Dynamic Correlation or Tail Dependence Hedging for Portfolio Selection

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Outline

- Motivation and objectives
- The model for asset prices - accounting for extreme asset co-movements through:
  - flexible tail dependence modeling
  - introducing observable factors to drive the dynamics of conditional asset return correlations
- The portfolio choice problem:
  - Market price of risk hedging demands due to increased tail dependence
  - Correlation hedging demands due to observable factors
- Conclusion

Risk factors and portfolio choice

- Identifiable observed factors
  - Replicable factors; traded and not traded factors (market incompleteness)
  - Stylized features of the data
  - Portfolio performance (benchmarking)
- Latent factors
  - account for omitted factors
  - allow for flexible modeling of asset co-movements
- Portfolio choice for traded securities, exposed to both observable and latent risk factors

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- Portfolio choice for traded securities, exposed to both observable and latent risk factors
- Hedging demands against stochastic changes of the conditional correlation of risky securities, driven by latent and observable factors

Asymmetries and downside risk

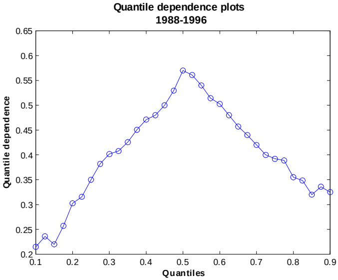
- Probability that assets in a portfolio will jointly decline
  - Is correlation enough?... Tail events (extreme moves) ask for different dependence measures
- Asymmetries:
  - Univariate case: skewness
  - Multivariate case: widespread evidence that correlations are higher in extreme market downturns than in extreme market upturns (Longin and Solnik (2001), Ang and Chen (2002), Poon, Rockinger, Tawn (2004))
  - Theoretical justification of this empirical fact: REE model, Ribeiro and Veronesi (2002)
- Wide-spread evidence that correlation risk is priced (Driessen et al, 2009) and commands sizeable hedging demands (Buraschi et al, 2009)

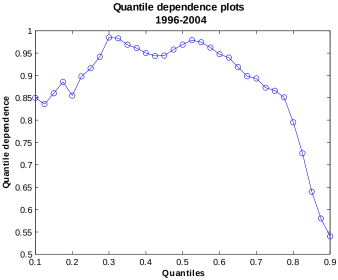
Evidence of dependence asymmetry

A 'near' tail dependence measure (Coles, Currie and Tawn, 1999):

the probability that one variable exceeds a certain quantile given that the other has exceeded it:

Plots of quantile dependence for the de-trended log-prices of S&P 500 vs. NASDAQ for the 1988-1996 and 1996-2004 subperiods.





Objectives

Model extreme asset co-movements, driven by latent and observable factors:

- We propose a model that is able to accommodate an extremal dependence structure in two methodologically distinct ways:
  - Through the **stationary distribution** of the process for asset prices (tail dependence)
  - Through a **dynamic conditional correlation** specification, driven by latent and observable factors
- . . . that also models in a tractable way the univariate asset return properties
- . . . while keeping a continuous time complete market setup for tractable portfolio solutions

Objectives

Correlation hedging due to latent and observable factors:

- Detect changes in the **portfolio composition**: a shift towards the risk-free asset in turmoil periods of increased dependence in the extreme
- Determine the **loss** in terms of wealth resulting from disregarding dependence during extreme return realizations
- Isolate **correlation hedging demands** due to observable factors that drive dependence between the assets in the portfolio

Related literature

- Modeling comovement asymmetries
  - GARCH-copula (Jondeau and Rockinger (2002,2005), Patton (2004))
  - Regime-Switching (Ang and Chen (2002), Ang and Bekaert (2002), Chesnay and Jondeau (2001))
  - Systemic jumps (Das and Uppal (2003))
  - Stock return correlations and the phase of the business cycle: Ledoit et al. (2003), Erb et al. (1994)
- Portfolio choice
  - Unconditional allocation (Patton (2004))
  - Conditional allocation and the hedging demands (Ang and Bekaert (2003), Das and Uppal (2004), Liu, Longstaff, and Pan (2003))
  - Correlation hedging: Buraschi et al. (2007)
- Solution methodology:
  - Monte Carlo with Malliavin Derivatives: Detemple, Garcia and Rindisbacher (2003)

