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Sector concentration risk in SME credit portfolios: a multifactor approach

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Introduction and Objectives

- Credit Risk:
 - > Defined as **portfolio wide** unexpected losses due to extreme events
 - > Depends (in particular) on the concentration of the portfolio

- Concentration risk in internal measurement of portfolio credit risk (Pillar 2 of Basel II – emphasized in 2008 by BCBS)

- What is concentration in retail banking ?
 - > Not a name concentration (granular portfolio)
 - > Refers to a potential situation of correlated defaults in sub-portfolios
 - Even if loans are small or medium-sized exposures in retail banking, there can be concentration risk in retail portfolios due to strong dependences across exposures (highly correlated defaults) within some portfolio's segments.

Objectives

- Concentration risk is a consequence of borrowers' heterogeneity:
 - > Borrowers may be affected by a same set of risk factors but react more or less strongly to variations of these factors.
 - > Borrowers may also differ because they are exposed to different systematic factors such as macroeconomic factors or segmentation factors (geography, wealth, profession...).

- Ignoring this heterogeneity could lead:
 - > To an underestimation of potential losses and, hence, of economic capital.
 - > To a misallocation of capital within the whole portfolio and biased investment choices and performance measurement (RAROC).

Introduction and Objectives

- Most internal models measuring economic capital as well as the formulas of regulatory charges in Basel II use an asymptotic single factor model (ASRF)
 - > What is measured in an ASFR model is the correlation to a latent (unobservable) risk factor assumed to represent general macroeconomic conditions

- But the ASFR model does not take into account borrowers heterogeneity. Only a multi-factor framework allows to take into account mortgage dependence to additional risk factors.
 - > But very few empirical works on multi-factor approach in retail banking

- In this paper, we use a multi-factor framework in order to estimate the dependence structure between exposures in order to:
 - > measure the level of economic capital charges,
 - > assess the degree of concentration risk

Contents

1. From the ASRF to the multi-factor Framework
2. Capital allocation and concentration risk



1. From the ASFR to the multi-factor Framework

2. Capital allocation and concentration risk

Credit losses within a multi-factor framework

- Hence, defining u_i as obligor's i loss given default (LGD) and $\mathbf{1}_{D_i}$ his default indicator, credit losses are defined as:

$$L = \sum_{i=1}^n u_i \mathbf{1}_{D_i}$$

- Assuming perfect granularity of the portfolio and multivariate normal distribution of risk factors, credit losses can be approximated by expected credit losses, hence:

$$L \approx \sum_{i=1}^n u_i \Phi \left[\frac{\Phi^{-1}(\bar{p}_i) - \mathbf{w}_i' s}{\sqrt{1 - \mathbf{w}_i' R \mathbf{w}_i}} \right]$$

- Assuming homogeneity within portfolio's segments, the loss variable becomes:

$$L \approx \sum_{i=1}^n u_i \Phi \left[\frac{\Phi^{-1}(\bar{p}_r) - \mathbf{w}_{ki}' s}{\sqrt{1 - \mathbf{w}_{ki}' R \mathbf{w}_{ki}}} \right]$$

Econometric methodology : GLMM

- A first and natural way to expand the ASRF model is to introduce additional (macroeconomic) risk factors
 - > Difficulty to identify pertinent factors and to access specific time series for retail loans.
 - > Indeed, retail markets are local and, possibly, highly heterogeneous markets.

- Alternative: expand the one-factor model by adding new latent factors that could be linked to observable characteristics of borrowers:
 - > Sector
 - > Size, Region, Profession...

- These additional factors could either reinforce or attenuate the effect of the general (systematic) risk factor on portfolio losses, introducing some level of heterogeneity within the measurement of credit risk.

- Invites to use a random effects specification by segmenting ratings histories according to a combination of factors.

Econometric methodology : GLMM

- The credit risk parameters (defaults thresholds and sensitivities to systematic risk factors) are estimated using a GLMM probit model.

- In this framework, the PD is modeled as:

$$P(\text{default} | \gamma_t) = \Phi[\mu_r + z'_{ii} \gamma_t]$$

random effects default threshold for rating r design matrix of random effects

- Specification where the default rate is explained by
 - > A fixed effect: the borrower's internal rating (μ_r)
 - > Random effects (γ_t):
 - A general latent factor
 - Augmented by a set of factors corresponding to a given segmentation of the portfolio. Here: industry.

