

Learning From Stock Prices

And Economic Growth

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Motivation

- What is the role of the stock market for economic growth?
 - Focus on informational frictions
 - Levine (1997): “Existing theories have not yet assembled the links in the chain from the functioning of stock markets, to information acquisition, and finally to aggregate long-run economic growth”
- Embed a stock market with informative stock prices into a neoclassical growth economy
 - Study the production of information and its dissemination through stock prices
 - Characterize various aspects of growth path: level and growth rate of income, precision of information, efficiency of investments, TFP, degree of economic specialization, wealth inequality, trading volume, liquidity, volatility

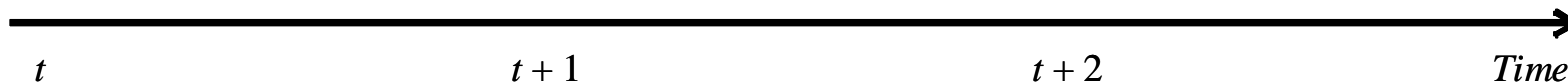
Sketch of the Model

- Backbone of the model is a neoclassical OLG model
 - Each generation = a continuum of agents
 - Endowed with 1 unit of labor time, and 1 unit of free time (for leisure or learning)
 - Two sectors:
 - * 1 final good (invest at $t \rightarrow$ yields at t)
 - * M intermediate goods (invest at $t \rightarrow$ yields at $t + 1$)
 - Young work in final good sector, invest wage in intermediate firms through SM
 - Agents derive utility from consumption of final goods and leisure
 - No technological progress, no population growth
- Embed a competitive stock market in the spirit of Grossman and Stiglitz (1980)
 - Intermediate firms (= stocks) are subject to random productivity shocks
 - Investors can acquire private signals about them at the expense of leisure time
 - Signals are revealed through stock prices, but only partially because of noise trading
 - No riskless asset

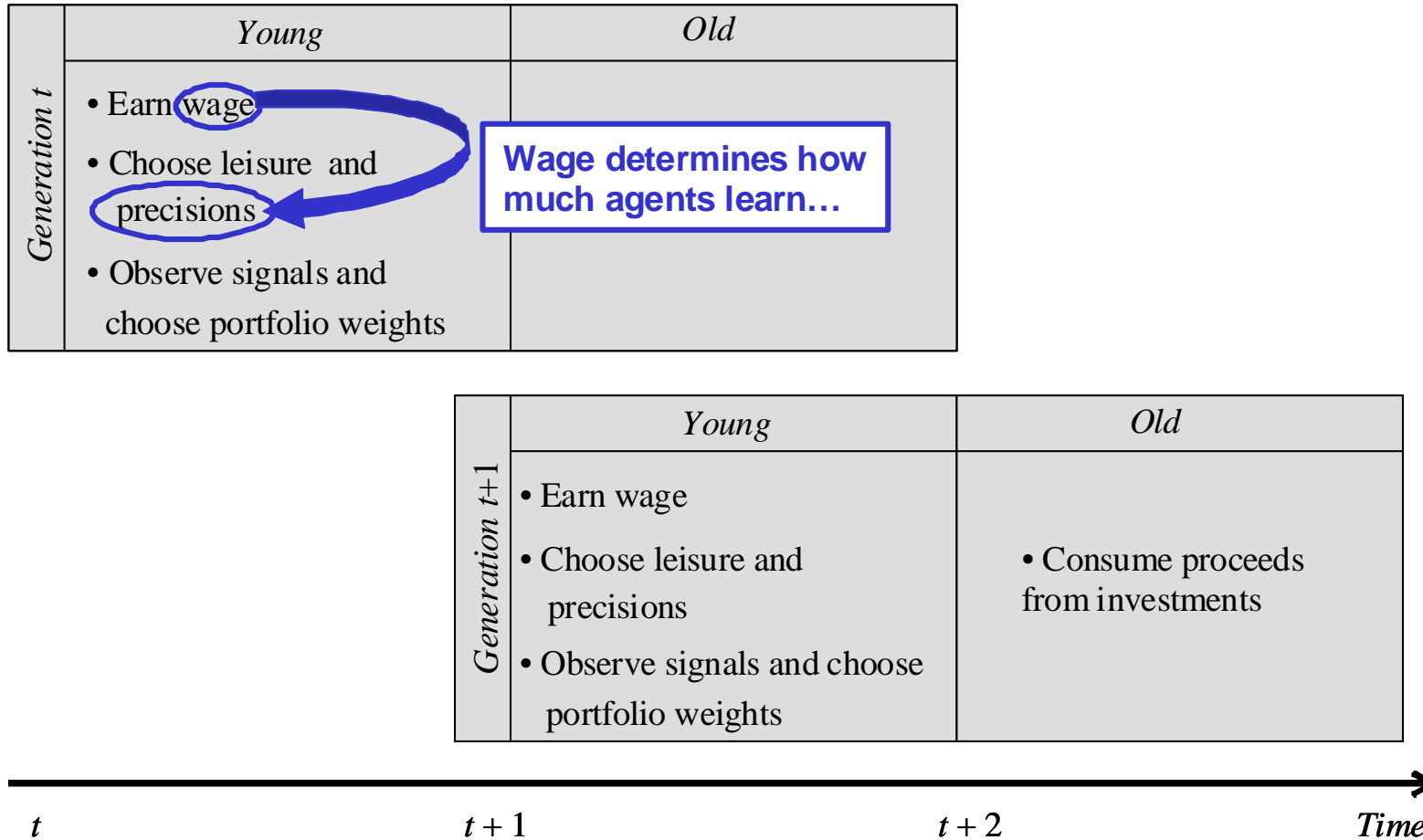
Timing

	<i>Young</i>	<i>Old</i>
<i>Generation t</i>	<ul style="list-style-type: none"> • Earn wage • Choose leisure and precisions • Observe signals and choose portfolio weights 	<ul style="list-style-type: none"> • Consume proceeds from investments

