Stress Testing & Reverse Stress Testing

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Defining halfspace depth

Given $\boldsymbol{y} \in \mathbb{R}^d$, and a direction $\boldsymbol{u} \in \mathbb{R}^d$, define the closed half space

$$H_{\boldsymbol{y},\boldsymbol{u}} = \{\boldsymbol{x} \in \mathbb{R}^d \text{ such that } \boldsymbol{u}'\boldsymbol{x} \leq \boldsymbol{u}'\boldsymbol{y}\}$$

and define depth at point $\boldsymbol{y}$ by

$$\text{depth}(\boldsymbol{y}) = \inf_{\boldsymbol{u},\boldsymbol{u}\neq\boldsymbol{0}} \{\mathbb{P}(H_{\boldsymbol{y},\boldsymbol{u}})\}$$

i.e. the smallest probability of a closed half space containing $\boldsymbol{y}$.

The empirical version is (see Tukey, 1975)

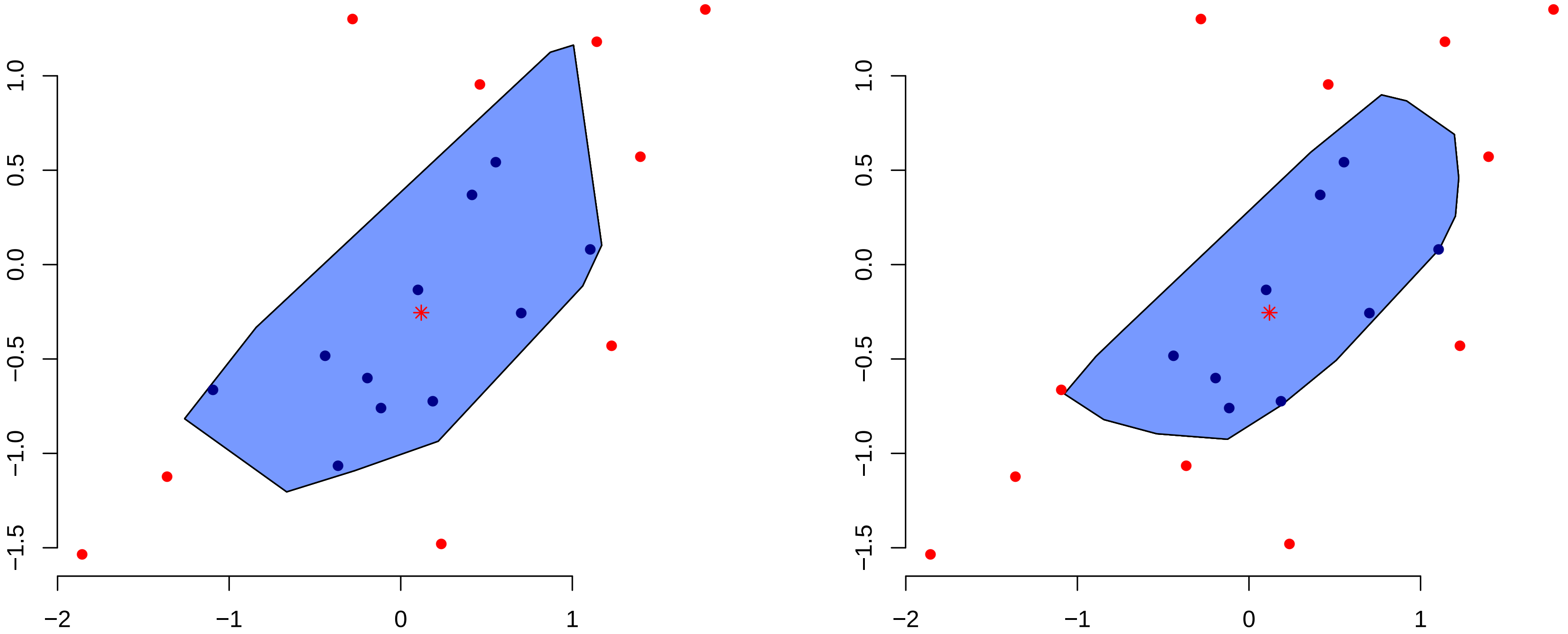
$$\text{depth}(\boldsymbol{y}) = \min_{\boldsymbol{u},\boldsymbol{u}\neq\boldsymbol{0}} \left\{ \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}(\boldsymbol{X}_i \in H_{\boldsymbol{y},\boldsymbol{u}}) \right\}$$

For $\alpha > 0.5$, define the depth set as

$$D_\alpha = \{\boldsymbol{y} \in \mathbb{R} \in \mathbb{R}^d \text{ such that } \geq 1 - \alpha\}.$$

The empirical version is can be related to the bagplot (Rousseeuw & Ruts, 1999).