Econometric methodology : GLMM

- Specification and estimation of the dependence structure of risk factors remains a theoretical and empirical challenge.
- Assumption about the dependence structure (covariance matrix of random effects):

$$G = \begin{bmatrix} \sigma_1^2 & 0 & \Lambda & \sigma_{1,q+1} \\ 0 & 0 & 0 & \sigma_{2,q+1} \\ M & K & \sigma_q^2 & M \\ \sigma_{1,q+1} & \sigma_{2,q+1} & K & \sigma_{q+1}^2 \end{bmatrix}$$

- > q independent industry effects
 - > Industry effects are correlated with the general ($q+1$) risk factor
- The additional (industry) factors reinforce (or attenuate) the effect of the general (systematic) risk factor on portfolio losses

- Complete ratings system provided by COFACE Services
 - > Ratings grouped in 7 rating grades plus default grade.

- More than 600.000 French SME, what allows to compute default rate by sector and size :
 - > 2 size classes :
 - very small firms with turnover lower than €750.000
 - small and medium-sized with turnover lower than 50 millions €.
 - > 12 industries

- 1999-2008 period

Table 1 Industry repartitions of firms and credit exposures

Industry	Number of firms		Total banking debt	
	SME	Very small firms	SME	Very small firms
Manufacturing	10,5%	10,5%	13,5%	6,4%
Construction	13,2%	14,4%	8,2%	8,1%
Retailing	36,1%	26,1%	21,7%	14,7%
Transport & logistics	5,1%	3,4%	4,1%	1,8%
Hotels & restaurants	3,7%	10,6%	4,7%	13,8%
Information & communication	3,9%	4,3%	2,6%	1,9%
Real estate	3,2%	6,3%	33,1%	36,5%
Scient. & tech. services to businesses	7,5%	11,1%	6,6%	10,0%
Administrative services	4,2%	4,6%	4,7%	3,3%
Education	0,7%	3,1%	0,2%	0,5%
Arts, culture, entertainment	0,8%	1,7%	0,5%	0,9%
Other services	0,7%	3,7%	0,2%	2,2%

Econometric analysis : Results

- Some industries are characterized by a high variance of industry factors and/or negative covariance between industry risk factors and the general risk factor.

Panel A: Very small firms												
Sect1	Sect2	Sect3	Sect4	Sect5	Sect6	Sect7	Sect8	Sect9	Sect10	Sect11	Sect12	Syst
0.06258												-0.00093
	0.009980											-0.00563
		0.01414										-0.00832
			0.02018									0.005808
				0.01401								0.004705
					0.006526							-0.00229
						0.09277						-0.00505
							0					0
								0.002610				-0.00017
									0.01155			-0.00050
										0.008196		0.003027
											0.003200	0.001089
0.00093	-0.00563	-0.00832	0.005808	0.004705	-0.00229	-0.00505	0	-0.00017	-0.00050	0.003027	0.001089	0.01407

Panel B, SME												
Sect1	Sect2	Sect3	Sect4	Sect5	Sect6	Sect7	Sect8	Sect9	Sect10	Sect11	Sect12	Syst
0.03102												0.01006
	0.009317											-0.00491
		0.02814										-0.00505
			0.004891									-0.00092
				0.1287								0.01601
					0.01754							-0.00425
						0.2372						0.007441
							0					0
								0.001050				-0.00011
									0.06322			0.01477
										0.008169		-0.00065
											0.04488	-0.00024
0.01006	-0.00491	-0.00505	-0.00092	0.01601	-0.00425	0.007441	0	-0.00011	0.01477	-0.00065	-0.00024	0.01370

Portfolio capital requirements

- Given credit risk parameters, computation of the PDF of losses by Monte Carlo simulation (assuming a 45% LGD).
- Using an homogeneous one factor model as a benchmark.

Table 3 Economic capital (% of total exposure)

Quantile	90%	95%	99%	99.5%	99.9%	99.97%
Panel A Very small firms						
One factor	0,85%	0,89%	0,98%	1,01%	1,08%	1,13%
Industry	0,96%	1,06%	1,28%	1,38%	1,61%	1,82%
Panel B SME						
One factor	0,70%	0,73%	0,82%	0,85%	0,91%	0,96%
Industry	0,73%	0,88%	1,33%	1,56%	2,14%	2,67%

- Introducing additional risk factors leads to a considerable increase in potential losses, underlining the importance of borrowers' heterogeneity in the measurement of credit risk.