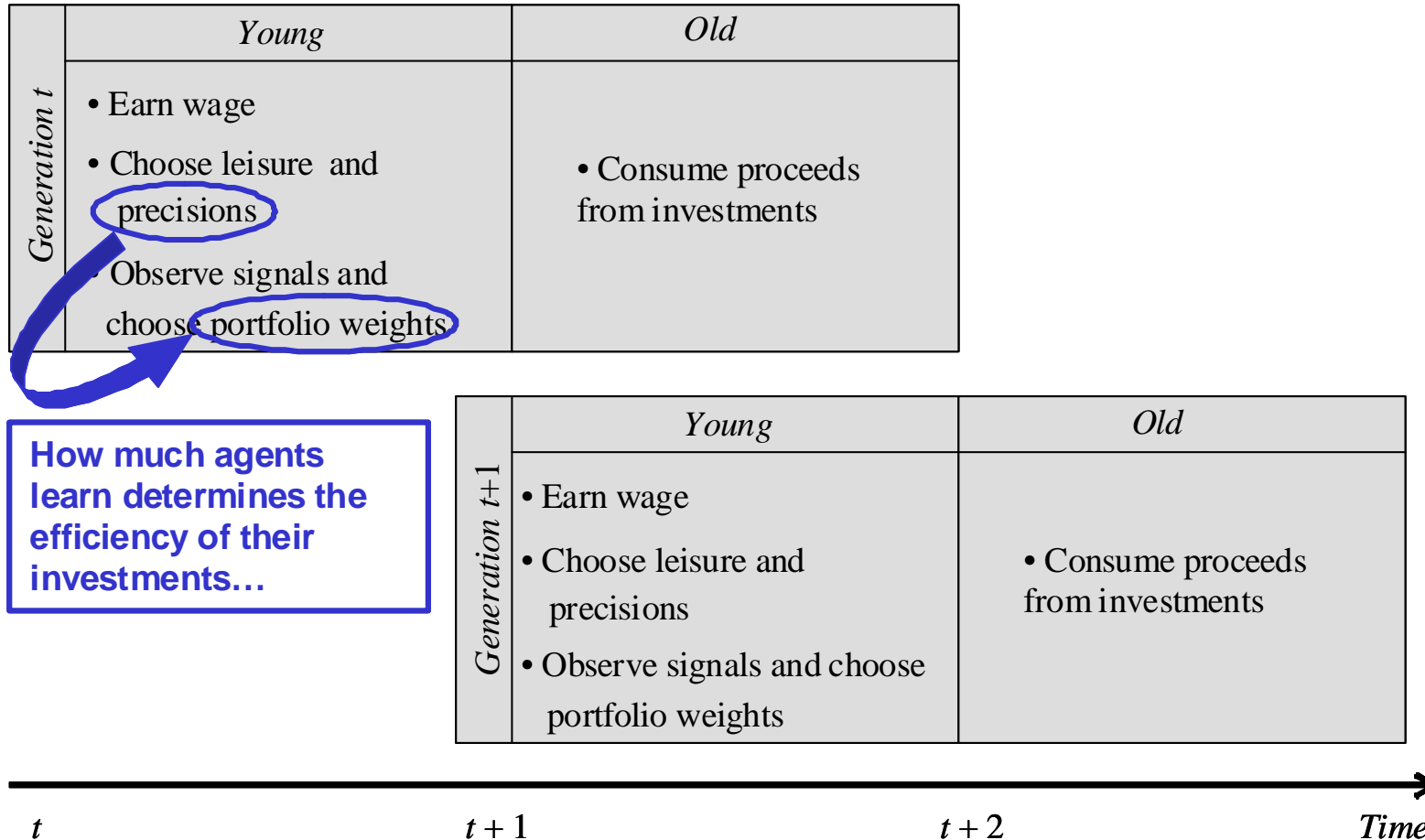
	<i>Young</i>	<i>Old</i>
<i>Generation t+1</i>	<ul style="list-style-type: none"> • Earn wage • Choose leisure and precisions • Observe signals and choose portfolio weights 	<ul style="list-style-type: none"> • Consume proceeds from investments



