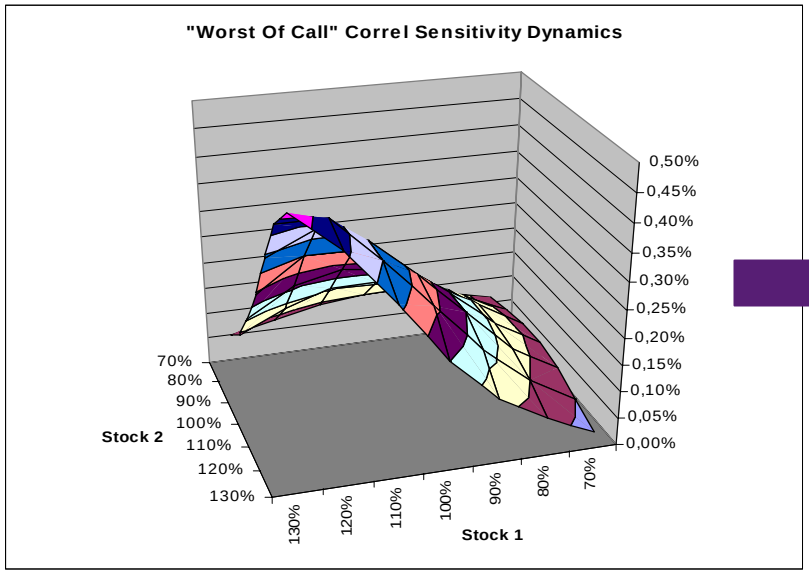
Case Study: Short Best Of Put **(Dealer aversion)**

MAX(0%, 100% - MAX(Perf 1 , Perf 2))

Initial correlation sensitivity increases in bearish market and decreases in bullish market.

=> Best Of Put is long correlation Skew





Case Study: Short Worst Of Call **(Dealer friendly)**

MAX(0%, MIN(Perf 1 , Perf 2) -100%)

Initial correlation sensitivity decreases in bearish market and increases in bullish market.

=> Worst Of Call is short correlation Skew

One looks like a natural hedge for the other as far as correl dynamics are concerned!

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