1. From the ASFR Framework to the multi-factor Framework

2. Capital allocation and concentration risk

Capital allocation and concentration risk

- Concentration risk: losses resulting from highly correlated defaults in sub-portfolios.
- Economic capital should in priority be allocated to the riskier segments of the portfolio, i.e. capital allocation should account for the portfolio's heterogeneity.
- Capital allocation rules should allow detecting and measuring potential concentration of losses within particular sub-segments.
- Properties of VaR allow to compute marginal contributions of sub-portfolios as partial derivatives of the portfolio VaR (Gouriéroux et al., 2000, Tasche, 1999).
- Computation of marginal contributions to VaR using the Tasche (2009) methodology, combining importance sampling and a kernel estimator for conditional expectations of individual losses.

VaR and marginal contributions: Results

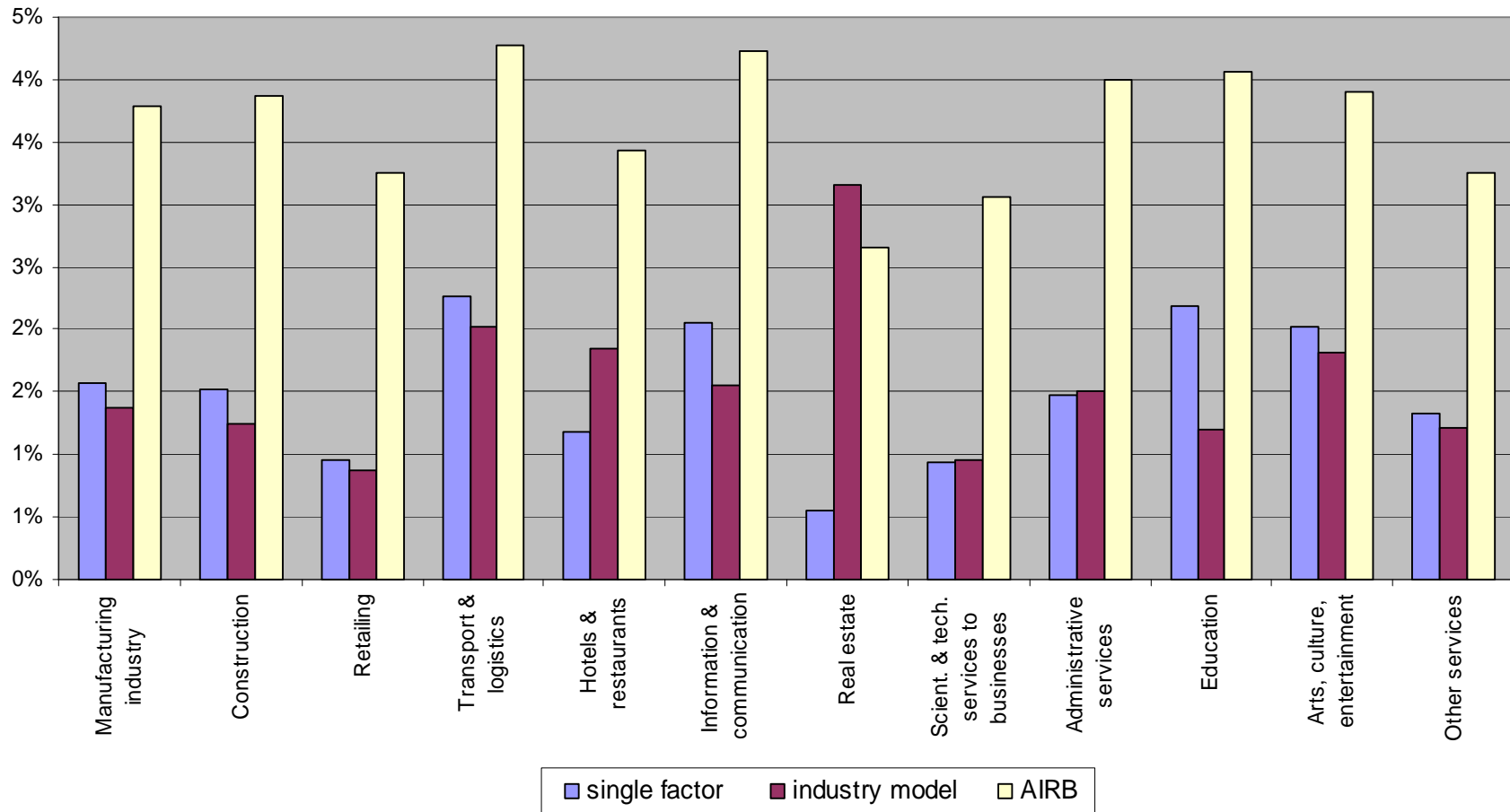
Table 4: Capital ratios by industry (99.97% confidence level).