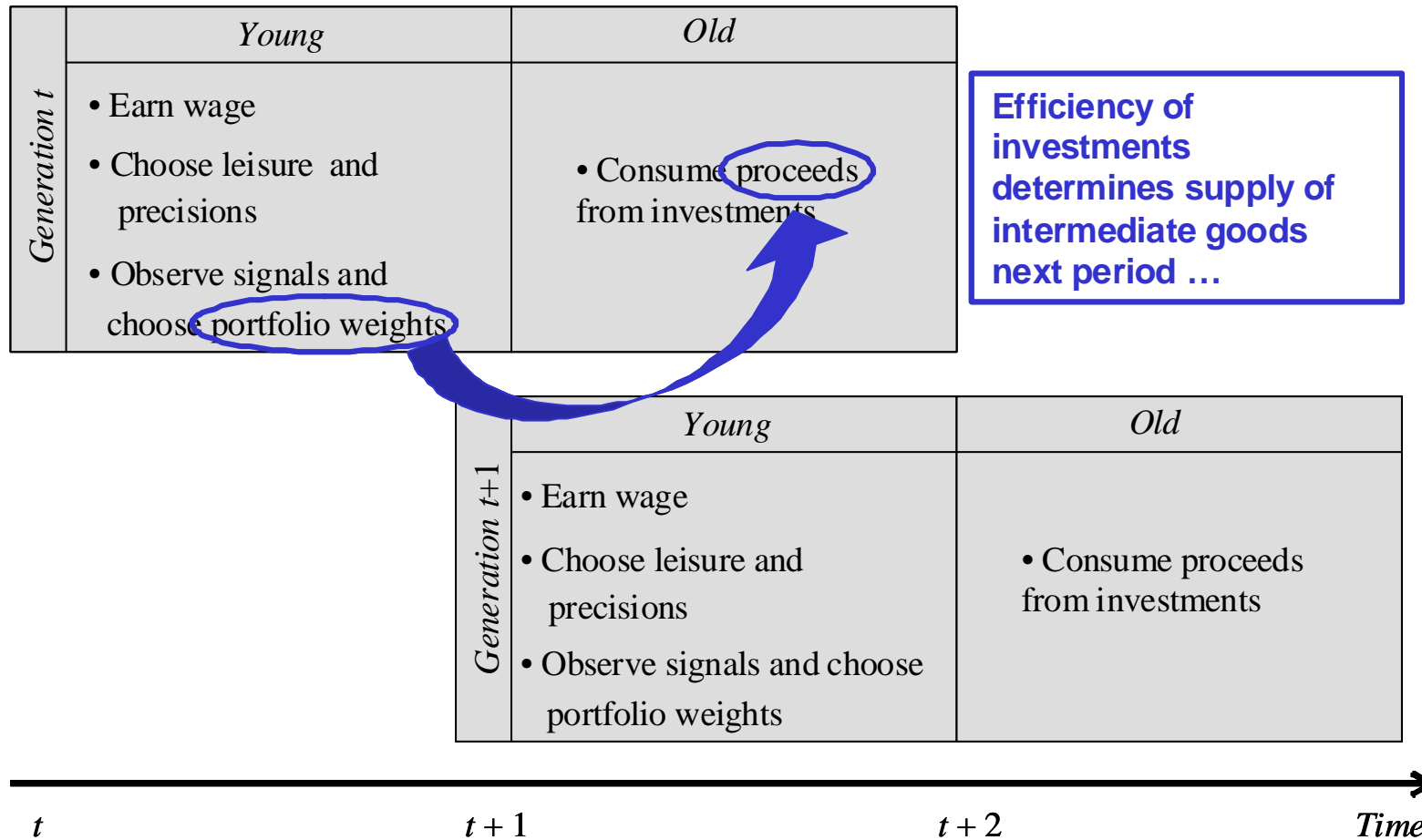
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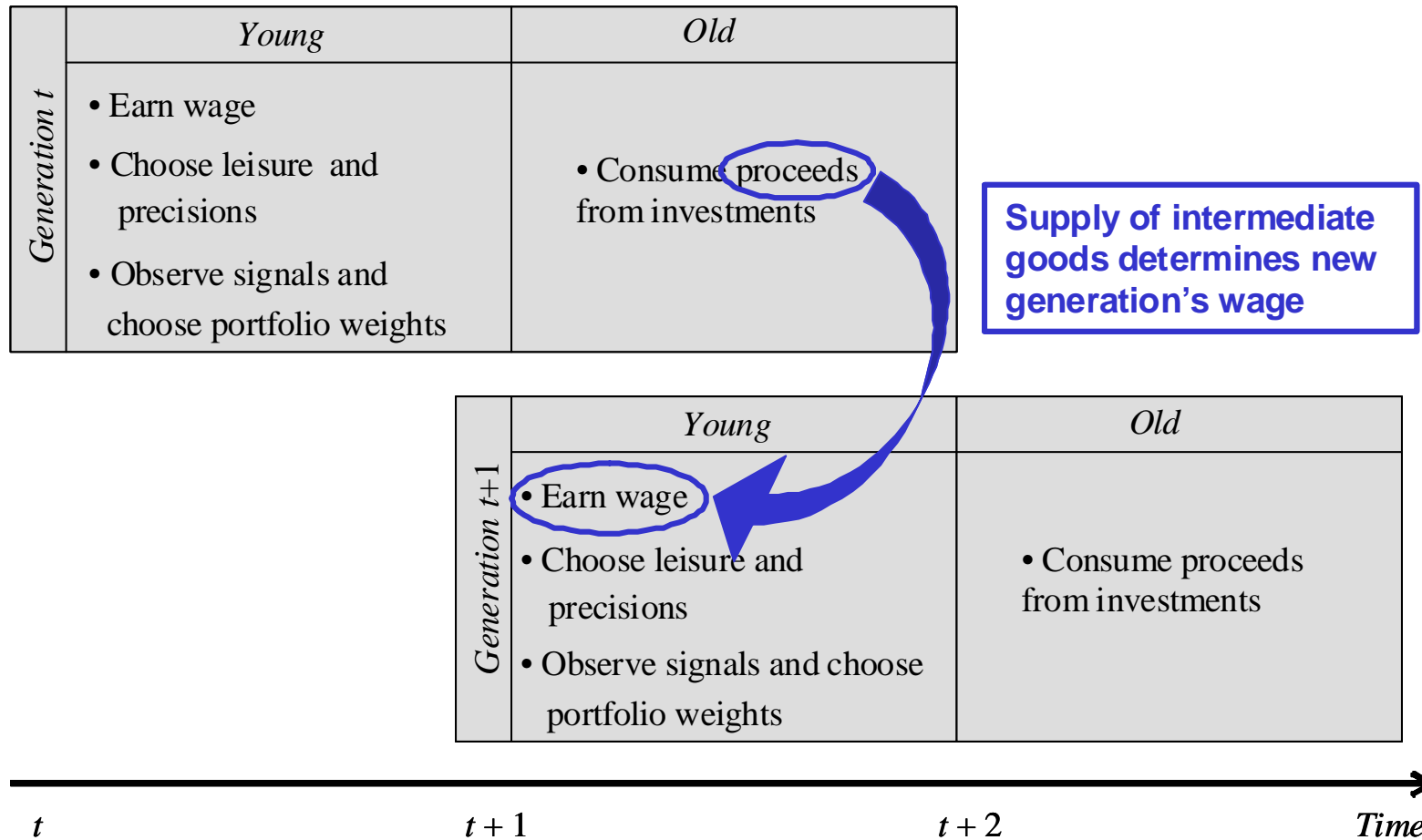
Timing



Timing



Timing



Main Results

- Dynamics of learning

- Better information at $t \Rightarrow$ More efficient investments at $t \Rightarrow$ Supply of interm. goods \nearrow at $t + 1 \Rightarrow$ M.P. of labor \nearrow at $t + 1$
- Income \nearrow at $t \Rightarrow$ Info. production at $t \left\{ \begin{array}{l} \nearrow \text{ through } \underline{\text{scale effect of info.}} \\ \text{But } \searrow \text{ through } \underline{\text{substitution effect}} \end{array} \right.$
- Impact of learning is only transitory

- Role of the stock market

- First best achieved if signals' precision is contractible, not achieved otherwise
- SM allows to share costly private signals in incentive-compatible way (noise trading)
- Ex post information sharing vs. ($>$) ex ante disincentive
- In the limit, when noise trading ≈ 0 , first best is reached

- Along the average growth path, when scale effect $>$ substitution effect:

- Capital efficiency \nearrow , TFP \nearrow , industrial specialization \nearrow , inequality, stock trading intensity and liquidity \nearrow then \searrow , stock price volatility \nearrow and return volatility \searrow

Related Literature

- Theoretical literature on finance and growth
 - Banks as selectors of the best entrepreneurs (e.g. King and Levine (1993), Acemoglu, Aghion and Zilibotti (2003) and Morales (2003))
 - Stock markets and their information processing role (Greenwood and Jovanovic (1990))
- Endogenous growth literature (e.g. Romer (1986, 1990), Aghion and Howitt (1992), Grossman and Helpman (1991))
 - Here, selection of technologies rather than their discovery
- Literature on trading under endogenous and asymmetric information
 - Real benefits of informational efficiency

Summary

- A growth model with information acquisition and dissemination through stock prices, and capital allocation
- The stock market allows investors to share their costly private signals in an incentive-compatible way when the signals' precision is not contractible
- Its impact is only transitory
- Tradeoffs
 - Learning: Substitution effect vs. scale effect of information
 - Stock market: *Ex post* information sharing vs. ($>$) *ex ante* disincentive
- Several predictions on the evolution of real and financial variables are derived
 - Capital efficiency, TFP, industrial specialization, inequality, stock trading intensity, liquidity, volatility

An Overlapping Generations Model...

