Improved Modeling of Double Default Effects in Basel II - An Endogenous Asset Drop Model without Additional Correlation

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Agenda

Double Default Effects and Basel II

IRB Treatment of Double Default Effects

Asset Drop Model

Summary

Credit Risk in Basel II

Internal Ratings Based (IRB) approach under Pillar 1 of Basel II:

Benchmark model to quantify minimal capital requirements for portfolio credit risk

- since 2007 binding for all banks in the European Union
- minimal capital requirement is the 99.9% value-at-risk of the credit portfolio loss distribution
- based on a conditional independence framework: Asymptotic Single Risk Factor (ASRF) model by Gordy (2003) where all idiosyncratic risk is assumed to be diversified away
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1. the obligor defaults AND
2. the guarantor defaults. Thus: **“double default”**

Hedging Instruments: Credit Derivatives such as CDS, collateral securitization, guarantees...

Treatment of Double Default Effects in Basel II:

- Original New Basel Accord (2003): Substitution approach
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2. propose a new asset drop model that addresses all mentioned weaknesses and which is
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Additional Correlation Approach under Basel II

The normally distributed asset returns r_n and r_{g_n} of obligor n and its guarantor are no more conditionally independent on the systematic risk factor X but

$$r_n = \sqrt{\rho_n}X + \sqrt{1-\rho_n}\left(\sqrt{\psi_{n,g_n}}Z_{n,g_n} + \sqrt{1-\psi_{n,g_n}}\epsilon_n\right)$$

ρ_n : asset correlation of obligor n
ψ_{n,g_n} : sensitivity of both n and g_n to stochastic factor Z_{n,g_n}
ϵ_n : idiosyncratic risk factor of obligor n.

This implies the double default probability

$$\begin{aligned}\mathbb{P}(\mathrm{DD}) &:= \mathbb{P}\left(\{\text{default of obligor n}\} \cap \{\text{default of guarantor } g_n\}\right)\\ &= \Phi_2\left(\Phi^{-1}(\mathrm{PD}_n), \Phi^{-1}(\mathrm{PD}_{g_n}); \rho_{n,g_n}\right).\end{aligned}$$

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Criticism of the additional correlation approach

1. Correlation, a symmetric measure of dependency, is used to describe an *asymmetric relationship*
2. What is an appropriate value for ρ_{n,g_n}?
   $\longrightarrow$ In Basel II set $\rho_{n,g_n} \equiv 0.5$ for all n and g_n.
   $\longrightarrow$ Grundke (2008) empirically evaluates this assumption
3. *Additional* correlation directly violates the conditional independence assumption of the ASRF model
4. Assumes that all guarantors are a) *distinct* and b) *external* to the portfolio
   $\longrightarrow$ no reflection of overly excessive contracting of the same guarantor
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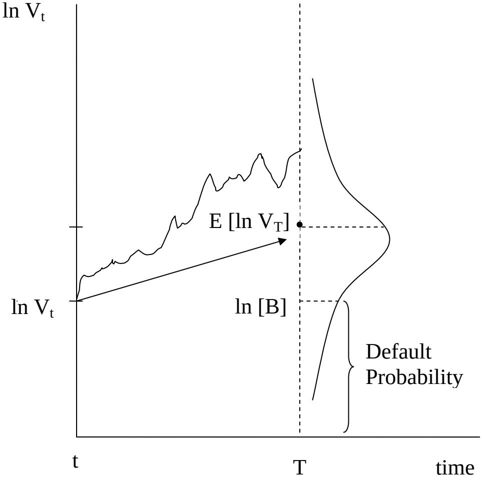
Motivation for Asset Drop Model: Merton Model



Asset Drop Model

Idea: Adjust PD_{g_n} appropriately to *effective default probability* PD'_{g_n}.

Within a structural model of default:

$$\mathrm{PD}_{g_n} = \mathbb{P}(V_{g_n}(T) < B_{g_n}),$$

$V_{g_n}(t)$: total asset value of g_n in period t, B_{g_n} : default threshold.

Denote with $\hat{E}_{n,g_n}$ the nominal g_n guarantees for n. Then

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$\longrightarrow$ Within Merton's model:

$$\mathrm{PD}'_{g_n} = 1 - \Phi\left(\frac{\ln\left(\frac{V_{g_n}(0)}{B_{g_n}+\hat{E}_{n,g_n}}\right) + (r - \frac{n}{2}\sigma_{g_n}^2)T}{\sigma_{g_n}\sqrt{T}}\right). \quad (2)$$