- Table 4 presents values of capital ratios by industry (all ratings taken together).
- Due to heterogeneity of portfolio risk parameters and exposures, capital ratios vary considerably across industries.

Industry	Very small firms	SME
Manufacturing	1,37%	1,14%
Construction	1,24%	0,75%
Retailing	0,87%	0,59%
Transport & logistics	2,03%	0,84%
Hotels & restaurants	1,84%	0,94%
Information & communication	1,56%	1,10%
Real estate	3,16%	5,53%
Scient. & tech. services to businesses	0,96%	0,67%
Administrative services	1,51%	1,06%
Education	1,20%	1,13%
Arts, culture, entertainment	1,81%	0,99%
Other services	1,21%	0,84%

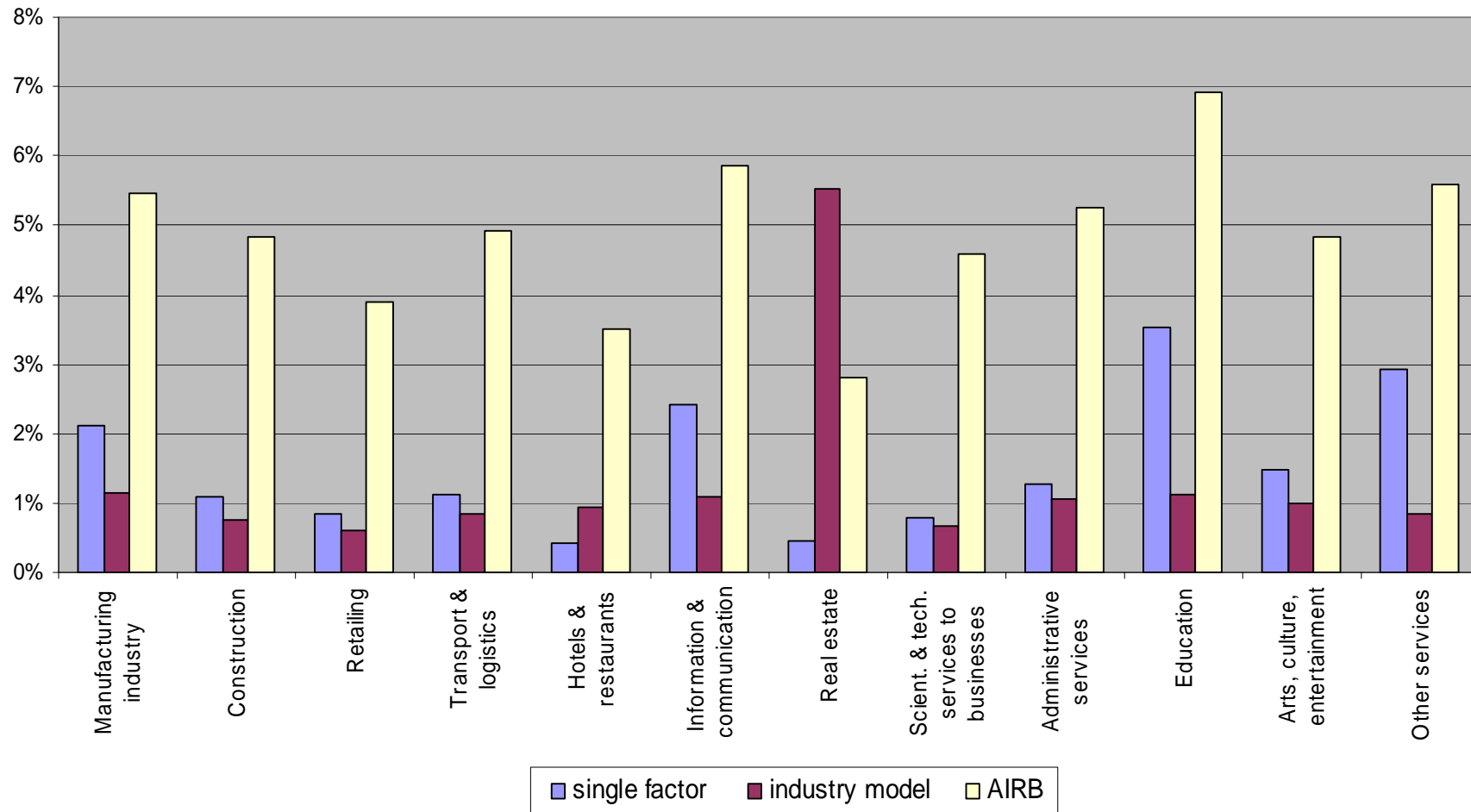
VaR and marginal contributions: Results

Graph 1A: Very small firms - Comparison of capital ratios single factor, multi factor and IRBA



VaR and marginal contributions: Results

Graph 1B: Small and medium-sized firms -
Comparison of capital ratios single factor, multi factor and IRBA



VaR and marginal contributions: Results

Table 5 Capital ratio distributions (%).

Panel A Very small businesses (turnover < 750.000 euros)					
Model	Mean	Std	Cv (%)	Min	Max
One factor	0,99%	0,18%	18,2	0,63%	1,34%
Industry	1,56%	0,61%	39,1	0,87%	3,16%
IRB	3,39%	0,31%	9,14	2,72%	3,84%
Panel B SME (750.000 < turnover < 50 millions euros)					
Model	Mean	Std	Cv (%)	Min	Max
One factor	0,86%	0,25%	29,8	0,47%	1,30%
Industry	1,40%	1,69%	120,7	0,59%	6,74%
IRB	4,68%	0,70%	14,9	3,54%	5,69%

Note: Cv is the percent coefficient of variation. Mean is the unweighted mean of capital ratios according to a given credit risk model and a given aggregation variable. All computations are based on a fixed *LGD* of 45%.

- Taking account for additional risk factors induces:
 - > An *increase* of economic capital charges in some portfolio segments, relative to the capital charge computed by using a single factor model, the additional charges coming from the introduction of previously 'missing' factors in the credit portfolio risk measurement
 - > A larger dispersion of capital charges across industries, reflecting the heterogeneity of credit risk.

Measuring concentration risk: a simulation

- Concentration risk is not solely linked to large industry credit exposures, but also to the industry's credit risk (PD and asset correlation).
 - > Thus, a portfolio's credit risk, measured by the level of its capital requirement, could stem from industries representing relative small shares of total credit exposures.
 - > Consequently, the concentration of exposures (at the industry level) of a portfolio might by itself not represent its credit concentration risk.
- One would expect the overall capital level to be related to the portfolio's concentration risk. In other words, we would expect capital concentration to be positively related to the capital level whereas the link between the capital level and the concentration of exposures could not ex ante be specified.
- Simulation exercise:
 - > Drawing random portfolios of, respectively, 10.000 and 20.000 exposures of SMEs and very small businesses.
 - > Computing the corresponding regulatory and economic capital ratios at the portfolio and the industry levels.
 - > Computing Herfindahl-Hirschmann indexes for credit exposures and capital (economic and regulatory).