- Two sectors

- M intermediate goods

$$\tilde{Y}_{t+1}^m \equiv \tilde{A}_t^m K_t^m \quad \text{for } m = 1, \dots, M$$

Firms raise capital by issuing 1 share at price P_t^m , liquidated after production

- One final good

$$G_t \equiv L^{1-\beta} \sum_{m=1}^M (Y_t^m)^\beta \quad \text{where } 0 < \beta < 1$$

- A continuum of agents, indexed by $l \in [0, L]$

- Endowed with 1 unit of labor time and 1 unit of free time (for leisure or learning)
- Young work in final good sector. Invest wage w_t in intermediate good sector

An Overlapping Generations Model...

- $U(g, j)$ = Utility from consumption of final goods g and leisure j

$$\tau(g) \equiv -\frac{\frac{\partial U}{\partial g}(g, 1)}{\frac{\partial^2 U}{\partial g^2}(g, 1)}$$

Absolute risk tolerance

↗ in g

$$\rho(g) \equiv \frac{\frac{\partial U}{\partial j}(g, 1)}{\frac{\partial U}{\partial g}(g, 1)}$$

Marginal rate of substitution
between g and j

↗ in g

- For example, Constant Elasticity of Substitution (CES):

$$U(g, j) \equiv (\varpi g^\sigma + (1 - \varpi)j^\sigma)^{1/\sigma}$$

where $0 < \varpi < 1$, $\sigma < 1$, $1/(1 - \sigma)$ = Elast. of subst. between goods and leisure

$$\tau(g) = \frac{g(\varpi g^\sigma + 1 - \varpi)}{(1 - \sigma)(1 - \varpi)}$$

$$\rho(g) = \frac{(1 - \varpi)}{\varpi} g^{1-\sigma}$$

...With a Competitive Stock Market

- Productivity shocks \tilde{A}_t^m are log-normally distributed
 - $\ln \tilde{A}_t^m \equiv \tilde{a}_t^m z$ where $\tilde{a}_t^m z$ i.i.d. $\sim N(\tilde{\alpha}_t^m z, \sigma_a^2 z)$
 - Mean shock, $\tilde{\alpha}_t^m$, i.i.d. $\sim N(0, \sigma_\alpha^2)$
 - z is small scaling factor (model solved in closed form for $z \approx 0$)
- Agents can acquire private information about average shock $\tilde{\alpha}_t^m$ at a cost

$$s_{l,t}^m = \beta \tilde{\alpha}_t^m + \tilde{\varepsilon}_{l,t}^m$$

- $\tilde{\varepsilon}_{l,t}^m$ i.i.d. $\sim N(0, 1/x_{l,t}^m)$: x = Precision of private information; X in equilibrium
- Cost $C(x_{l,t}^m)z$ units of free time
- C continuous, increasing, convex and $C(0) = C'(0) = 0$
- Noise trading
 - Fraction q of agents believes expected return on stock m equals $\tilde{\theta}_t^m$, exogenous
 - $\tilde{\theta}_t^m$ i.i.d. $\sim N(0, \sigma_\theta^2)$
 - No riskless asset

Equilibrium Concept

1. Market clearing in the intermediate goods sector

$$\tilde{w}_{t+1} = (1 - \beta) \sum_{m=1}^M (\tilde{Y}_{t+1}^m / L)^\beta \quad \text{and} \quad \tilde{\rho}_{t+1}^m = \beta (L / \tilde{Y}_{t+1}^m)^{1-\beta}$$

$\tilde{\rho}_{t+1}^m$ = price of intermediate good m ; $\tilde{\Pi}_{t+1}^m = \tilde{\rho}_{t+1}^m \tilde{Y}_{t+1}^m$ = firm m 's profit

2. Capital allocation

$$\max_{\{f_{l,t}^m\}} E [U(\tilde{g}_{l,t+1}, j_t) \mid \mathcal{F}_{l,t}] \quad \text{subject to} \quad \left\{ \begin{array}{l} \tilde{g}_{l,t+1} = w_t \tilde{R}_{l,t+1} \\ \tilde{R}_{l,t+1} = \sum_{m=1}^M f_{l,t}^m \tilde{R}_{t+1}^m \\ \sum_{m=1}^M f_{l,t+1}^m = 1 \end{array} \right.$$

$f_{l,t}^m$ = Agent l 's portfolio weights; $\mathcal{F}_{l,t} \equiv \{s_{l,t}^m, P_t^m \text{ for } m = 1 \text{ to } M\}$, $\tilde{R}_{l,t+1}$ = Portfolio return; $\tilde{R}_{t+1}^m = \tilde{\Pi}_{t+1}^m / P_t^m$ = Return on stock m ; X_t^m = Average precision of private information about firm $m \implies \underline{U_0(\{x_t^m, X_t^m\}, j_t, w_t)}$ = Value function for this problem

Equilibrium in the stock market:

$$\int_l w_t f_{l,t}^m = K_t^m = P_t^m \quad \text{for } m = 1, \dots, M$$

3. Precision choice

Optimal precision $x_t^m = x(w_t, \{X_t^m\})$ solves:

$$\max_{\{j_t \geq 0, x_t^m \geq 0\}} E[U_0(\{x_t^m, X_t^m\}, j_t, w_t)] \quad \text{subject to} \quad \sum_{m=1}^M C(x_t^m)z + j_t = 1$$

taking average precisions $\{X_t^m\}$ as given

In equilibrium, the average and optimal precisions must be consistent:

$$X_t^m = x(w_t, \{X_t^m\}) \quad \text{for } m = 1, \dots, M$$

Notation

For any firm-specific variable ψ_t^m , $\bar{\psi}_t \equiv \frac{1}{M} \sum_{m=1}^M \psi_t^m$ and $\Delta\psi_t^m \equiv \psi_t^m - \bar{\psi}_t$

First Best

- Agents collect signals of infinitesimal precisions (≈ 0 but > 0) and announce them
 - Law of Large Numbers \Rightarrow Productivity shocks are revealed
 - Capital allocation: Firm m 's capital stock equals $K_t^{mFB} \approx \frac{Lw_t}{M} \exp(\Delta k_t^{mFB} z)$

$$\text{where } \Delta k_t^{mFB} = \frac{1}{1 - \beta} \Delta \beta \tilde{\alpha}_t^m$$

- Standard neoclassical economy: Income \nearrow at a decreasing rate until it reaches a steady-state:

$$E(\tilde{w}_{t+1}) \approx \underbrace{\Lambda \exp(\lambda^{FB} z^2)}_{\text{a constant}} w_t^\beta$$

- First Best is what G&J (1990) consider (endowed signals)
 - First Best is not a Nash equilibrium when precisions are not contractible
 - Agents' best response is to set their precisions to zero and report noise
 \Rightarrow No learning

Agents' Portfolio Weights

Guess that firm m 's capital stock takes the form $K_t^m \approx \frac{Lw_t}{M} \exp(\Delta k_t^m z)$ where

$$\Delta k_t^m \equiv k_{\alpha t}^m (\beta \Delta \tilde{\alpha}_t^m + \mu_t^m \Delta \tilde{\theta}_t^m) \text{ and } \mu_t^m \text{ is a deterministic scalar ('noisiness')}$$

