DISCUSSION ON THE PAPERS

"Inflation-Hedging Portfolios in Different Regimes"

by M. BRIERE and O. SIGNORI

and

"Optimal Investment and Capital Management Decisions for a Non-Life Insurance Company"

by S. MANKAÏ and C. BRUNEAU

Christian Y. Robert - ENSAE ParisTech

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Portfolio optimization has a long history starting with Markowitz's mean-variance theory.

However the mean-variance theory had some limitations when the random variables of interest follow a non-normal distribution.

Although in this case the expected utility function could be used to optimize portfolio, practitionners have had the tendency during the last fifteen years to keep the concepts of *reward* and *risk* of a portfolio separated.

As with the mean-variance theory, there are efficient portfolios and efficient frontiers, but their characteristics depend on the definition of risk being used.

A *risk-reward* criteria represents the preference relation of an agent (individual investor/financial institution) using the Pareto preference.

- The *reward* function measures the desirability of the portfolio/the retained earning: *reward*$(A) \geq$*reward*(B) implies that A is preferred to B if the investor is indifferent to risk.

- The *risk* function measures the dangerousness of the portfolio/the retained earning: if *reward*$(A)=$*reward*(B), then a risk averse person will select A if *risk*$(A) \leq$*risk*(B).

If the risk function is convex and the reward function is concave with respect to the allocation vector, then the expected utility is only a special case of the risk-reward framework. Moreover the efficient frontier is convex.

As usual, the most difficult task is to select the adequate *risk* and *reward* measures of a portfolio to approximate the investors' behavior accurately.

Some common measures of reward includes:

- the mean,

- the median (however in practice such a choice increases the unreliability of optimization algorithms).

Some common measures of risk includes pseudo-coherent and coherent measures:

- A risk measure is pseudo-coherent if it is homogeneous, monotoneous and invariant by translation.

Example: VaR (not convex in general).

- A risk measure is coherent if it is pseudo-coherent and sub-additive.

Examples: Variance, Expected Shortfall, TailVaR, Conditional Tail expectation, Conditional Var...

Both papers consider a static portfolio optimization problem in a *risk-reward* framework.

- Paper 1 ("Inflation-Hedging Portfolios in Different Regimes") presents the optimal asset allocation of an investor seeking to attain a fixed target for real returns and to hedge inflation risk for a one period, subject to a shortfall probability constraint.

- Paper 2 ("Optimal Investment and Capital Management Decisions for a Non-Life Insurance Company") proposes to optimize in a one period framework the return on risk-adjusted capital (RORAC) of a non-life insurance company subject to a zero-conditional value-at-risk shortfall constraint.

The insurance company has to find simultaneously the optimal investment portfolio and the level of capital in the presence of capital adjustement costs.

Reward measures:

- Paper 1: An investor faces inflation risk and wants to attain a fixed target for real returns of his portfolio. His reward measure is the

$$\text{Expected portfolio return } (\mathbf{w}) \text{ - Expected inflation rate}$$

where $\mathbf{w}$ is the vector of the fractions of capital invested in the assets.

- Paper 2: A non-life insurance company wants to maximize the ratio of the expected net profit to the economic capital

$$\frac{\text{Expected net profit } (\mathbf{x}, \Delta C)}{\text{Economic capital } (\Delta C)}$$

where $\mathbf{x}$ is the vector of the fractions of the economic capital and the premiums invested in the assets and ΔC are the funds obtained by issuing or retiring stocks from the initial capital C. A piecewise linear cost function for ΔC is assumed.

Two cases are considered: ΔC is totally invested in a risk free asset, ΔC is allocated to all the assets.

Risk measures:

- Paper 1: The probability that

$$\text{Portfolio return - Inflation rate} \ \leq \ \text{Target real return}$$

is smaller than α. This is equivalent to

$$VaR(\text{Portfolio return - Inflation rate;}\alpha) \ \leq \ \text{Target real return.}$$

The target real return and α are chosen by the investor.

- Paper 2: The Conditional Tail Expectation (at level α) of the loss function (the difference at the end of the period between the future losses and expenses, and the market value of assets) is smaller than 0:

$$CTE(\text{Loss function;}\alpha) \ \leq \ 0$$

The level α may be imposed by the regulator.

Parameter sets:

- Paper 1: short sales are disallowed ($w_i > 0$), $\sum_i w_i = 1$.

- Paper 2: short sales are disallowed ($x_i > 0$), $\sum_i x_i \leq 1$ (there exists a risk free asset), $C + \Delta C > 0$ (positive economic capital).

Existence of a unique solution:

- Paper 1: not proved (but it is maybe true for the Gaussian case)

- Paper 2:

- ΔC is totally invested in a risk free asset: a unique global optimum (simultaneous and sequential optimization programs)

- ΔC is allocated to all the assets: a unique global optimum (simultaneous and sequential optimization programs).

Simulated data:

- Paper 1:

A VAR model with Gaussian innovations for the asset returns (Cash, Nominal bonds, IL bonds, Equities, Real Estates, Commodities) and state variables (dividend yield, term spread, inflation).

$\Rightarrow$ two periods 1973-1990 (volatile economic environment) and 1991-2009 (more stable economic environment).

- Paper 2:

  - Shifted gamma distributions + a hierarchical copula for the 4 lines of business (Motor, Third party, Fire, Property damage),

  - Gaussian distribution for the two risky assets (CAC 40 Index, a French stock fund),

  - Franck copula between agregate claims and rate of returns of stock fund.

Computational methods:

- Paper 1 and Paper 2: not mentioned.

⇒ Risk gradient optimization (recursive approach, optimization with biased gradient estimators, optimization with unbiased gradient estimators,...)

⇒ Quantile based optimization (Brute force method, Mixed integer programming method, the Greedy linear programming,...)

Instability erratic efficient frontier in the case of the VaR risk measure?

Sensitivity analysis:

- Paper 1: two regimes, number of years for the investment horizon (1, 3, 5, 10, 20, 30 years), parameters (?).

- Paper 2: capital adjustment costs, initial capital level, agregate loss volatility, dependence structure between agregate claims and rate of returns of stock fund.

Comments and questions:

- Paper 1:

  - What is a reasonnable level of required real return?

  - For the 20 next years, will the economic environment change? How to take into account this change?

  - Is the assumption of Gaussian innovations strong?

- Paper 2:

  - Is it also possible to optimize the insurance liabilities (fractions of the premiums for each LoB)?

  - For solvency purposes (Solvency II), $C + \Delta C > 0$ should be replaced by $C + \Delta C > C_0(\mathbf{w})$ where $C_0(\mathbf{w})$ is the minimum capital requirement.

  - In the definition of RORAC, the denominator should be adjusted for risk.