The model for stock prices

- The general model with latent and observable factors:

$$S_{it} = S_{0t} \exp\left(\alpha_i X_t + \beta_i F_t + k_i t\right) \qquad , i = 1...d$$

where

$$\begin{aligned} \text{latent factors} \quad &: \quad dX_t = \mu^X\left(X_t, F_t\right) dt + \Lambda^X\left(X_t, F_t\right) dW_t^X \\ \text{observed factors} \quad &: \quad dF_t = \mu^F\left(F_t\right) dt + \Lambda^F\left(F_t\right) dW_t^F \end{aligned}$$

- An intuitive analogy with GBM
- Incorporate thick tails and dependence in extreme realizations in the stationary distribution of the state variable process
- Significance from the perspective of an investor with a long-term investment horizon

The model for stock prices (cont.)

Incorporating tail dependence

Need a link between the stationary distribution of the process and its diffusion specification (Chen, Hansen, Scheinkman (2005)):

$$dX_t = \mu^X (X_t, F_t)\, dt + \Lambda^X (X_t, F_t)\, dW_t^X$$

$$\mu_{ij}^X = \frac{1}{2q} \sum_{i=1}^{d} \frac{\partial (v_{ij} q)}{\partial x_i} \qquad \Sigma = \Lambda^X \left(\Lambda^X\right)' \text{ with entries } v_{ij}$$

Thick tails through the marginals (NIG) and tail dependence through the (asymmetric) copula specification of the stationary density:

$$q\left(x_{1,\dots,x_d} | \theta, \phi\right) \equiv \widetilde{c}\left(x_{1,\dots,x_d} | \theta\right) \prod_{i=1}^{d} \widetilde{f}^i \left(x_i | \phi\right)$$

Conditional volatility and correlation dynamics:

$$v_{ij} = \rho_{ij} (X_t, F_t)\, \sigma_i^X \sigma_j^X \qquad \sigma_i^X = \sigma_i \left[\widetilde{f}^i (x_i)\right]^{-\frac{1}{2}\kappa_i}$$

- Correlation hedging demands implied by:

1. latent factors (with tail dependence)
2. observable factors (macroeconomic conditions (CFNAI index) and market-wide volatility (the VIX)

$$\text{conditional correlation} \quad : \quad \rho_{ij}(X_t, F_t) = A\left(h_{ij}(X_t, F_t)\right)$$

$$\begin{aligned} h_{ij}(X_t, F_t) &= \gamma_{ij,0} + \gamma_{ij,1} \max\left(\sigma_1^X(X_t), \ldots, \sigma_d^X(X_t)\right) \\ &\quad + \gamma_{ij,2} \prod_{k=1}^{d} \widetilde{F}(X_{kt}) \\ &\quad + \gamma_{ij,3} F_t^{VIX} + \gamma_{ij,4} F_t^{CFNAI} \end{aligned}$$

- Benchmark: constant conditional correlation: $\gamma_{ij,1} = \gamma_{ij,2} = \gamma_{ij,3} = \gamma_{ij,4} = 0$

Portfolio choice in the presence of extremal dependence

The portfolio decomposition formula:

- **Mean-variance term** (MV) isolated from intertemporal hedges
- Explicit **interest rate hedging demands** (IRH) and **market price of risk hedging demands** (MPRH) in terms of conditional expectations of the state variables Y_t and their Malliavin derivatives

$$\alpha_t = \left(\Lambda_t\left(Y_t\right)'\right)^{-1}\left[\begin{array}{c} \frac{1}{R\left(\omega_t\right)}\theta\left(Y_t\right)MV\left(Y_t,\omega_t\right) - IRH\left(Y_t,\omega_t\right) \\ -MPRH\left(Y_t,\omega_t\right) \end{array}\right]$$