- Portfolio weights (the fraction of agent l 's income invested in stock m) :

$$\frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t)} \frac{E(\Delta r_{t+1}^m \mid \text{Info.}_{l,t})}{\text{Var}(\Delta r_{t+1}^m \mid \text{Info.}_{l,t})} \quad \text{where } \varphi(w) \equiv \beta M^{1-\beta} w^\beta$$

– For rational traders $\approx \frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t) \beta^2 \sigma_a^2} \{ \text{Weight}_s * \Delta s_{l,t}^m + \text{Weight}_k * \Delta k_t^m \}$

– For noise traders $\approx \frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t) \beta^2 \sigma_a^2} \Delta \tilde{\theta}_t^m$

- Under CES:

– $\frac{\tau(\varphi(w))}{\varphi(w)}$ (= Relative risk tolerance) \nearrow with w if $\sigma > 0$, \searrow with w if $\sigma < 0$

– independent of w if $\sigma = 0$ (Cobb-Douglas = CRRA: $U(g, j) \equiv g^\varpi j^{1-\varpi}$)

Capital Allocation

Noisiness μ_t^m is given. There exists a log-linear rational expectations equilibrium.

Capital equals $K_t^m \approx \frac{Lw_t}{M} \exp(\Delta k_t^m z)$

$$\text{where } \Delta k_t^m \equiv k_\alpha(\mu_t^m)(\beta \Delta \tilde{\alpha}_t^m + \mu_t^m \Delta \tilde{\theta}_t^m)$$

- X_t^m (Average precision of information about stock m) and μ_t^m (its noisiness) are related through:

$$X(\mu) \equiv \frac{H(\mu)}{\frac{1-q}{q}\mu - 1} \quad \text{where} \quad H(\mu) \equiv \frac{1}{\beta^2 \sigma_\alpha^2} + \frac{1}{\mu^2 \sigma_\theta^2}$$

- Total precision of investor's information: $H(\mu) + X(\mu)$
- Elasticity of stock price to prod. shock : $k_\alpha(\mu) \equiv \frac{1}{1-\beta} \left(1 - \frac{1}{\beta^2 \sigma_\alpha^2 (H(\mu) + X(\mu))} \right)$

Information Acquisition: First-Order Condition

- μ_t^m (noisiness of stock m 's price) is given. An investor sets the precision of her private signal about stock m , x_t^m , such that

$$\rho(\varphi(w_t))C'(x_t^m) = \tau(\varphi(w_t))\frac{M-1}{2M\beta^2\sigma_a^2}\left(\frac{1}{H(\mu_t^m)+x_t^m}\right)^2$$

Marginal cost of information

↗ in income w_t
(substitution effect)

Marginal benefit of information

↗ in income w_t
(scale effect of information)

Information Acquisition: Properties of Individual Precisions x_t^m

- Agents collect more precise signals ($x_t \nearrow$) when:
 - priors are less informative ($\sigma_\alpha^2 \nearrow$)
 - noise trading intensifies (μ_t^m or $\sigma_\theta^2 \nearrow$)
 - conditional variance of productivity shocks falls ($\sigma_a^2 \searrow$)
 - marginal cost of information falls ($C' \searrow$)
- x_t is non-monotonic in β (factor share of intermediate goods)
 - Lower $\beta \Rightarrow$ Investors' share of GDP \searrow ($\varphi(w_t)$ term) $\Rightarrow \frac{\partial U}{\partial g} \nearrow \Rightarrow \rho$ and $\tau \nearrow$
 - Lower $\beta \Rightarrow$ Stocks are less sensitive to productivity shocks
 - * $\Rightarrow \tilde{\alpha}_t^m$ (component of shock that can be learnt) has smaller impact on profit \Rightarrow Info less valuable
 - * \Rightarrow Stock less risky (component of shock that cannot be learnt) \Rightarrow Larger positions \Rightarrow Info more valuable

Information Acquisition: Equilibrium

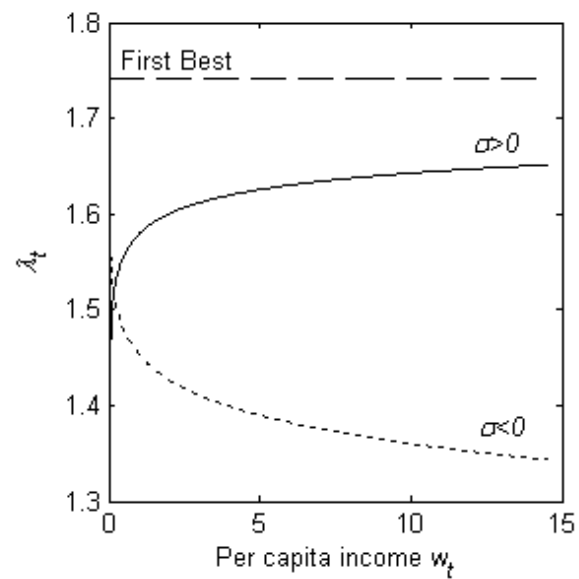
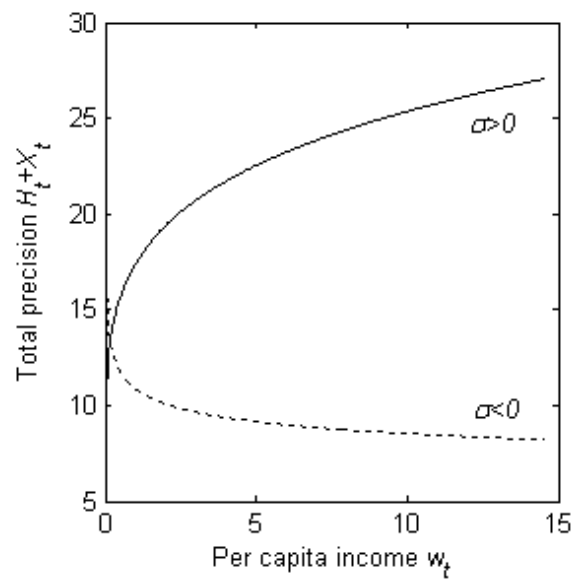
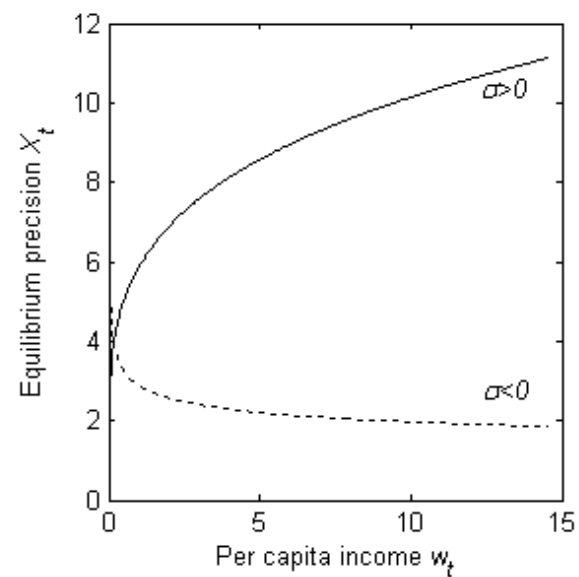
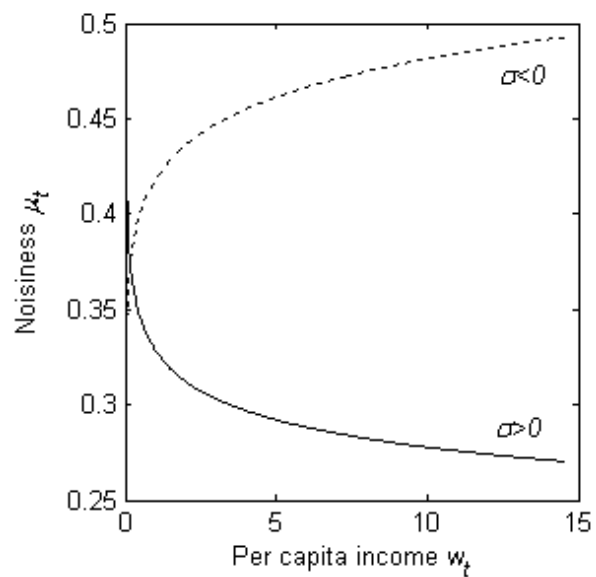
- In equilibrium, the noisiness of stock prices, μ_t , is the unique solution to:

$$\rho(\varphi(w_t))C' \left(\frac{H(\mu_t)}{\frac{1-q}{q}\mu_t - 1} \right) = \tau(\varphi(w_t)) \frac{M-1}{2M\beta^2\sigma_a^2} \left(\frac{1 - \frac{q}{(1-q)\mu_t}}{H(\mu_t)} \right)^2$$

- Properties of μ_t : $\mu_t \searrow$ (i.e. stock prices are more informative) when:
 - priors are more accurate ($\sigma_\alpha^2 \searrow$)
 - variance of noise trades increases ($\sigma_\theta^2 \nearrow$)
 - conditional variance of productivity shocks falls ($\sigma_a^2 \searrow$)
 - marginal cost of information falls ($C' \searrow$)
 - fraction of noise traders falls ($q \searrow$, direct effect of q on μ_t dominates its indirect effect through X_t)

Information Acquisition: Impact of Income

- Current income $w_t \Rightarrow$ Current noisiness μ_t
 - If $\tau/\rho \nearrow$ in consumption (e.g. $\sigma > 0$ under CES utility), then noisiness declines with income (precision of info. rises)
 - If $\tau/\rho \searrow$ (e.g. $\sigma < 0$), then noisiness rises with income
 - Two competing forces: Substitution effect vs. Scale effect of information
- Current noisiness $\mu_t \Rightarrow$ Future expected income $E(\tilde{w}_{t+1})$
 - Income is larger on average when agents are better informed (noisiness lower)
 - Indeed: Noisiness $\searrow \Rightarrow$ More efficient investments \Rightarrow Supply of intermediate goods \nearrow on average \Rightarrow Marginal product of labor \nearrow on average



Dynamics of Income (Along the Average Growth Path)

Income converges to a steady-state in which it no longer grows:

$$E(\tilde{w}_{t+1}) \approx \Lambda w_t^\beta \exp(\lambda(w_t) z^2)$$

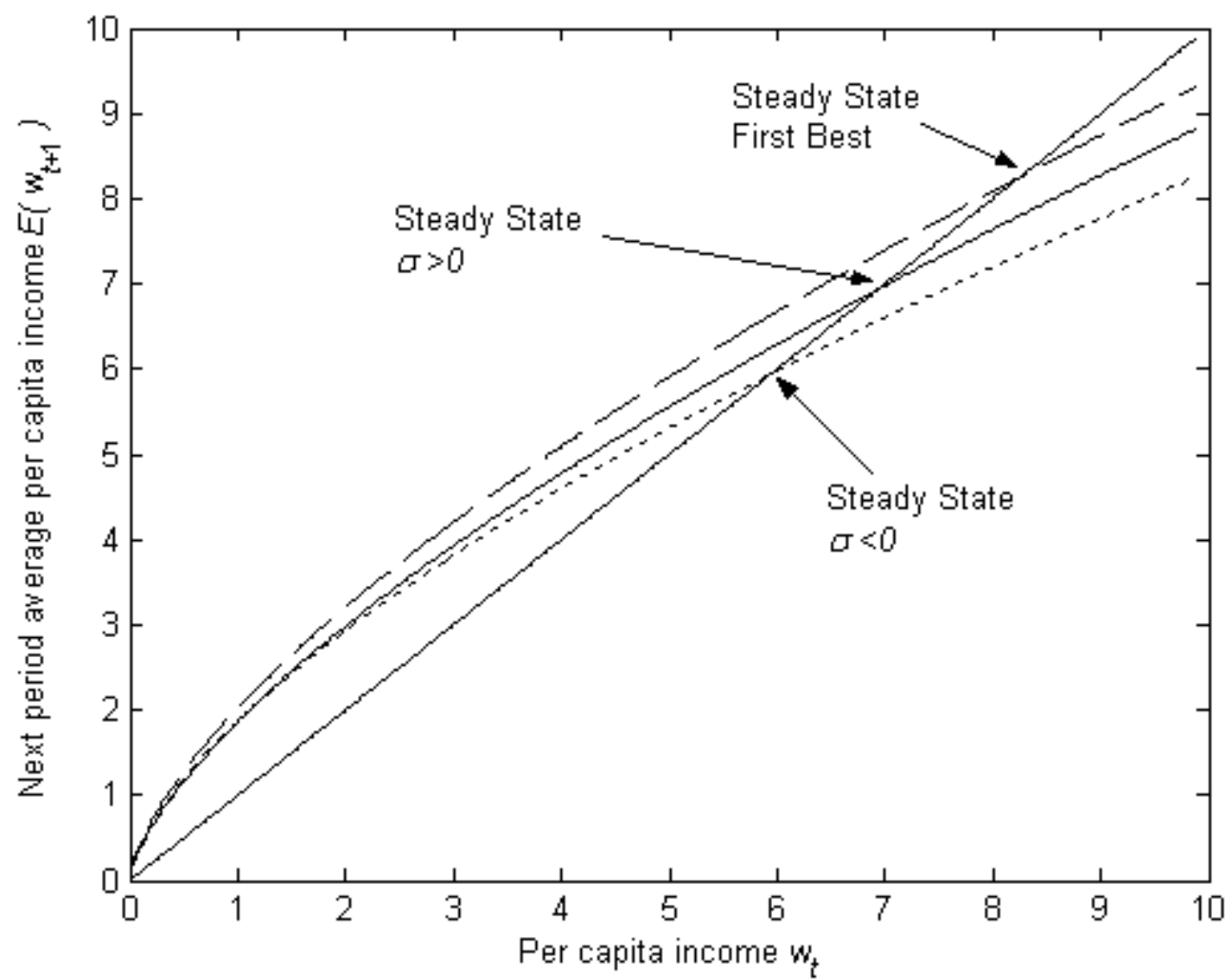
where
$$\lambda(w_t) \equiv \frac{M-1}{M} \beta^2 \left(k_\alpha(\mu_t) \beta \sigma_\alpha^2 + \frac{k_\alpha(\mu_t)^2}{2} (\beta^2 \sigma_\alpha^2 + \mu_t^2 \sigma_\theta^2) \right) > 0$$