Empirical sets extremely sentive to the algorithm



where the blue set is the empirical estimation for D_α, $\alpha = 0.5$.

The bagplot tool

The depth function introduced here is the multivariate extension of standard univariate depth measures, e.g.

$$\text{depth}(x) = \min\{F(x), 1 - F(x^-)\}$$

which satisfies $\text{depth}(Q_\alpha) = \min\{\alpha, 1-\alpha\}$. But one can also consider

$$\text{depth}(x) = 2 \cdot F(x) \cdot [1 - F(x^-)] \text{ or } \text{depth}(x) = 1 - \left|\frac{1}{2} - F(x)\right|.$$

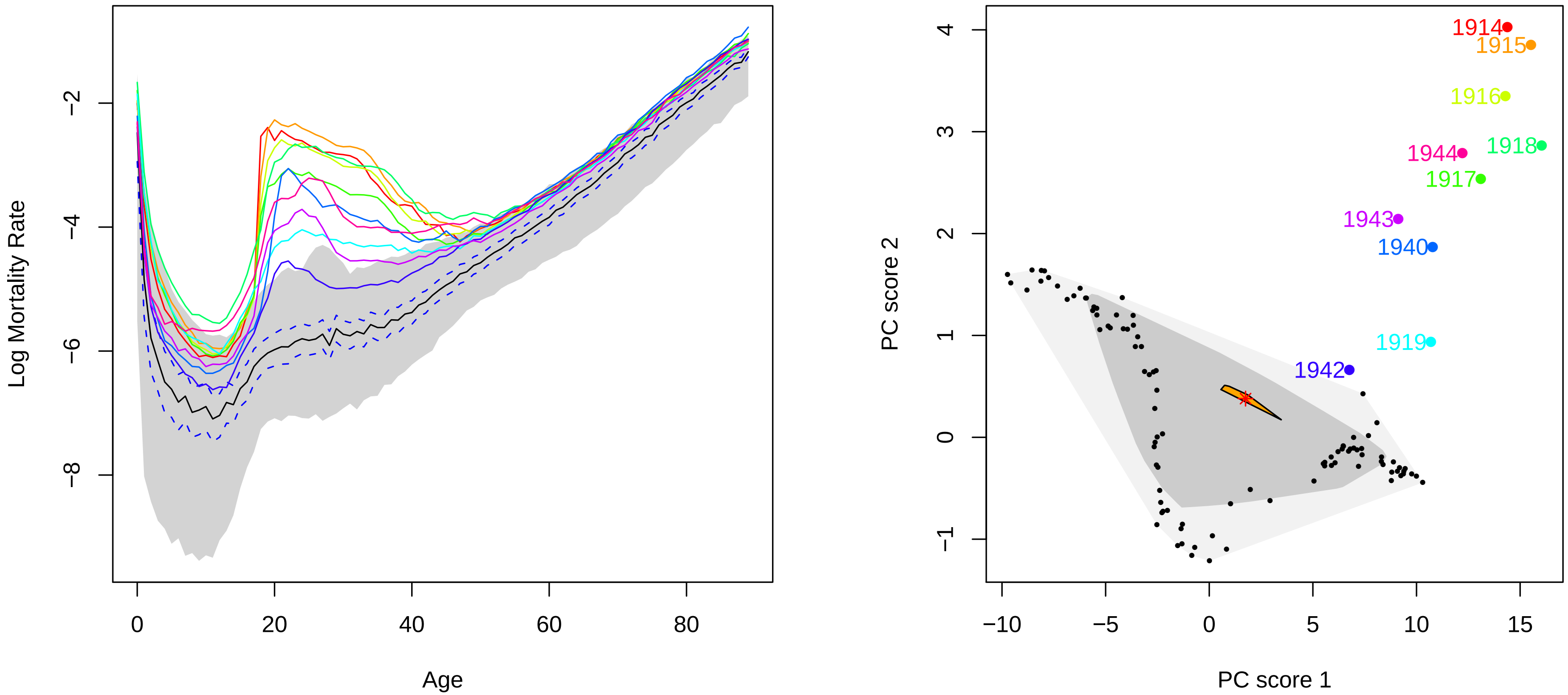
Possible extensions to *functional bagplot*. Consider a set of functions $f_i(x)$, $i = 1, \cdots, n$, such that

$$f_i(x) = \mu(x) + \sum_{k=1}^{n-1} z_{i,k}\varphi_k(x)$$

(i.e. principal component decomposition) where $\varphi_k(\cdot)$ represents the eigenfunctions. Rousseeuw et al., 1999 considered bivariate depth on the first two scores, $\boldsymbol{x}_i = (z_{i,1}, z_{i,2})$. See Ferraty & Vieu (2006).