Measuring concentration risk: a simulation

■ Assessing the impact of concentration risk

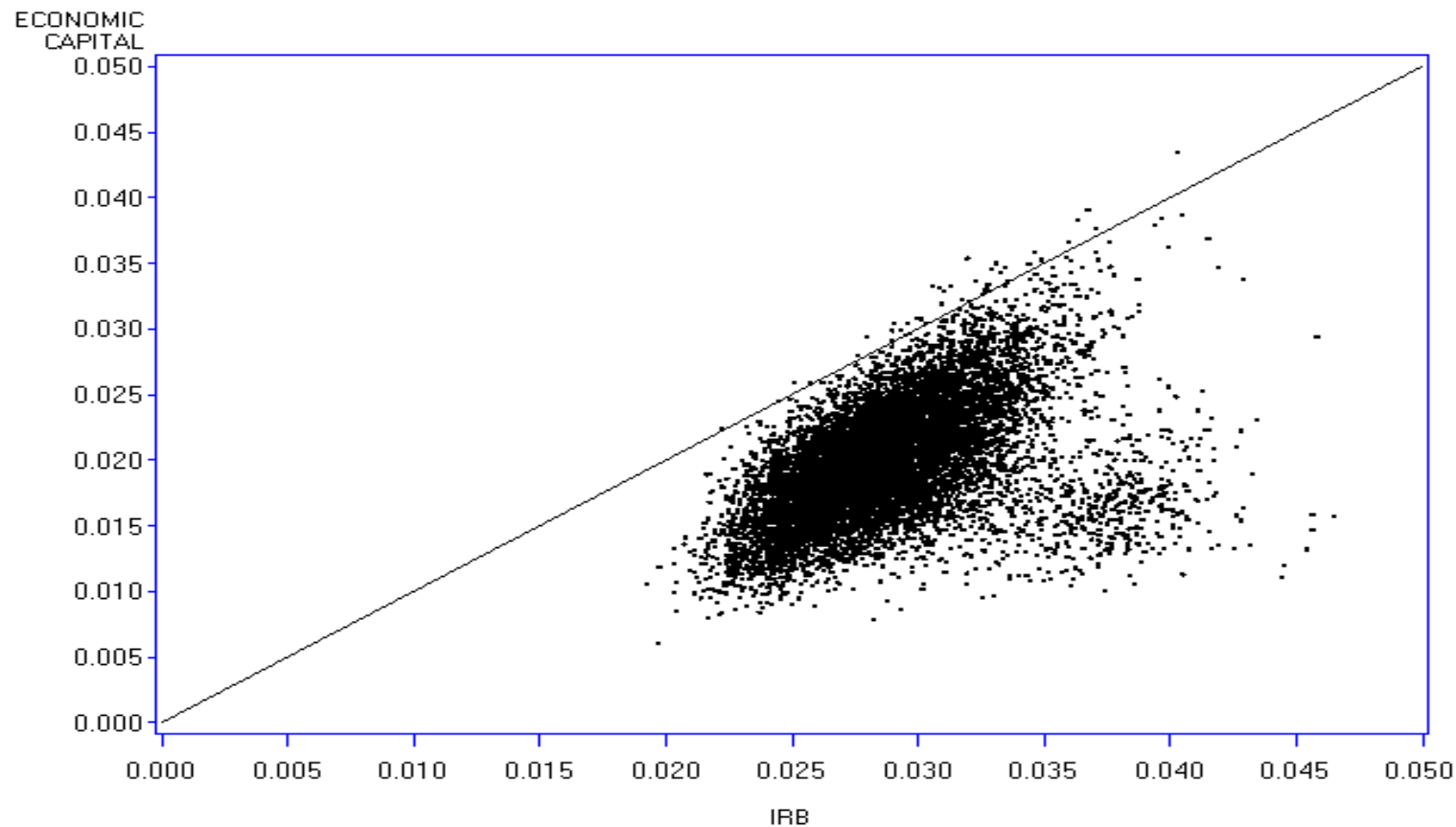
Table 6 Simulation results

Panel A SME (750.000 < turnover < 50 millions euros)						
	IRB	EC	HHI IRB	HHI EC	HHI	
IRB	1					
EC	0.46	1				
HHI IRB	0.26	0.44	1			
HHI EC	-0.21	0.66	0.45	1		
HHI	-0.20	0.28	0.74	0.50	1	
Panel B Very small businesses (turnover < 750.000 euros)						
	IRB	EC	HHI IRB	HHI EC	HHI	
IRB	1					
EC	0.83	1				
HHI IRB	0.05	0.30	1			
HHI EC	0.36	0.78	0.55	1		
HHI	-0.43	-0.22	0.81	0.17	1	

Note: Table shows the correlation matrices between regulatory capital requirements (IRB), the 99,97% economic capital level (EC), the corresponding industry concentration of capital (HHI_IRB and HHI_EC) as well as the

Measuring concentration risk: a simulation

- The underestimation of concentration risk by the IRB formulas could lead to an underestimation of capital.



Conclusion

- Single-factor (regulatory) model of portfolio credit risk is insufficient to implement an economic capital allocation policy that reflects concentration risk.
- Multi-factor model allows to detect and manage potential concentration in retail loans portfolios
- However, the capacity to measure dependence structure across SME loans depends of the choice of additional systematic risk factors
- Complexity of this problem : a real theory of firm default - that could help to identify these factors - is still missing
- Active capital management needs additional research on the nature of risk factors in such portfolios