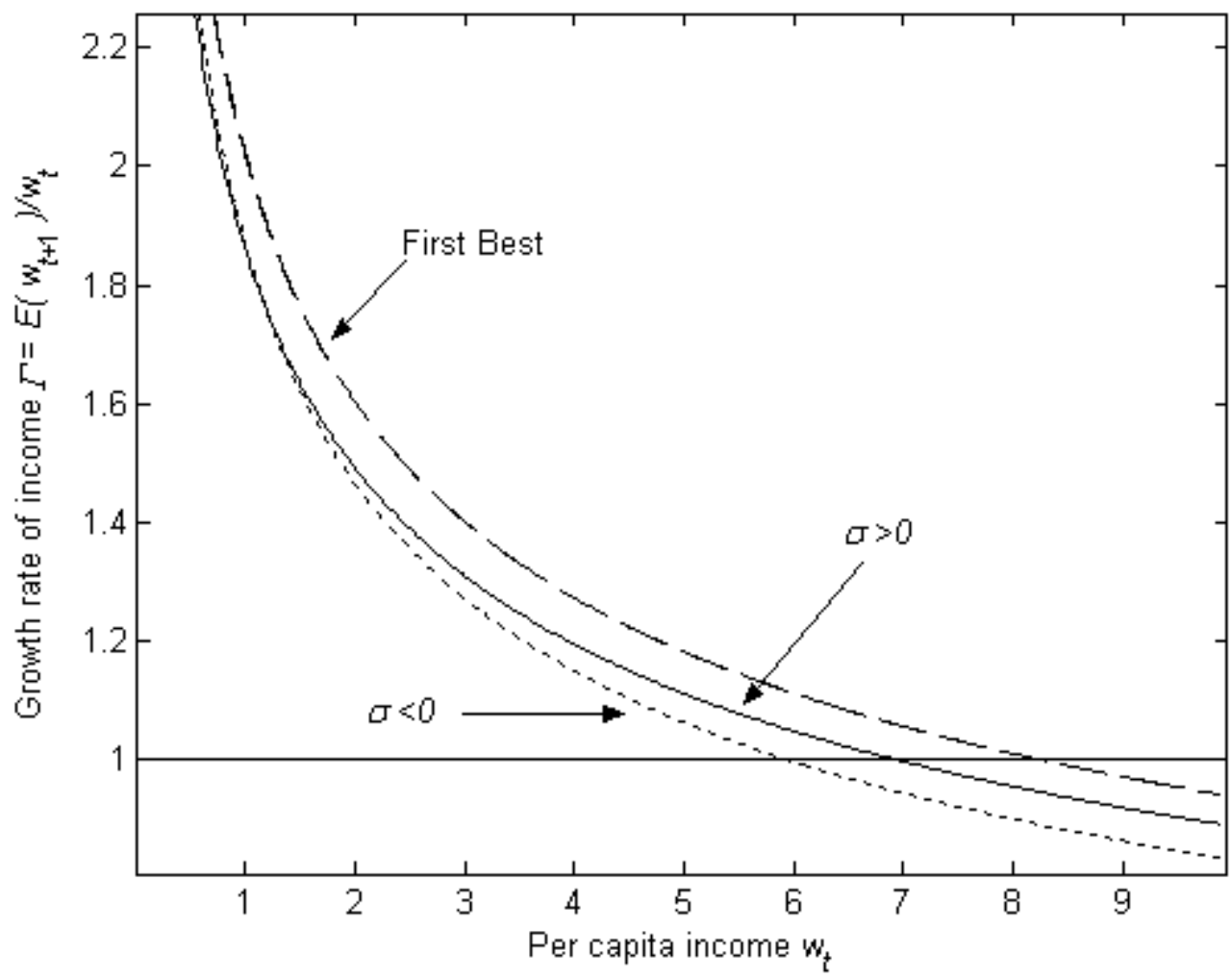
- If $\tau/\rho \nearrow$ (e.g. $\sigma > 0$), then $\lambda \nearrow$ with w (to λ^{FB}) so $E(\tilde{w}_{t+1})/w_t$ (growth rate of income) \searrow less fast than in FB:

$$\frac{d \ln (E(\tilde{w}_{t+1})/w_t)}{d \ln w_t} = -(1 - \beta) + \frac{d\lambda(w_t)}{d \ln w_t} z^2 > -(1 - \beta)$$

- If $\tau/\rho \searrow$ (e.g. $\sigma < 0$), then $\lambda \searrow$ and $E(\tilde{w}_{t+1})/w_t \searrow$ at a faster rate than in FB:

$$\frac{d \ln (E(\tilde{w}_{t+1})/w_t)}{d \ln w_t} = -(1 - \beta) + \frac{d\lambda(w_t)}{d \ln w_t} z^2 < -(1 - \beta).$$





The Role of the Stock Market

- When the fraction of noise traders q \searrow :
 - Less information is produced but more is shared through stock prices
 - Net effect = improvement in total info. and in efficiency of investments ($k_{at} \nearrow$)