- **Effect of the dependence structure:** in the MPR hedge through the process of the market price of risk θ and its Malliavin derivative:

$$\begin{aligned} &\mathrm{MPRH}(Y_t,\omega_t) \\ &= \mathrm{E}_t\left[\xi_{t,T}\frac{\omega_T}{\omega_t}\left(1 - R\left(\omega_T\right)^{-1}\right) I_{\omega_T>0}\int_t^T \left(dW_s + \theta_s ds\right)' D_t\theta_s\right] \end{aligned}$$

Correlation hedging demands

The market price of risk hedging term:

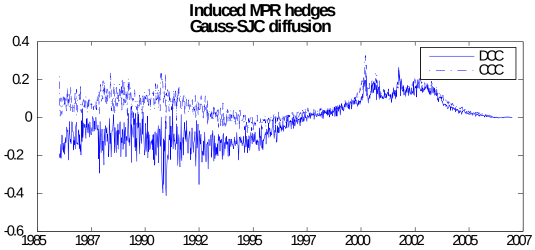
- Driven by the sensitivity of the latent factors X and the observables factors F to uncertainty shocks (i.e. using their Malliavin derivatives):

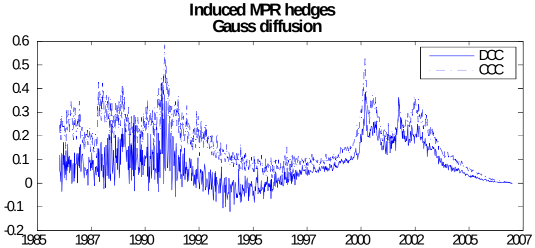
$$
\begin{aligned}
H_t^{\Theta} &= \int_t^T \Psi_s D_t Y_s \\
\text{where } \Psi_t &= (dW_t + \Theta(t, Y_t)ds)^{\intercal} \partial_2 \Theta(t, Y_t) \\
H_{t,T,i}^{\Theta} &= \int_t^T (\Psi_{1,s} D_{i,t} X_{1,s} + \ldots + \Psi_{d,s} D_{i,t} X_{d,s}) \\
&\quad + \int_t^T \left(\Psi_{d+1,s} D_{i,t} F_s^{VIX} + \Psi_{d+2,s} D_{i,t} F_s^{CFNAI} \right)
\end{aligned}
$$