The bagplot tool for mortality models

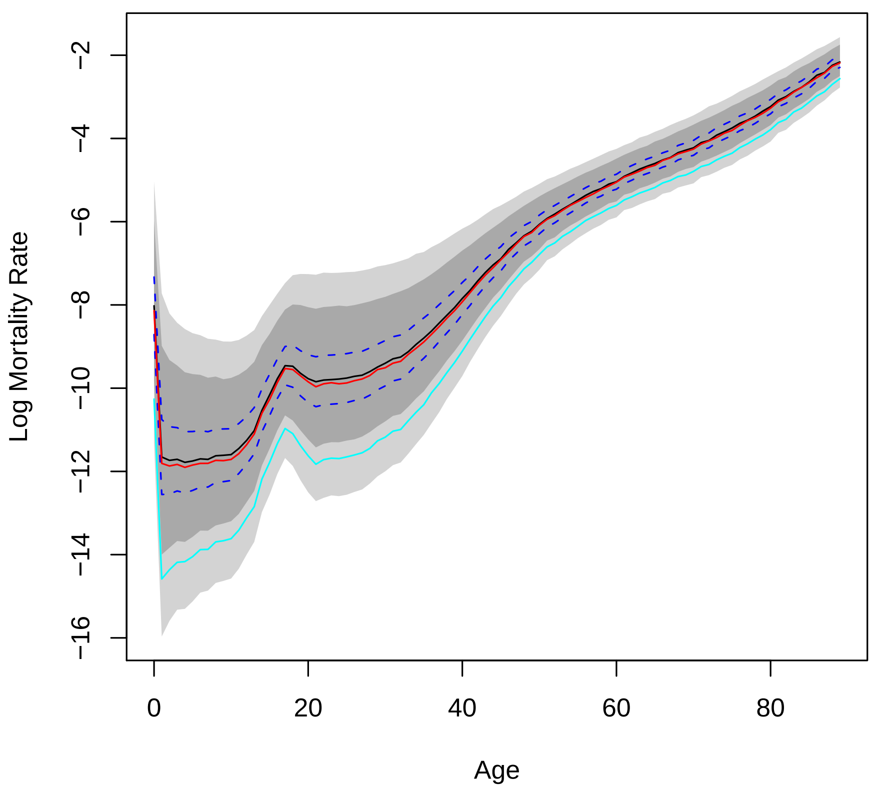
On the a French dataset, we have the following *past* outliers,

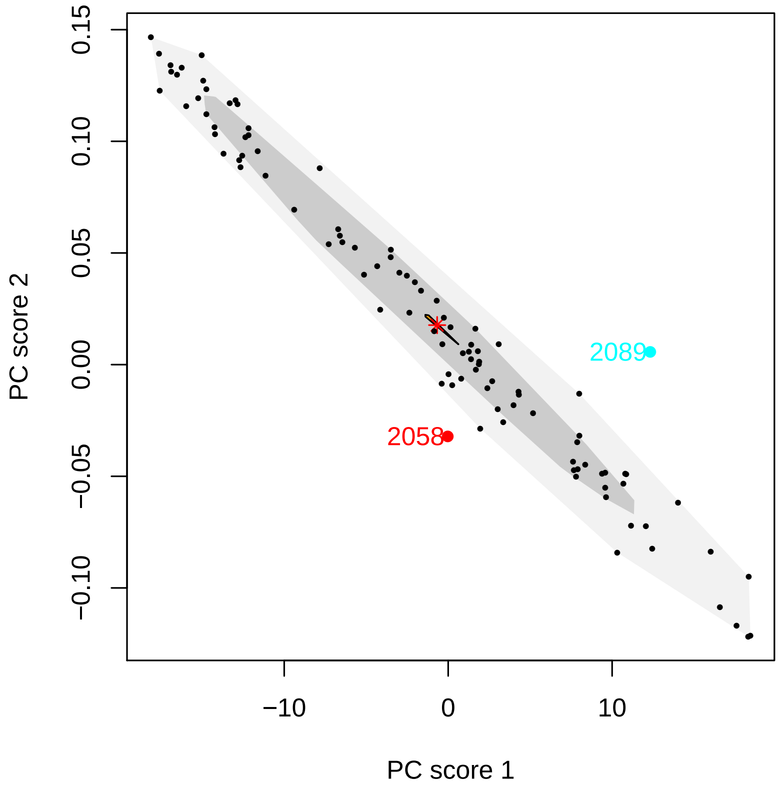


(here male log-mortality rates in France from 1899 to 2005).

The bagplot tool for mortality models

Using functional bagplot techniques it is also possible to identify outliers in stochastic scenarios,





Further references

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