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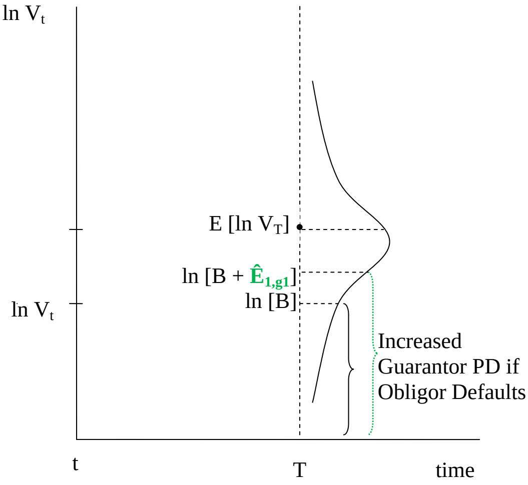
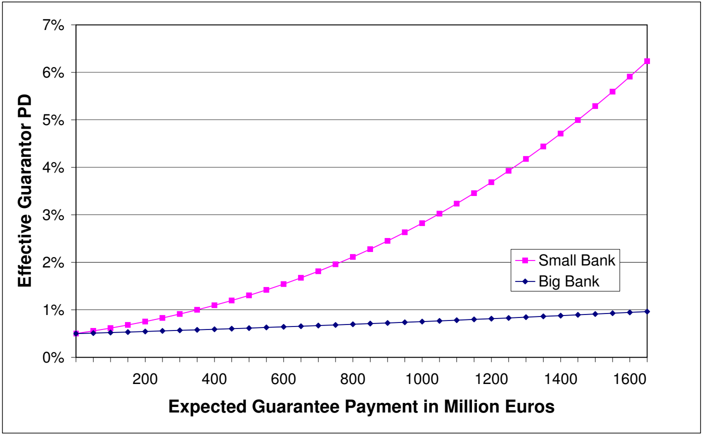


Example 1: PD increase

Consider two guarantors g_1 ("big bank") and g_2 ("small bank").



Here: $V_{g_1}(0) = 50$ and $V_{g_2}(0) = 10$ billion Euros, respectively, $\sigma^2_{g_1} = \sigma^2_{g_2} = 30\%$, $T = 1$, $r = 0.02\%$ and $\text{PD}_{g_1} = \text{PD}_{g_2} = 0.5\%$ (implies $B_{g_1} = 22.5$ and $B_{g_2} = 4.5$ billion Euros.)

Treatment of Different Hedging Constellations

⟶ Convexity punishes overly excessive contracting of the same guarantor

⟶ Treatment of guarantor within the portfolio: Joint loss distribution L_{1,g_1} of obligor 1 and its guarantor g_1:

$$\mathbb{P}(L_{1,g_1} = l) = \begin{cases} \mathrm{PD}'_{g_1}\,\mathrm{PD}_1 & \text{for } l = s_1\,\mathrm{ELGD}_1\,\mathrm{ELGD}_{g_1} + s_{g_1}\,\mathrm{ELGD}_{g_1} \\ \mathrm{PD}_{g_1}(1-\mathrm{PD}_1) & \text{for } l = s_{g_1}\,\mathrm{ELGD}_{g_1} \\ (1-\mathrm{PD}'_{g_1})\,\mathrm{PD}_1 + (1-\mathrm{PD}_{g_1})(1-\mathrm{PD}_1) & \text{for } l = 0. \end{cases}$$

implies

$$\mathbb{E}[L_{1,g_1}] = s_{g_1}\,\mathrm{ELGD}_{g_1}\overbrace{\mathrm{PD}_{g_1}(1+\mathrm{PD}_1\,\lambda_{1,g_1})}^{\text{adjusted }\mathrm{PD}_{g_1}} + s_1\,\mathrm{ELGD}_1\,\mathrm{ELGD}_{g_1}\overbrace{\mathrm{PD}_1\,\mathrm{PD}'_{g_1}}^{\mathbb{P}(\mathrm{DD})}$$

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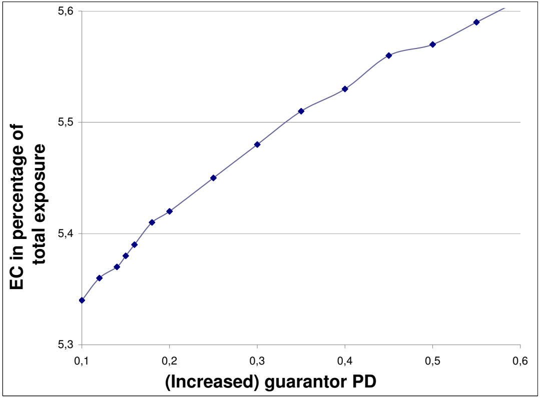
Example 2: Economic Capital (EC)

With IRB treatment of double default effects: 5.40% of total exposure (99.9% VaR) level. With asset drop technique:



Portfolio with 110 obligors, each has exposure 1, maturity 1 year. The first ten are hedged by the last ten (guarantors are in the portfolio). For obligors PD = 1%, LGD = 45%. For guarantors PD = 0.1%, LGD = 100%.

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We criticize the IRB double default treatment for

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