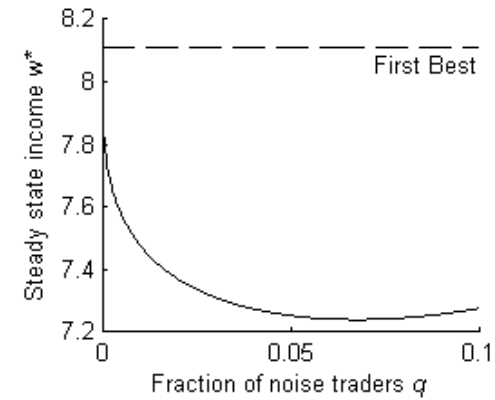
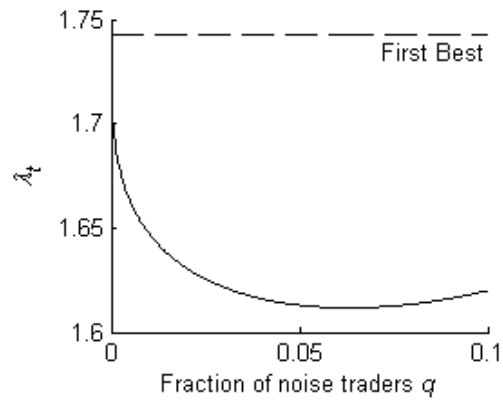
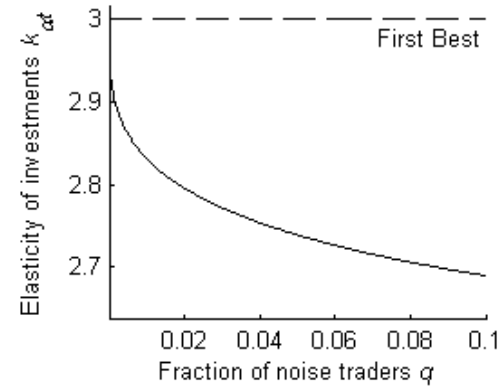
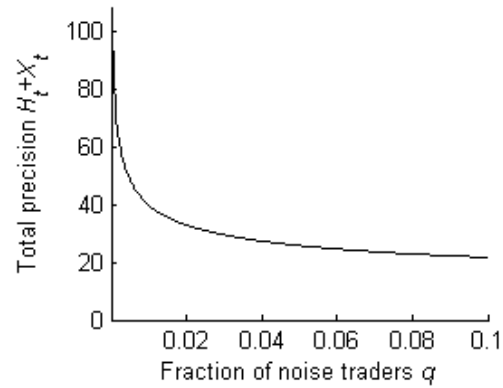
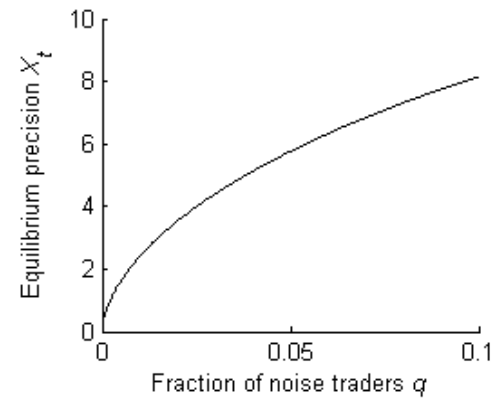
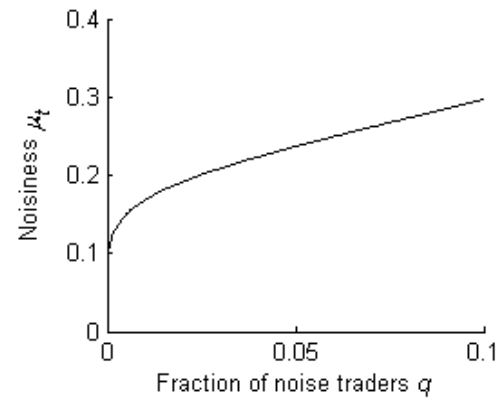
$$\begin{aligned}
 \frac{d(H(\mu_t) + X(\mu_t))}{dq} &= \underbrace{\frac{\partial H_t}{\partial \mu_t(X_t)} * \frac{\partial \mu_t}{\partial q(X_t)}}_{< 0} + \underbrace{\frac{\partial H_t}{\partial \mu_t(X_t)} * \frac{\partial \mu_t}{\partial X_t(q)} * \frac{dX_t}{dq} + \frac{dX_t}{dq}}_{> 0} \\
 &\qquad \qquad \qquad \text{Ex post information sharing} \qquad \qquad \text{Ex ante disincentive}
 \end{aligned}$$

- As the fraction of noise traders q approaches zero:

- The allocation of capital converges to the FB allocation

$$\lim_{q \rightarrow 0 (q > 0)} k_t^m = k_t^{mFB} \quad \text{for } m = 1, \dots, M$$

- SS level of income and transitory growth rate converge to those of FB



Real Variables Along the Average Growth Path

Assume scale effect $>$ substitution effect (e.g. $\sigma > 0$) \implies Noisiness $\mu_t \searrow$ with w_t

- Elasticity of investments to productivity shocks and TFP increase as the eco. grows
 - TFP defined from economy-wide production function
 - Empirically: Elasticity of investments to value added \nearrow with informativeness of a country's stock market (Wurgler (2000)). Stock markets promote growth in TFP (Levine and Zervos (1998))
- Capital and profits are more concentrated across firms as the economy grows
 - Herfindahl index: $Her(K_t^m) \equiv E(K_t^{m2})/[E(K_t^m)]^2$
 - Empirically: Sectoral concentration \searrow at first, then \nearrow with GDP (Imbs and Wacziarg (2003)). Industrial specialization is positively related to share of financial sector in GDP (Kalemli-Ozcan, Sørensen and Yosha (2003))

Real Variables Along the Average Growth Path

- Wealth inequality follows a “Kuznets curve”, \nearrow at first, then \searrow as eco. grows
 - Inequality driven by disagreement between investors about stocks
 - * At early stage, private signals are imprecise \implies Follow mostly price signals \implies Low disagreement
 - * Precision $\nearrow \implies$ Private signals receive more weight \implies Disagreement \nearrow
 - * More precise private signals are also more similar \implies Disagreement \searrow beyond a point
 - Empirically: Mixed evidence in favor of Kuznets curve

Financial Variables Along the Average Growth Path (e.g. $\sigma > 0$)

- As the economy grows, trading on the equity market and liquidity ↗ at first, then ↘
 - As with inequality, driven by disagreement
 - Empirically: Share turnover on the stock market is positively related to output growth (Levine and Zervos (1998), Rousseau and Wachtel (2000))
- Impact on stocks' volatility as the economy grows:
 - In current setup (1 trading round), stock price volatility ↗, idiosyncratic and total return volatility ↘, market volatility constant
 - Conjecture: With 2 trading rounds, return volatility ↗
 - Empirically: Stock prices are less synchronous in richer economies (Morck, Yeung and Yu (2000)). Idiosyncratic return volatility ↗ in the U.S. from 1962 to 1997, while the volatility of the market remained stable (Campbell, Lettau, Malkiel and Xu (2001))