- Explicitly isolate correlation hedging demands due to observable factors

The evolution of portfolio hedging terms

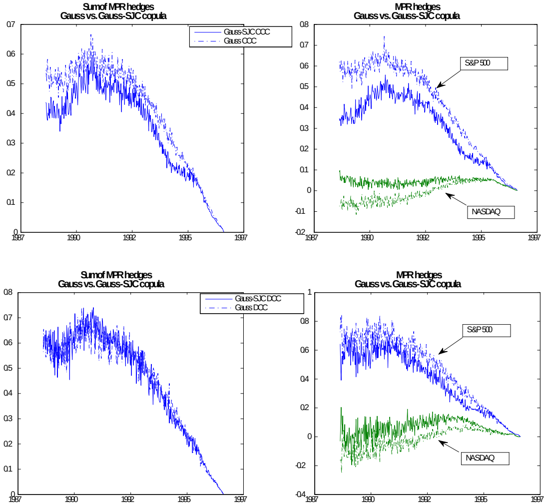
Induced hedging demands





The evolution of portfolio hedging terms

Along realized paths: calm sub-period

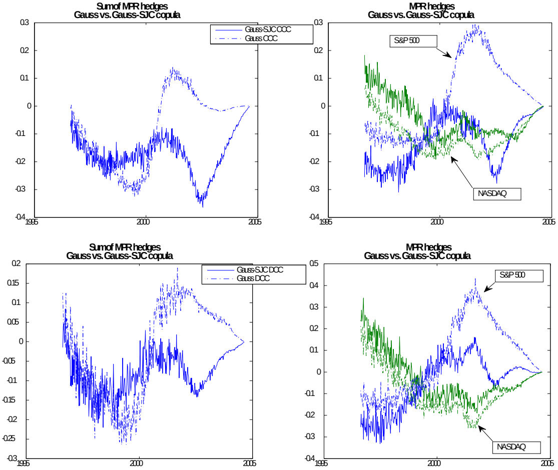


CCC

DCC

The evolution of portfolio hedging terms

Along realized paths: turmoil sub-period



CCC

DCC

Hedging effect of tail dependence

out-of-sample

Horizon	5 years		
	MPRH Sum	CorrH F^M	CorrH F^V*100
CRRA, γ=5	0.9605	-0.0127	0.1512
CRRA, γ=10	0.8905	0.0237	-0.2819
HARA, γ=5,b=-0.2	0.9353	-0.0026	0.0358
HARA, γ=10,b=-0.2	0.8752	0.0292	-0.3400

Gaussian DCC

Horizon	5 years		
	MPRH Sum	CorrH F^M	CorrH F^V*100
CRRA, γ=5	0.8419	-0.0245	0.1429
CRRA, γ=10	0.7708	0.0226	-0.1317
HARA, γ=5,b=-0.2	0.8166	-0.0110	0.0664
HARA, γ=10,b=-0.2	0.7552	0.0303	-0.1761

Gaussian − SJC DCC

Correlation hedging effect

out-of-sample

Horizon	1 year	5 years	
	$\sum$ MPRH	$\sum$ MPRH	
CRRA, γ=5	0.1056	0.8623	*Gaussian – SJC CCC*
CRRA, γ=10	0.0643	0.7341	
HARA, γ=5,b=-0.2	0.0873	0.8201	
HARA, γ=10,b=-0.2	0.0539	0.7113	

Horizon	1 year	5 years	
	$\sum$ MPRH	$\sum$ MPRH	
CRRA, γ=5	0.0268	0.8419	*Gaussian – SJC DCC*
CRRA, γ=10	0.0140	0.7708	
HARA, γ=5,b=-0.2	0.0206	0.8166	
HARA, γ=10,b=-0.2	0.0107	0.7552	

The cost of disregarding dependence between extreme realizations

The Certainty Equivalent Cost is given in cents per dollar. Investment horizon is 5 years.

Panel A. The cost of disregarding tail dependence						
	(Gaussian alternative, DCC)			(Gaussian alternative, CCC)		
	HARA b=-0.2	CRRA b=0	HARA b=0.2	HARA b=-0.2	CRRA b=0	HARA b=0.2
$\gamma = 2$	1.3153	1.5158	1.7162	3.2467	3.8692	4.4916
$\gamma = 6$	0.3912	0.4619	0.5326	0.4602	0.6562	0.8523
$\gamma = 10$	0.1902	0.2327	0.2751	0.0000	0.0507	0.1664

NB. The true data generating process has both tail dependence and dynamic correlation

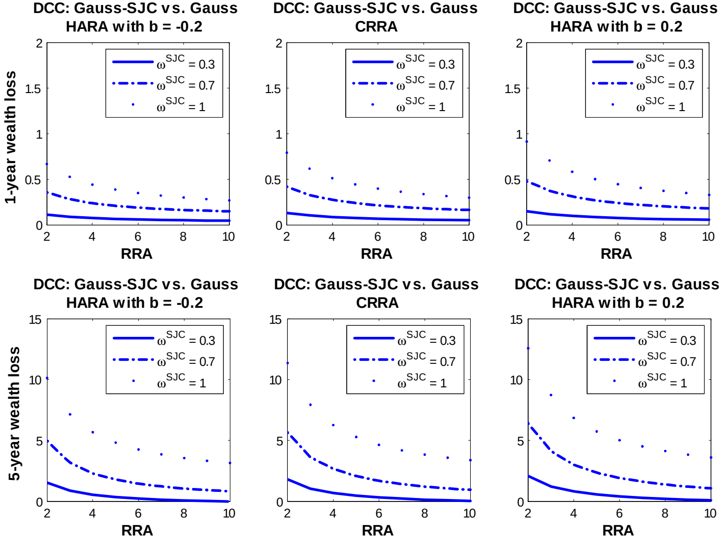
The cost of disregarding dependence between extreme realizations

The Certainty Equivalent Cost is given in cents per dollar. Investment horizon is 5 years.

Panel B. The cost of disregarding asymmetric tail dependence						
	(Student's t alternative, DCC)			(Student's t alternative, CCC)		
	HARA	CRRA	HARA	HARA	CRRA	HARA
	b=-0.2	b=0	b=0.2	b=-0.2	b=0	b=0.2
$\gamma = 2$	0.1886	0.1696	0.1506	0.5891	0.6486	0.7081
$\gamma = 6$	0.4259	0.4403	0.4546	0.3960	0.4260	0.4559
$\gamma = 10$	0.3999	0.4106	0.4213	0.3224	0.3411	0.3598

NB. The true data generating process has both tail dependence and dynamic correlation

...increases with growing dependence in the extremes



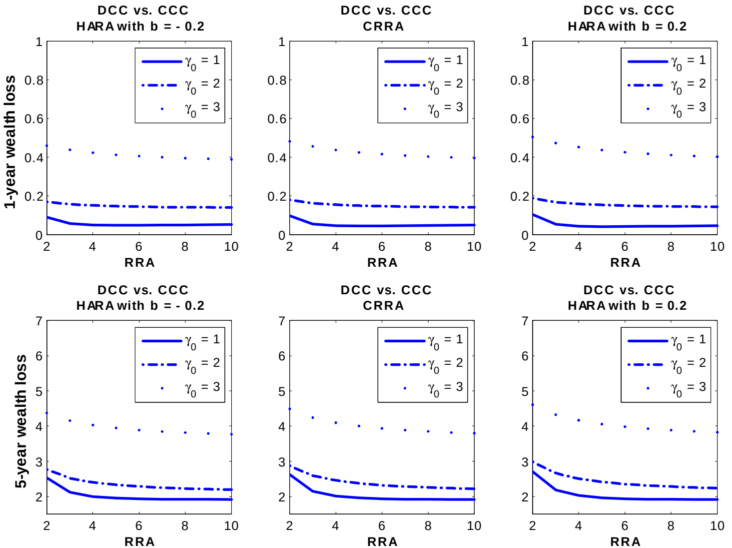
The cost of disregarding dynamics of conditional correlation

The Certainty Equivalent Cost is given in cents per dollar. Investment horizon is 5 years.

Panel C. The cost of disregarding DCC			
	HARA, $b = -0.2$	CRRA	HARA, $b = 0.2$
$\gamma = 2$	2.3054	2.4039	2.5024
$\gamma = 6$	1.7983	1.8216	1.8449
$\gamma = 10$	1.7289	1.7419	1.7549

NB. The true data generating process has both tail dependence and dynamic correlation

...increases with increased levels of conditional correlation



Conclusion

What we have found so far:

- The portfolio solution methodology allows us to isolate:
  - correlation hedging demands due to observable factors
  - the impact of tail dependence on market price of risk hedging terms
- Correlation hedging demands and intertemporal demands due to high level of tail dependence have a distinct impact on the optimal portfolio behavior:
  - both in terms of portfolio composition
  - and economic significance

Further developments

- Introduce stochastic variations in the dependence structure of the process for the latent state variables, driven by observable factors
- Traded and non-traded risk factors (market incompleteness)
- Systematic correlation risk factor